



## OBJECTIVE AND MAIN RESULTS

The objectives are the following:

- to propose a low complexity algorithm that jointly estimates unknown location and scale parameters of a symmetric unimodal distribution. The estimation is based on quantized measurements from this distribution. The quantizer has adjustable input gain and offset.
- Adjustable quantizer gain and offset are dynamically updated using the estimates to enhance asymptotic estimation performance.

The following results are obtained:

- by choosing optimal algorithm parameters, the asymptotic performance is shown to attain the Cramér–Rao bound (CRB) for static offset and gain set with the true location-scale values.
- Location and scale estimation performance are shown to be asymptotically decoupled, thus no degradation of asymptotic performance occurs when the scale is unknown.
- Simulation under Gaussian and Cauchy noise shows that the estimation performance with only a few quantization bits ( $N_B = 4$  or  $5$ ) is very close to the continuous measurement CRB.

## PROBLEM STATEMENT AND QUANTIZATION SCHEME

Estimate the location  $x$  and scale  $\delta$  of the marginal CDF  $F\left(\frac{y-x}{\delta}\right)$  of an i.i.d. sequence

$$\mathbf{Y} = [Y_1 \ Y_2 \ \dots \ Y_k]^T.$$

The PDF  $f$  is symmetric and unimodal.

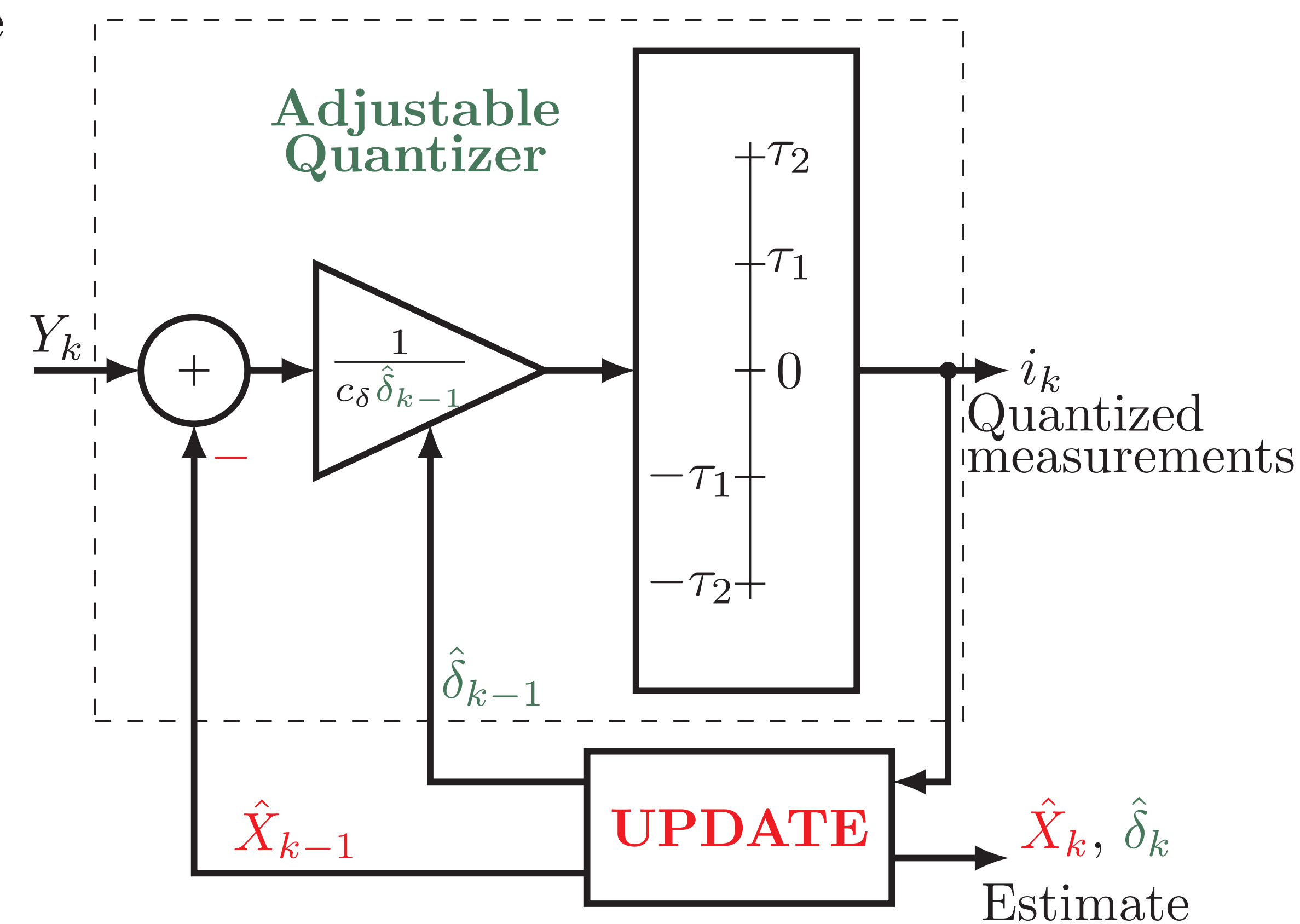
Rate constraints impose the measurements to be quantized

$$i_k = Q\left(\frac{Y_k - b_k}{\Delta_k}\right),$$

$Q$   $\left\{ \begin{array}{l} \text{is adjustable through } b_k, \Delta_k, \\ \text{has symmetric thresholds } \tau_i. \end{array} \right.$

To enhance estimation performance:

$$b_k = \hat{X}_{k-1}, \quad \Delta_k = c_\delta \hat{\delta}_{k-1}.$$



## ALGORITHM

Adaptive algorithm – **UPDATE**

$$\begin{bmatrix} \hat{X}_k \\ \hat{\delta}_k \end{bmatrix} = \begin{bmatrix} \hat{X}_{k-1} \\ \hat{\delta}_{k-1} \end{bmatrix} + \frac{\mathbf{\Gamma}}{k} \hat{\delta}_{k-1} \begin{bmatrix} \eta_x(i_k) \\ \eta_\delta(i_k) \end{bmatrix}.$$

- Online algorithm.
- Low complexity.
- Low memory requirements.

Parameters

- $\mathbf{\Gamma}$  – constant matrix gain,
- $\eta_x(i_k)$  –  $\eta_x(i_k) = -\eta_x(-i_k)$ ,  
 $\eta_x(i_k) > 0$  for  $i_k > 0$ ,
- $\eta_\delta(i_k)$  –  $\eta_\delta(i_k) = \eta_\delta(-i_k)$ ,  
 $\eta_\delta(1) < 0$ .

## ASYMPTOTIC PERFORMANCE AND OPTIMAL PARAMETERS

Asymptotic mean (ODE method)

$$\frac{d}{dt} \begin{bmatrix} \hat{x} \\ \hat{\delta} \end{bmatrix} = \mathbf{\Gamma} \mathbf{h}(\hat{x}, \hat{\delta}),$$

$$t_k = \sum_{j=1}^k \frac{1}{j} \text{ and } \mathbf{h}(\hat{x}, \hat{\delta}) = \mathbb{E} \begin{bmatrix} \hat{\delta} \eta_x \left( Q \left( \frac{y-\hat{x}}{c_\delta \hat{\delta}} \right) \right) \\ \hat{\delta} \eta_\delta \left( Q \left( \frac{y-\hat{x}}{c_\delta \hat{\delta}} \right) \right) \end{bmatrix}.$$

Optimal coefficients

$$\begin{aligned} \eta_x &= -\mathbf{F}_d^{-1} \mathbf{f}_d^{(x)}, \\ \eta_\delta &= -\mathbf{F}_d^{-1} \mathbf{f}_d^{(\delta)}. \end{aligned}$$

Asymptotic error covariance matrix

$$k^{\frac{1}{2}} \boldsymbol{\epsilon}_k \underset{k \rightarrow \infty}{\rightsquigarrow} \mathcal{N}(0, \mathbf{P}),$$

$\mathbf{P}$  – covariance matrix given by the optimal gain  $\mathbf{\Gamma}^*$ .

$$\mathbf{P} = \frac{\delta^2}{2} \begin{bmatrix} \frac{\eta_x^T \mathbf{F}_d \eta_x}{(\eta_x^T \mathbf{f}_d^{(x)})^2} & 0 \\ 0 & \frac{\eta_\delta^T \mathbf{F}_d \eta_\delta}{(\eta_\delta^T \mathbf{f}_d^{(\delta)})^2} \end{bmatrix}, \quad \mathbf{\Gamma}^* = -\frac{1}{2} \begin{bmatrix} \frac{1}{\eta_x^T \mathbf{f}_d^{(x)}} & 0 \\ 0 & \frac{1}{\eta_\delta^T \mathbf{f}_d^{(\delta)}} \end{bmatrix}.$$

Optimal gain and covariance

$$\mathbf{P} = \delta^2 \mathbf{\Gamma}^* = \frac{\delta^2}{2} \begin{bmatrix} \frac{1}{\mathbf{f}_d^{(x)T} \mathbf{F}_d^{-1} \mathbf{f}_d^{(x)}} & 0 \\ 0 & \frac{1}{\mathbf{f}_d^{(\delta)T} \mathbf{F}_d^{-1} \mathbf{f}_d^{(\delta)}} \end{bmatrix}.$$

## SIMULATION

MSE for the location parameter – Optimal uniform thresholds

