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**Weighted completion time minimization on a  
single-machine with a fixed non-availability interval:  
differential approximability**

**Imed Kacem, Vangelis Th. Paschos**



# Weighted Completion Time Minimization on a Single-Machine with a Fixed Non-Availability Interval: Differential Approximability\*

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## Abstract

This paper is the first successful attempt on differential approximability study for a scheduling problem. Such a study considers the weighted completion time minimization on a single machine with a fixed non-availability interval. The analysis shows that the Weighted Shortest Processing Time (WSPT) rule cannot yield a differential approximation for the studied problem in the general case. Nevertheless, a slight modification of this rule provides an approximation with a differential ratio of  $\frac{3-\sqrt{5}}{2} \approx 0.38$ .

## 1 Introduction

In this paper, we study the differential approximability of a well-known scheduling problem. Our work is motivated by the fact that the differential approximability concept has not yet been investigated for scheduling problems. Contrary to the standard approximability based on the comparison in the worst case of a heuristic solution to the optimal one, the differential approximability principle consists in comparing the heuristic solution to both the optimal and the worst solutions. More precisely, we say that heuristic  $A$  is an  $\alpha$ -differential approximation for problem  $\Pi$  if for every instance  $I$  of  $\Pi$  the following relation holds  $f(A(I)) \leq \alpha f(opt(I)) + (1 - \alpha)f(wst(I))$ , where  $f$  is the objective function to be minimized in problem  $\Pi$  and the

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values  $opt(I)$ ,  $A(I)$  and  $wst(I)$  denote respectively the values of an optimal solution, of an approximate solution and of a worst solution. This last solution is defined as the optimal solution of a problem having the same instances and set of constraints with the initial problem but the opposite goal (i.e., max, if the initial problem is a minimization one and min if the initial problem is a maximization problem. Let us also note that worst solutions are not always easy to compute. For instance, for the minimization version of travelling salesman problem, the worst solution is a Hamiltonian cycle of maximum total distance, i.e., the optimum solution of maximum travelling salesman problem. The computation of such a solution is not trivial since the latter problem is as hard as the former one. On the contrary, examples of problems for which a worst solution is easily computed are maximum independent set where the worst solution is the empty set, minimum vertex cover, where this solution is the whole vertex-set of the input graph, or, even, minimum graph-coloring, where the worst solution consists of taking a color per vertex of the input graph. The value  $\alpha$  is called the differential ratio and it belongs to  $(0, 1)$ . For more details on these approaches, the reader is invited to consult Ausiello and Paschos [2] and Demange and Paschos [4].

In the studied problem we have a set of independent jobs to be performed on a single machine under the constraint of a fixed non-availability interval. The objective is to minimize the total weighted completion time under the non-resumable scenario. This problem has been proved to be NP-Hard by Adiri et al. [1] and Lee [13] and it has been studied in the literature under various criteria. Several standard approximations have been proposed. A sample of them include the worst-case analysis of heuristic methods (see for example Adiri et al. [1]; Lee and Liman [15]; Sadfi et al. [16]; He et al. [5]; Wang et al. [19] and Breit [3]; Kacem and Chu [7]; Kacem [9]; Kellerer and Strusevich [12]). Efficient standard approximation schemes were also published in Kellerer and Strusevich [11]; Kacem and Mahjoub [10] and He et al. [5]. Other exact methods to solve this problem have been proposed in Kacem et al. [8]-[6]. For more detail on scheduling problems under non-availability constraints, we refer to the state-of-the-art papers by Lee [14] and Schmidt [17].

The review of the related literature shows that no differential approximation has been proposed to this problem according to the best of our knowledge. In a more general way, we did not find any work dedicated to the differential approximation to scheduling problems. For these reasons, this paper is a first successful attempt to develop a polynomial  $\frac{3-\sqrt{5}}{2}$ -differential approximation algorithm for the studied problem.

The paper is organized as follows. In Section 2, we present a description of the studied problem. Section 3 provides the differential analysis of the Weighted Shortest Processing Time heuristic (*WSPT*). In Section 4, we show that the modification of the above heuristic yields a differential ratio

of  $\frac{3-\sqrt{5}}{2}$ . Finally, Section 5 concludes the paper.

## 2 Problem Formulation

We have to schedule a set of  $n$  jobs  $J = \{1, 2, \dots, n\}$  on a single machine. Every job  $i$  has a processing time  $p_i$  and a weight  $w_i$ . The machine is unavailable between  $t_1$  and  $t_2$  and it can process at most one job at a time. The fixed non-availability interval length is denoted by  $\Delta t$  where  $\Delta t = t_2 - t_1$ . Let  $C_i(\sigma)$  denote the completion time of job  $i$  in a feasible schedule  $\sigma$ . The aim is to find a schedule  $\sigma^*$  that minimizes the total weighted completion time  $\sum_{i=1}^n w_i C_i(\sigma^*)$ . With no loss of generality, we consider that all data are integers and that jobs are indexed according to the *WSPT* rule (i.e.,  $\frac{p_1}{w_1} \leq \frac{p_2}{w_2} \leq \dots \leq \frac{p_n}{w_n}$ ). Due to the dominance of the *WSPT* order (see Smith [18]), an optimal schedule is composed of two sequences of jobs scheduled in nondecreasing order of their indexes (one sequence will be performed before  $t_1$  and another after  $t_2$ ).

If all the jobs can be inserted before  $t_1$ , the problem studied ( $\mathcal{P}$ ) has obviously a trivial optimal solution obtained by the *WSPT* rule (Smith [18]). We therefore consider only the problems in which all the jobs cannot be scheduled before  $t_1$ . The worst solution can be naturally defined as the solution *WST* consisting of scheduling all the jobs after  $t_2$  in the *WSPT* order. Figure 1 illustrates the two sequences *WSPT* and *WST* and the related notations.

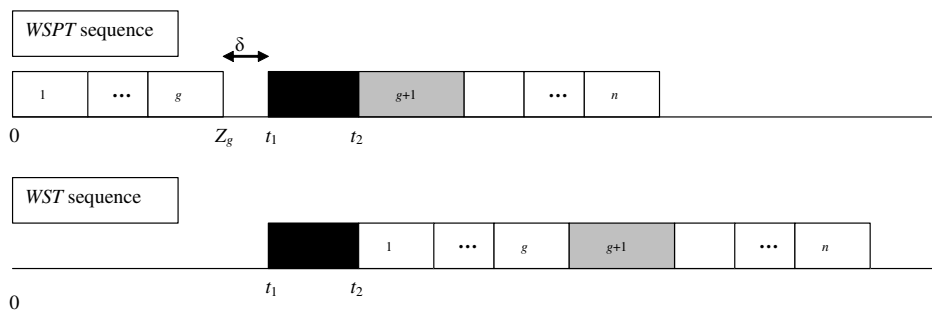


Figure 1: Illustration of *WSPT* and *WST* solutions

In the remainder of the paper, we define  $Z_k = \sum_{i=1}^k p_i$  for every  $k \in \{1, 2, \dots, n\}$ . Job  $g$  is identified by  $Z_g \leq t_1$  and  $Z_{g+1} > t_2$ . Variable  $\delta$  denotes the idle-time between  $t_1$  and the completion time of  $g$  in the *WSPT* sequence (i.e.,  $\delta = t_1 - Z_g$ ). Moreover,  $\varphi^*(\mathcal{P})$  denotes the minimum weighted sum of the completion times for problem  $\mathcal{P}$  and  $\varphi_\sigma(\mathcal{P})$  is the weighted sum of the completion times of schedule  $\sigma$  for problem  $\mathcal{P}$ .

### 3 WSPT Analysis

In this section, we are interested in the differential approximability of the *WSPT* rule. We recall that the absolute worst-case performance ratio of this rule can be arbitrarily large [13], but not smaller than 3 under some conditions [7]. Our analysis is based upon the comparison of *WSPT* to *WST* and it uses a lower bound introduced in Kacem and Chu [7] (page 1083, Equation (6)).

**Lemma 1** [7] *The following relation holds:*

$$\varphi^*(\mathcal{P}) \geq \sum_{i=1}^g w_i Z_i + w_{g+1} \frac{\Delta t}{p_{g+1}} (Z_g + \Delta t) + \sum_{i=g+1}^n w_i (Z_i + \Delta t) - w_{g+1} \frac{\Delta t}{p_{g+1}} t_2$$

From the definition of *WSPT* and *WST* solutions (Figure 1), the following proposition can directly be established.

**Proposition 2**

$$\varphi_{WST}(\mathcal{P}) - \varphi_{WSPT}(\mathcal{P}) = t_2 \sum_{i=1}^g w_i + Z_g \sum_{i=g+1}^n w_i$$

**Proposition 3**

$$\varphi_{WST}(\mathcal{P}) - \varphi^*(\mathcal{P}) \leq t_2 \sum_{i=1}^g w_i + t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta$$

**Proof.** From Figure 1, we can establish that:

$$\varphi_{WST}(\mathcal{P}) = \sum_{i=1}^n w_i (t_2 + Z_i) \quad (1)$$

By combining Lemma (1) and Equation (1), we obtain:

$$\begin{aligned} \varphi_{WST}(\mathcal{P}) - \varphi^*(\mathcal{P}) &\leq \sum_{i=1}^n w_i (t_2 + Z_i) - \sum_{i=1}^g w_i Z_i - w_{g+1} \frac{\Delta t}{p_{g+1}} (Z_g + \Delta t) \\ &\quad - \sum_{i=g+1}^n w_i (Z_i + \Delta t) + w_{g+1} \frac{\Delta t}{p_{g+1}} t_2 \\ &= t_2 \sum_{i=1}^g w_i + t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} (t_2 - (Z_g + \Delta t)) \\ &= t_2 \sum_{i=1}^g w_i + t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta \end{aligned}$$

as claimed. ■

The following well-known lemma will be used in what follows (see also Kacem [9]).

**Lemma 4** Let  $a_i$  and  $b_i$  be positive numbers ( $i = 1, 2, \dots, k$ ) such that  $b_i > 0$  for every  $i$  and  $\frac{a_1}{b_1} \geq \frac{a_2}{b_2} \geq \dots \geq \frac{a_k}{b_k}$ . The following relation holds:

$$\frac{\sum_{i=1}^{k-1} a_i}{\sum_{i=1}^{k-1} b_i} \geq \frac{a_k}{b_k} \quad (2)$$

**Theorem 5** Let  $\rho$  be a positive number such that  $\rho \in (0, 1)$ . If  $\delta \leq \rho t_1$ , then  $WSPT$  is a  $(1 - \rho)$ -differential approximation for  $\mathcal{P}$ , i.e.,

$$\varphi_{WSPT}(\mathcal{P}) \leq (1 - \rho) \varphi^*(\mathcal{P}) + \rho \varphi_{WST}(\mathcal{P}) \quad (3)$$

**Proof.** Let  $X$  and  $Y$  be defined as follows:

$$\begin{aligned} X &= t_2 \sum_{i=1}^g w_i + Z_g \sum_{i=g+1}^n w_i \\ Y &= t_2 \sum_{i=1}^g w_i + t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta \end{aligned}$$

Hence,

$$\begin{aligned} \frac{Y}{X} &= \frac{t_2 \sum_{i=1}^g w_i + t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta}{t_2 \sum_{i=1}^g w_i + Z_g \sum_{i=g+1}^n w_i} = 1 + \frac{\delta \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta}{t_2 \sum_{i=1}^g w_i + Z_g \sum_{i=g+1}^n w_i} \\ &\leq 1 + \frac{\rho t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta}{(1 - \rho) t_1 \sum_{i=g+1}^n w_i + t_2 \sum_{i=1}^g w_i} \\ &= 1 + \frac{\rho t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\Delta t}{p_{g+1}} \delta}{(1 - \rho) t_1 \sum_{i=g+1}^n w_i + t_2 \left( \sum_{i=1}^g p_i \right) \left( \frac{\sum_{i=1}^g w_i}{\sum_{i=1}^g p_i} \right)} \end{aligned}$$

By Lemma 4, it can be established that

$$\frac{w_{g+1}}{p_{g+1}} \leq \frac{w_g}{p_g} \leq \left( \frac{\sum_{i=1}^g w_i}{\sum_{i=1}^g p_i} \right)$$

Moreover, we know that  $\Delta t \leq t_2$  and  $\sum_{i=1}^g p_i = Z_g$ . Hence, we deduce that:

$$\begin{aligned} \frac{Y}{X} &\leq 1 + \frac{\rho t_1 \sum_{i=g+1}^n w_i + t_2 \frac{w_{g+1}}{p_{g+1}} \delta}{(1-\rho) t_1 \sum_{i=g+1}^n w_i + t_2 \frac{w_{g+1}}{p_{g+1}} Z_g} \\ &\leq 1 + \frac{\rho t_1 \sum_{i=g+1}^n w_i + t_2 \frac{w_{g+1}}{p_{g+1}} \rho t_1}{(1-\rho) t_1 \sum_{i=g+1}^n w_i + t_2 \frac{w_{g+1}}{p_{g+1}} (1-\rho) t_1} \\ &\leq 1 + \frac{\rho t_1 \left( \sum_{i=g+1}^n w_i + t_2 \frac{w_{g+1}}{p_{g+1}} \right)}{(1-\rho) t_1 \left( \sum_{i=g+1}^n w_i + t_2 \frac{w_{g+1}}{p_{g+1}} \right)} = \frac{1}{(1-\rho)} \end{aligned}$$

From the last result:

$$\begin{aligned} \frac{\varphi_{WST}(\mathcal{P}) - \varphi_{WSPT}(\mathcal{P})}{\varphi_{WST}(\mathcal{P}) - \varphi^*(\mathcal{P})} &\geq \frac{X}{Y} \geq 1 - \rho \\ \Rightarrow \varphi_{WSPT}(\mathcal{P}) &\leq (1 - \rho) \varphi^*(\mathcal{P}) + \rho \varphi_{WST}(\mathcal{P}) \end{aligned}$$

as claimed. ■

Let us note that, in the general case, *WSPT* can be arbitrarily bad when  $\delta$  is large (compared to  $t_1$ ). As an example, let us consider the tow-job instance with  $p_1 = \varepsilon$ ,  $w_1 = \varepsilon$ ,  $p_2 = t$ ,  $w_2 = t - \varepsilon$ ,  $t_1 = t$  and  $t_2 = t + t^2$  (with  $t \gg \varepsilon$ ). Figure 2 illustrates the *WSPT*, *WST* and *OPT* solutions. For this instance we have  $\varphi_{WSPT}(\mathcal{P}) \approx t^3$ ,  $\varphi_{WST}(\mathcal{P}) \approx t^3$  whereas  $\varphi^*(\mathcal{P}) \approx t^2$ . In other words, the differential approximation ratio in this case tends to 0.



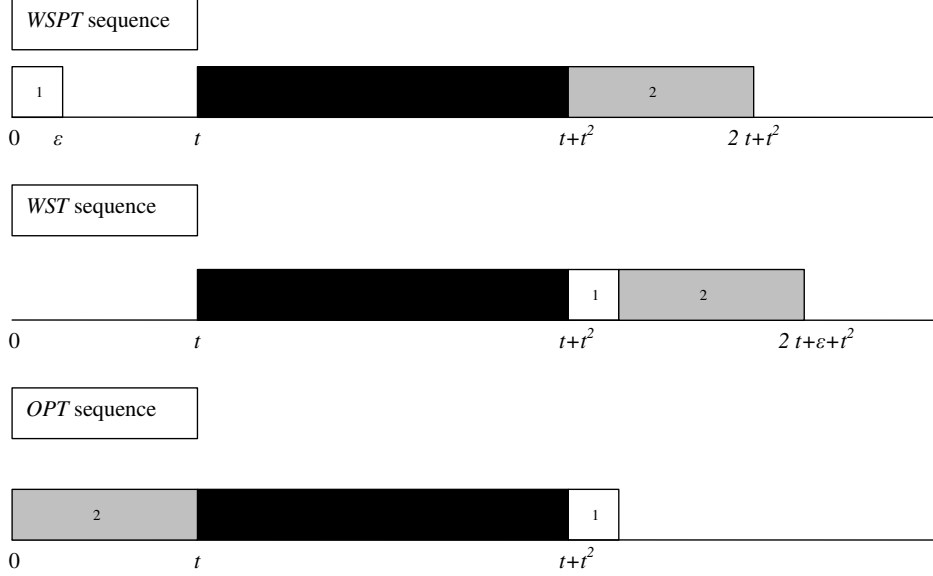


Figure 2: Worst-case example for *WSPT*

#### 4 Modifying the WPST Rule to Get a $\frac{3-\sqrt{5}}{2}$ -Differential Approximation

Based upon the previous analysis of *WSPT* rule, it appears that the wrong scheduling of job  $g + 1$  can be the origin of its possible weakness. Hence, we investigate the modification of the *WSPT* sequence based upon the following algorithm *H*, which tests the two possibilities of scheduling job  $g + 1$  before and after the non-availability interval. This algorithm generates two sequences. The first one is the *WSPT* sequence. In the second one (denoted as *WSPT2*), job  $g + 1$  is scheduled before  $t_1$  and the other jobs are scheduled in the *WSPT* order. The output of this algorithm is the best generated sequence.

HEURISTIC *H*

- (i) Construct the sequence *WSPT* and compute  $\varphi_{WSPT}(\mathcal{P})$ .
- (ii) Let  $y$  be the  $y^{th}$  job in  $J - \{g + 1\}$  according to the *WSPT* order ( $y < g$ ) such that  $\sum_{i=1}^y p_i + p_{g+1} \leq t_1$  and  $\sum_{i=1}^{y+1} p_i + p_{g+1} > t_1$ . Construct the sequence:

$$WSPT2 = \langle 1, 2, \dots, y, g + 1, y + 1, y + 2, \dots, g, g + 2, g + 3, \dots, n \rangle$$

and, if feasible, compute  $\varphi_{WSPT2}(\mathcal{P})$ .

- (iii) Output the best among the solutions obtained in steps (i) and (ii) of value  $\varphi_H(\mathcal{P}) = \min\{\varphi_{WSPT}(\mathcal{P}), \varphi_{WSPT2}(\mathcal{P})\}$ .

It can be easily seen that Heuristic  $H$  can be implemented in  $O(n \log(n))$  time.

To illustrate Heuristic  $H$ , let us consider the following four-job instance:  $p_1 = 1; w_1 = 2; p_2 = 2; w_2 = 3; p_3 = 3; w_3 = 4; p_4 = 1; w_4 = 1; t_1 = 4; t_2 = 7$ . Given this instance, we have:  $\varphi_{WSPT}(\mathcal{P}) = 62; \varphi_{WSPT2}(\mathcal{P}) = 55$  and  $\varphi_H(\mathcal{P}) = 55$ . Figure 3 illustrates the obtained schedules. In this case, we have  $g = 2$  and  $y = 1$ .

Schedule  $WSPT$



Schedule  $WSPT2$



Figure 3: Illustration of  $H$

**Proposition 6**

$$\varphi_{WSPT}(\mathcal{P}) - \varphi_{WSPT2}(\mathcal{P}) > w_{g+1} \frac{\delta}{p_{g+1}} \Delta t + \delta \sum_{i=g+1}^n w_i \quad (4)$$

**Proof.** From Figure 4, the value  $\varphi_{WST}(\mathcal{P}) - \varphi_{WSPT2}(\mathcal{P})$  can be computed:

$$\begin{aligned}
\varphi_{WST}(\mathcal{P}) - \varphi_{WSPT2}(\mathcal{P}) &= t_2 \sum_{i=1}^y w_i + Z_y \sum_{i=y+1}^g w_i + w_{g+1} \left( t_2 + \sum_{i=y+1}^g p_i \right) \\
&\quad + (Z_y + p_{g+1}) \sum_{i=g+2}^n w_i \\
&> w_{g+1} t_2 + p_{g+1} \sum_{i=g+2}^n w_i \\
&> w_{g+1} (t_1 + \Delta t) + \delta \sum_{i=g+2}^n w_i \\
&> w_{g+1} \Delta t + \delta \sum_{i=g+1}^n w_i > w_{g+1} \frac{\delta}{p_{g+1}} \Delta t + \delta \sum_{i=g+1}^n w_i
\end{aligned}$$

as claimed. ■

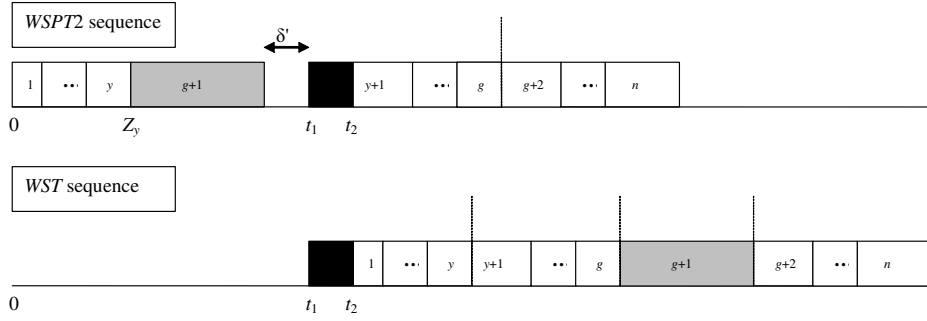


Figure 4: Comparison of  $WSPT2$  with  $WST$

**Theorem 7** Let  $\rho$  be a positive number such that  $\rho \in (0, 1)$ . If  $\delta > \rho t_1$ , then  $H$  is a  $\frac{\rho}{(1+\rho)}$ -differential approximation for  $\mathcal{P}$ , i.e.,

$$\varphi_H(\mathcal{P}) \leq \frac{\rho}{(1+\rho)} \varphi^*(\mathcal{P}) + \frac{1}{(1+\rho)} \varphi_{WST}(\mathcal{P}) \quad (5)$$

**Proof.** By definition:

$$\begin{aligned}
\varphi_H(\mathcal{P}) &= \min \{ \varphi_{WST}(\mathcal{P}), \varphi_{WSPT2}(\mathcal{P}) \} \\
&\leq \frac{\rho}{(1+\rho)} \varphi_{WST}(\mathcal{P}) + \frac{1}{(1+\rho)} \varphi_{WSPT2}(\mathcal{P})
\end{aligned}$$

Hence,

$$\begin{aligned}\varphi_{WST}(\mathcal{P}) - \varphi_H(\mathcal{P}) &\geq \frac{\rho}{(1+\rho)} (\varphi_{WST}(\mathcal{P}) - \varphi_{WSPT}(\mathcal{P})) \\ &\quad + \frac{1}{(1+\rho)} (\varphi_{WST}(\mathcal{P}) - \varphi_{WSPT_2}(\mathcal{P}))\end{aligned}$$

From Propositions 2 and 6, we can deduce the following inequality:

$$\begin{aligned}\varphi_{WST}(\mathcal{P}) - \varphi_H(\mathcal{P}) &\geq \frac{\rho}{(1+\rho)} \left( t_2 \sum_{i=1}^g w_i + Z_g \sum_{i=g+1}^n w_i \right) \\ &\quad + \frac{1}{(1+\rho)} \left( w_{g+1} \frac{\delta}{p_{g+1}} \Delta t + \delta \sum_{i=g+1}^n w_i \right)\end{aligned}$$

By assumption,  $\delta > \rho t_1$ . Therefore, we deduce that:

$$\begin{aligned}\varphi_{WST}(\mathcal{P}) - \varphi_H(\mathcal{P}) &> \frac{\rho}{(1+\rho)} \left( t_2 \sum_{i=1}^g w_i + Z_g \sum_{i=g+1}^n w_i \right) \\ &\quad + \frac{1}{(1+\rho)} \left( w_{g+1} \frac{\delta}{p_{g+1}} \Delta t + \rho t_1 \sum_{i=g+1}^n w_i \right) \\ &> \frac{\rho}{(1+\rho)} \left( t_2 \sum_{i=1}^g w_i \right) \\ &\quad + \frac{\rho}{(1+\rho)} \left( w_{g+1} \frac{\delta}{p_{g+1}} \Delta t + t_1 \sum_{i=g+1}^n w_i \right) \\ &> \frac{\rho}{(1+\rho)} \left( t_2 \sum_{i=1}^g w_i + t_1 \sum_{i=g+1}^n w_i + w_{g+1} \frac{\delta}{p_{g+1}} \Delta t \right)\end{aligned}$$

Finally, from Proposition 3 we obtain:

$$\varphi_{WST}(\mathcal{P}) - \varphi_H(\mathcal{P}) \geq \frac{\rho}{(1+\rho)} (\varphi_{WST}(\mathcal{P}) - \varphi^*(\mathcal{P})),$$

and then, Equation (5) is proved. ■

**Theorem 8** *Heuristic H is a  $\frac{3-\sqrt{5}}{2}$ -differential approximation for problem  $\mathcal{P}$ .*

**Proof.** Let  $\rho$  be a positive number such that  $\rho \in (0, 1)$ . By combining Theorems 5 and 7, Heuristic  $H$  is a  $\frac{\rho}{(1+\rho)}$ -differential approximation for  $\mathcal{P}$  (if  $\delta > \rho t_1$ ) and a  $(1-\rho)$ -differential approximation for  $\mathcal{P}$  (if  $\delta \leq \rho t_1$ ).

Hence, by taking  $\rho \in (0, 1)$  such that  $\frac{\rho}{(1+\rho)} = 1 - \rho$ , we obtain  $\rho = \frac{\sqrt{5}-1}{2}$ . Therefore, Heuristic  $H$  is a  $\frac{3-\sqrt{5}}{2}$ -differential approximation for problem  $\mathcal{P}$  in the general case:

$$\varphi_H(\mathcal{P}) \leq \frac{3-\sqrt{5}}{2}\varphi^*(\mathcal{P}) + \frac{\sqrt{5}-1}{2}\varphi_{WST}(\mathcal{P})$$

that completes the proof. ■

## 5 Conclusion

Motivated by the absence in the literature of differential approximability analysis for scheduling problems, this paper aims to investigate this new direction. The considered study is related to the weighted completion time minimization on a single machine with a fixed non-availability interval. The analysis of the Weighted Shortest Processing Time (*WSPT*) rule shows that this rule cannot yield a differential approximation for the studied problem in the general case. Nevertheless, a slight modification of this rule provides a  $\frac{3-\sqrt{5}}{2}$ -differential approximation.

Ongoing research will aim at designing more efficient differential approximations for scheduling problems (in particular, differential approximation schemes).

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