

Adaptive Estimation Based on Quantized Measurements

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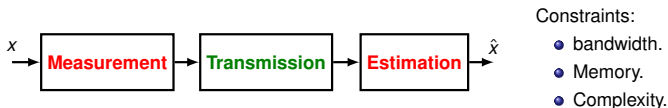
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Outline

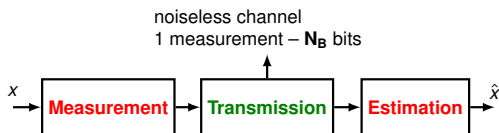
- 1 Introduction
- 2 Problem statement
- 3 Adaptive algorithm
- 4 Performance
- 5 Conclusions

Estimation under communication constraints

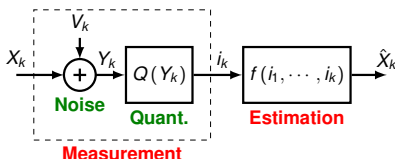
Sensor networks \longrightarrow Remote estimation



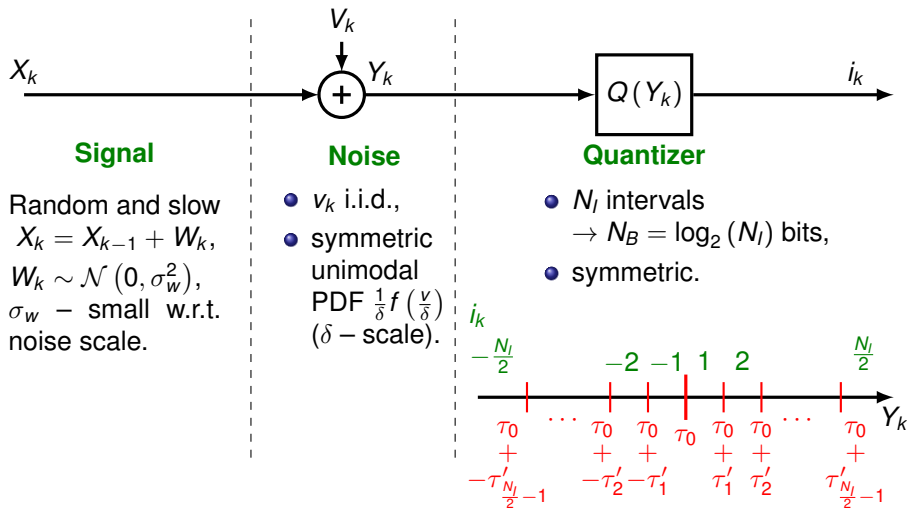
Bandwidth constraint \longrightarrow Rate constraint



Rate constraint \longrightarrow Scalar quantization

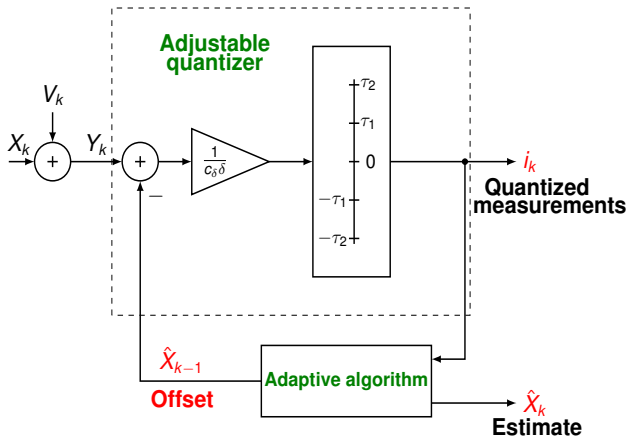


Signal and measurement models



Adaptive algorithm

Estimation performance — for most noise distributions, degrades when τ_0 is far from x_k . — Solution — **Adaptive scheme, set $\tau_0 = \hat{X}_{k-1}$.**



Adaptive algorithm:

- Low complexity.
- Low memory requirements.

It can be implemented at sensor level.

Update = Prediction + **Correction**

$$\hat{X}_k = \hat{X}_{k-1} + \gamma \eta \left[Q \left(\frac{Y_k - \hat{X}_k}{C_\delta \delta} \right) \right]$$

γ algorithm step,

η [i] correction coefficients:

$$\begin{aligned} \eta[i] &= -\eta[-i], \\ \eta[i] &> 0, \text{ for } i > 0. \end{aligned}$$

Theoretical performance and optimal parameters

Mean error – Bias (ODE method)

ODE approximation

For small γ , $\mathbb{E} \left(\hat{X}_k \right) \approx \hat{x}(t_k)$

$$\frac{d\hat{x}}{dt} = \mathbb{E} \left[\eta \left(Q \left(\frac{x - \hat{x} + V}{c_\delta \delta} \right) \right) \right] = h(\hat{x}),$$

$$t_k = k\gamma, V \sim \frac{1}{\delta} f \left(\frac{v}{\delta} \right).$$

Bias

$$\epsilon_k = \mathbb{E} \left(\hat{X}_k - X_k \right) \approx \epsilon(t_k)$$

$$\frac{d\epsilon}{dt} = \tilde{h}(\epsilon),$$

$$\tilde{h}(\epsilon) = h(\epsilon + x).$$

- Using ODE stability analysis
 $\epsilon \rightarrow 0$ when $t \rightarrow \infty$.

**Asymptotic
unbiasedness**

Theoretical performance and optimal parameters

Asymptotic MSE

Using general results for adaptive algorithms¹

Optimal gain and asymptotic MSE

$$\gamma^* = \frac{\sigma_w}{\sqrt{2\eta^T \mathbf{F}_d \eta}}.$$

$$\text{MSE}_\infty = \frac{\sigma_w \sqrt{2\eta^T \mathbf{F}_d \eta}}{2\eta^T \mathbf{f}_d}.$$

Optimal η , γ and MSE

$$\boldsymbol{\eta}^* = \arg \max_{\boldsymbol{\eta}} \left\{ \frac{[\boldsymbol{\eta}^T \mathbf{f}_d]^2}{\boldsymbol{\eta}^T \mathbf{F}_d \boldsymbol{\eta}} \right\} = \mathbf{F}_d^{-1} \mathbf{f}_d.$$

$$\text{MSE}_\infty^* = \gamma^* = \sigma_w \left(2 \sum_{i=1}^{N_I} \frac{f_d^2[i]}{F_d[i]} \right)^{-\frac{1}{2}}.$$

$$\boldsymbol{\eta} = \left[\eta[1] \cdots \eta\left[\frac{N_I}{2}\right] \right]^T,$$

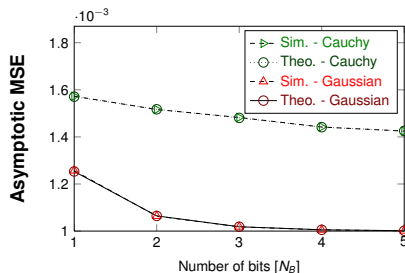
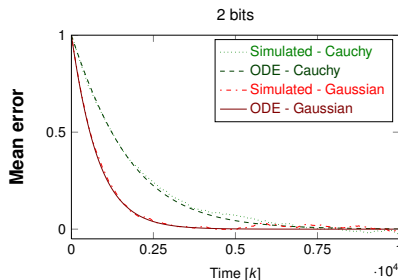
$$\mathbf{F}_d = \text{diag} \left[F_d[1], \dots, F_d\left[\frac{N_I}{2}\right] \right], \quad F_d[i] = \mathbb{P} \left(i_k = i | \hat{X}_k, X_k \right) \Big|_{\hat{X}_k = X_k},$$

$$\mathbf{f}_d = \left[f_d[1] \cdots f_d\left[\frac{N_I}{2}\right] \right]^T, \quad f_d[i] = \frac{\partial \mathbb{P}(i_k = i | \hat{X}_k, X_k)}{\partial \hat{X}_k} \Big|_{\hat{X}_k = X_k}.$$

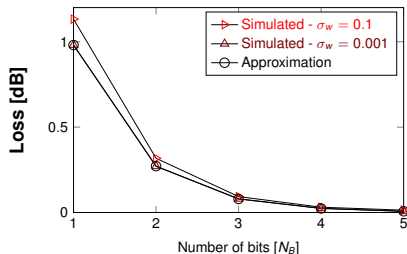
¹A. Benveniste, M. Métivier, and P. Priouret. *Adaptive algorithms and stochastic approximations*. Springer-Verlag New York, Inc., 1990.

Simulation

- Optimal η ,
- optimal uniform quantization
- **Gaussian** $f(v) = \frac{1}{\sqrt{\pi}} e^{-v^2} - \delta = \sqrt{2}$,
($\sigma = 1$)
- **Cauchy** $f(v) = \frac{1}{\pi(1+v^2)} - \delta = \sqrt{1}$.
- $\sigma_w = 0.001$.



Loss due to quantization



Continuous case (Kalman filter)

$$\text{MSE}_{\infty}^c = \frac{\sqrt{\sigma_w^4 + 2\delta^2\sigma_w^2} - \sigma_w^2}{2}.$$

Loss

$$L = 10 \log_{10} \left(\frac{\text{MSE}_{\infty}^*}{\text{MSE}_{\infty}^c} \right).$$

Slow variations ($\sigma_w \ll \delta$)

$$L \approx -5 \log_{10} \left(\delta^2 \sum_{i=1}^{\frac{N_L}{2}} \frac{f_d^2[i]}{F_d[i]} \right).$$

Main points and further work

- Low complexity algorithm for estimation under constrained rate.
Simplified signal model, but noise is not assumed to be Gaussian.
(slow random walk)
- Negligible loss for $N_B = 4$ and 5 \rightarrow

Multiple sensors, low resolution

X

Single sensor, high resolution

Remark: for $N_B = 1$, this scheme is similar to Δ modulation, but the performance is obtained without quantization noise simplifications.

Further work

- $X_k = \alpha X_{k-1} + W_k$ ($\alpha \approx 1$, $\alpha < 1$) $\rightarrow \hat{X}_k = \alpha \hat{X}_{k-1} + \gamma \eta(i_k)$
- σ_w or δ unknown $\rightarrow \hat{X}_k = \hat{X}_{k-1} + \gamma \left(i_k, \hat{X}_{k-1} \right) \eta(i_k)$

Thank you!

Questions?