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MODÈLE TEMPOREL SOUS FORME DE REPRÉSENTATION D'ÉTAT POUR UN HOULOGÉNÉRATEUR

Approximated State-Space Model for a Partially Submerged Wave Energy Converter

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Dans cet article, nous présentons un modèle dynamique pour un récupérateur d'énergie des vagues ou houlogénérateur dans le domaine temporel. Le mouvement relatif entre deux corps oscillant dans la houle, l'un flottant et l'autre immergé, est exploité afin d'extraire la puissance incidente de la vague. Un système mécanique direct de type pignon-crémaillère convertit le mouvement linéaire en un mouvement de rotation permettant ainsi d'entraîner une génératrice synchrone à aimants permanents. Sous l'hypothèse de la théorie potentielle, le problème de tenue à la mer est résolu dans le domaine fréquentiel par une méthode dite semi-analytique et les paramètres hydrodynamiques (i.e. les masses ajoutées, les amortissements de radiation et les efforts d'excitation) sont ainsi déterminés. Une fois ces paramètres obtenus, nous pouvons utiliser la formulation de Cummins pour décrire le mouvement de la structure en houle irrégulière. Le principal inconvénient de cette forme intégral-différentielle est le traitement numérique des termes de convolution décrivant les efforts de radiation et les efforts d'excitation. Un algorithme d'indentification est donc mis en place dans le domaine fréquentiel afin d'identifier un modèle équivalent du noyau de convolution sous forme de représentation d'état.

Mots-clefs : Houlogénérateur, théorie potentielle, formulation de Cummins, représentation d'état, identification fréquentielle.

In this paper, we present a time-dependant model based on the Cummins formulation for a two body wave energy converter that consists of a cylindrical buoy sliding along a partially submerged platform made up of a plate and a column. This model is intended to be used for WEC control purposes in irregular wave conditions. Linear potential is assumed and a semi-analytical approach is proposed for the computation of the hydrodynamic coefficients and the excitation forces due to heave motion in finite depth water. Given a frequency response for both radiation and excitation force, one can use a frequency domain identification algorithm to obtain a parametric model in the form of state-space representation which is more commonly used in control field.

Key-words: WEC, potential theory, Cummins formulation, state-space model, frequency domain identification.

I INTRODUCTION

This work was motivated from the need to provide a time-dependant model for a wave energy converter (WEC) in order to analyse its behavior in irregular waves. The WEC device schematically depicted in Figure 1 consists of a cylindrical buoy riding in waves and sliding along a partially submerged platform made up of a plate and a column. Energy resulting from the relative motion between the two concentric bodies is extracted from the incoming wave by a gearbox and a permanent magnet synchronous generator.

According to [Yu and Falnes, 1995] and more recently to [Taghipour et al., 2008] and [Perez and Fossen, 2011], linear models based on the Cummins formulation [Cummins, 1962] are a good starting point for modelling the response of a marine structure in waves. Not only that, but in the context of wave energy converter, authors in [Perez and Fossen, 2011] defined this model as a basis for initial design and prediction of power capture. Cummins equation is an integro-differential equation which relates the motion of the marine structure to the incoming wave. Moreover once this model is obtained, nonlinear terms as viscous damping or mooring line, can be easily introduced to correct and improve the quality of the model. However the main drawback of this formulation comes from the two convolution terms, one representing fluid memory effects associated with the radiation forces and the other representing the wave excitation force due to an incoming wave [Taghipour et al., 2008].

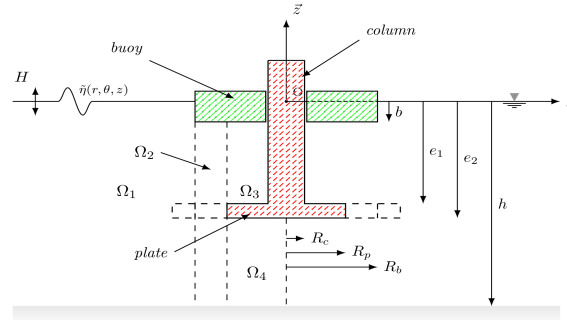


Figure 1. Definition sketch of the WEC.

Direct computation based on a discrete time approximation of the convolution terms requires to save enough past data to evaluate the integral at each step of simulation. This approach can be time consuming in simulations and may require significant amounts of computer memory. An approach to overcome this problem is the use of a parametric models that approximate the convolution terms in the Cummins equation. From the literature review several identification schemes have been investigated either in frequency domain or in time domain. Authors in [Taghipour et al., 2008] and [Perez and Fossen, 2008] provide a review of different methods in both domains but most attention have been reported on radiation force model estimation. Taghipour shows that it is better to use frequency domain identification because we can reinforce the identification algorithm by adding prior knowledges. In the next sections will give an overview of the whole process to achieve this time-domain modelling.

II MATHEMATICAL MODELLING BACKGROUND

Under the assumption of linear wave potential theory, equations of motion for the six degrees of freedom are given in an earthbound reference frame coordinating system with its origin O located at the intersection of the undisturbed free surface level with cylinders axes and the z -axis is positive upward. From the Newton's second law and using matrix notations we have

$$\mathbf{M} \ddot{\boldsymbol{\xi}}(t) = \mathbf{F}(t) \quad (1)$$

where

- \mathbf{M} is the rigid-body generalised inertia matrix .
- $\boldsymbol{\xi} = [x \ y \ z \ \varphi \ \theta \ \psi]^T$ is the generalized position vector of the body-fixed frame with respect to the equilibrium frame with the three translations (i.e. x -surge, y -sway, z -heave) and the three rotations expressed in terms of Euler's angle (i.e. φ -roll, θ -pitch, and ψ -yaw). $\dot{\boldsymbol{\xi}}$ and $\ddot{\boldsymbol{\xi}}$ denote respectively the generalized velocity and acceleration vectors.
- \mathbf{F} is the generalised force vector .

The generalised force vector can be expressed in term of several components such as

$$\mathbf{F}(t) = \mathbf{F}_{exc}(t) + \mathbf{F}_r(t) + \mathbf{F}_s(t) + \mathbf{F}_{gen}(t) \quad (2)$$

Where

- \mathbf{F}_{exc} is the wave excitation force associated with the scattering problem which can be expressed in the time-domain as

$$\mathbf{F}_{exc}(t) = \int_{-\infty}^{\infty} \mathbf{H}_{exc}(t - \tau) \eta(0, \tau) d\tau \quad (3)$$

with $\eta(0, t)$ the wave elevation at the origin and $\mathbf{H}_{exc}(t)$ is the impulse response matrix of the wave excitation force [Yu and Falnes, 1995].

- \mathbf{F}_r is the radiation force associated with the radiation problem. In linear potential theory it is conventional to decompose this force into two parts which are frequency dependent. One is proportional to the acceleration of body and another is proportional to its velocity and are respectively referenced as added mass and radiation damping matrix

$$\mathbf{F}_r = -\mathbf{A}(\omega) \ddot{\boldsymbol{\xi}}(t) - \mathbf{B}(\omega) \dot{\boldsymbol{\xi}}(t) \quad (4)$$

- \mathbf{F}_s is the restoring force which is proportional to the displacement of the body structure from its equilibrium position. The coefficient of proportionality denoted and \mathbf{K}_s is referenced as the buoyancy stiffness matrix

$$\mathbf{F}_s = -\mathbf{K}_s \boldsymbol{\xi} \quad (5)$$

- \mathbf{F}_{gen} is the force due to the generator .

Regarding the radiation force, Cummins [Cummins, 1962] shows that it can be approximated by the following representation in the time domain for the case of zero forward speed

$$\mathbf{F}_r(t) = -\mathbf{A}_\infty \ddot{\boldsymbol{\xi}}(t) - \int_0^t \mathbf{K}(t-\tau) \dot{\boldsymbol{\xi}}(\tau) d\tau \quad (6)$$

where \mathbf{A}_∞ is the infinite-frequency added mass matrix defined as

$$\mathbf{A}_\infty = \lim_{\omega \rightarrow \infty} \mathbf{A}(\omega) \quad (7)$$

The second terms is referenced as the fluid memory effect and capture energy transfert from the motion of the structure to the radiated waves. By replacing and combining terms in (1) by (5) and (6), we obtain the Cummins equation:

$$(\mathbf{M} + \mathbf{A}_\infty) \ddot{\boldsymbol{\xi}}(t) + \int_0^t \mathbf{K}(t-\tau) \dot{\boldsymbol{\xi}}(\tau) d\tau + \mathbf{K}_s \boldsymbol{\xi}(t) = \mathbf{F}_{exc}(t) + \mathbf{F}_{gen}(t) + \mathbf{F}_{NL}(t) \quad (8)$$

where we add a term, $\mathbf{F}_{NL}(t)$, modelling nonlinear effects such as viscous damping or mooring line although. In the following we will not investigate this aspect. As already mentioned in introduction, one of the main drawback of this formulation is the convolution terms which are not convenient for time-domain analysis. In the next section we give a short introduction on how to approximate these convolution integrals.

III CUMMINS APPROXIMATED STATE-SPACE MODEL

In both cases (i.e. for the wave excitation and fluid memory terms), the main idea is to approximate the convolution terms by a linear-time-invariant parametric model using state-space representation. For the case of fluid memory terms we are looking for a model such as

$$\boldsymbol{\mu} = \int_0^t \mathbf{K}(t-\tau) \dot{\boldsymbol{\xi}}(\tau) d\tau \simeq \begin{cases} \dot{\mathbf{x}} = \hat{\mathbf{A}}_r \mathbf{x} + \hat{\mathbf{B}}_r \boldsymbol{\xi} \\ \hat{\boldsymbol{\mu}} = \hat{\mathbf{C}}_r \mathbf{x} \end{cases} \quad (9)$$

where matrices $\hat{\mathbf{A}}_r$, $\hat{\mathbf{B}}_r$, and $\hat{\mathbf{C}}_r$ are constant and approximate the convolution kernel (or impulse response) matrix which is the inverse Fourier transform of the frequency response function [Perez and Fossen, 2011] defined by

$$\mathbf{K}(i\omega) = \mathbf{B}(\omega) + i\omega[\mathbf{A}(\omega) - \mathbf{A}_\infty] \quad (10)$$

This function depends on the radiation damping, the added mass, and the infinite-frequency added mass matrix. The latter are not always available (depend on the theory used to solve the boundary value problem) and then it have to be estimated together with the fluid memory term. In [Perez and Fossen, 2008], authors suggest the used of the rational transfert matrix, which from (9), is given by

$$\mathbf{K}(i\omega) \simeq \hat{\mathbf{K}}(i\omega) = \hat{\mathbf{C}}_r \left[i\omega \mathbf{I} - \hat{\mathbf{A}}_r \right]^{-1} \hat{\mathbf{B}}_r \quad (11)$$

and for which, we have for each entry

$$k_{ij}(s) = \frac{\hat{P}_{ij}(s)}{\hat{Q}_{ij}(s)} = \frac{p_r s^r + p_{r-1} s^{r-1} + \dots + p_0}{s^n + q_{n-1} s^{n-1} + \dots + q_0} \quad (12)$$

Therefore the identification problem focus on the determination of the transfert function $\hat{K}_{ij}(s)$ and can be formulated in term of least-square problem, in the frequency domain, as

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} \sum_l w_l \epsilon_l^* \epsilon_l \quad (13)$$

with

$$\epsilon_l = K_{ij}(i\omega_l) - \hat{K}_{ij}(i\omega_l, \boldsymbol{\theta}) \quad (14)$$

where \mathbf{I} means that we use a finite set of frequencies, w_l are weighting coefficients, and $\boldsymbol{\theta}$ is the parameter vector defined as

$$\boldsymbol{\theta} = [p_r, \dots, p_0, q_{n-1}, \dots, q_0]^T \quad (15)$$

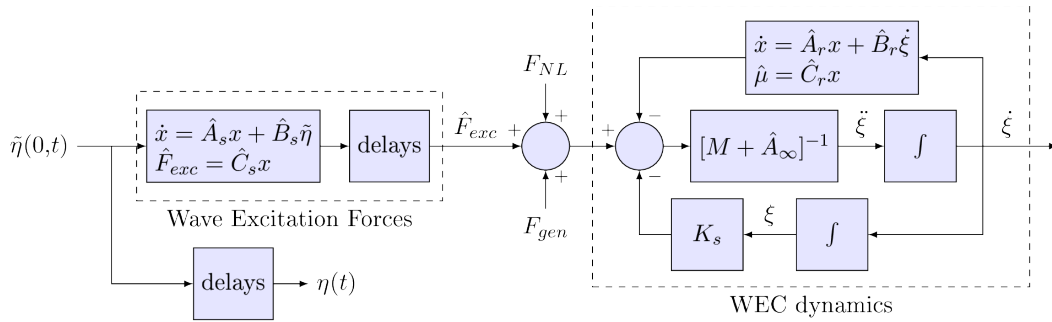


Figure 2. Bloc diagram representation of the Cummins formulation taking into account non-causality for wave excitation forces.

Perez et Fossen (2009) produce a MATLAB ToolBox which is freely accessible [Perez and Fossen, 2009] for the identification of the fluid memory term using this approach. For the case of wave excitation forces, a similar procedure is applied. However Falnes, in [Falnes, 1995], shows that the impulse response is not necessarily causal and then we are looking for an approximated model such as

$$\mathbf{F}_{exc}(t) = \int_{-\infty}^{\infty} \mathbf{H}_{exc}(t - \tau) \eta(0, \tau) d\tau \simeq \begin{cases} \dot{\mathbf{x}} = \hat{\mathbf{A}}_s \mathbf{x} + \hat{\mathbf{B}}_s \eta(0, t) \\ \hat{\mathbf{F}}_{exc}(t) = \hat{\mathbf{C}}_s \mathbf{x}(t + t_c) \end{cases} \quad (16)$$

where the main difficulties comes from the causalizing time shift t_c identification. In Figure 2 we give a bloc diagram representation of the Cummins approximated state-space model used for simulations.

Then it follows that frequency response function $\mathbf{F}_{exc}(i\omega)$ and $\mathbf{K}_{ij}(i\omega)$ (therefore $\mathbf{A}_{ij}(i\omega)$ and $\mathbf{B}_{ij}(i\omega)$) are the starting point to generate the data used in the identification problem and their determination is discussed in the next section.

IV HYDRODYNAMIC PARAMETERS DETERMINATION

Hydrodynamic parameters (i.e. added mass, radiation damping, and wave excitation force) are usually determined using numerical software such as WAMIT which is based on the boundary integral equation method or more recently using a CFD program. However, in both cases this can take amount of time for numerical computations. Due to the simplicity of the model geometry an alternative to these approaches is the use of a semi-analytical method which is probably the fastest and most reliable way to get them. More explanations about mathematical theory will be found in [Molin, 2002], [Linton and McIver, 2001], [Falnes, 2002]. Regarding the specific structure depicted in Figure 1 details will be found in [Olaya et al., 2013] for the heaving mode. In the following we give an overview of the method.

Considering a cylindrical coordinate system (r, θ, z) with its origin \mathbf{O} located at the intersection of the undisturbed free surface level with cylinders axes and the z -axis is positive upward. Assuming linear wave theory and supposing that the fluid is inviscid, incompressible, and its motion is irrotational, fluid flow can be described, using a complex representation, by the velocity potential

$$\Phi(r, \theta, z, t) = \text{Re}\{\phi(r, \theta, z)e^{-i\omega t}\} \quad (17)$$

where $\text{Re}\{\}$ denotes the real part of the complex expression, ω is the angular frequency, t is the time dependency. The spatial part of the velocity potential $\phi(r, \theta, z)$ has to satisfy the following boundary value problem

- Governing equation in the whole fluid

$$\Delta\phi = 0 \quad (18)$$

- Free surface boundary condition

$$\omega^2\phi - g\frac{\partial\phi}{\partial z}\Big|_{z=0} = 0 \quad (19)$$

- Seabed boundary condition

$$\frac{\partial\phi}{\partial z}\Big|_{z=-h} = 0 \quad (20)$$

- Body surface boundary condition

$$\nabla\phi \cdot \vec{n} = \vec{U}_k \cdot \vec{n} \quad (21)$$

- Sommerfeld radiation condition

$$\lim_{r \rightarrow +\infty} \sqrt{r} \left(\frac{\partial \phi}{\partial r} - ik_e \phi \right) = 0 \quad (22)$$

In the solution procedure, analytical expressions for velocity potentials are obtained using the variable separation method. Assuming angular independency due to the axysymmetric configuration, velocity potentials are expressed as infinite series of orthogonal functions into each subdomain (see Figure. 1) fulfilling all boundary conditions. The infinite series are expressed in terms of unknown Fourier coefficients and it remains to determine those coefficients. We have therefore used the matching eigenfunction expansion method which impose conditions of pressure and normal velocity continuity at different imaginary interfaces as well as the body surface boundary condition (21) at each body vertical walls.

V CONCLUSION

When studying the response of an offshore structure in irregular waves, for example to a wave energy converter to predict its power capture, time-domain modelling appears to be necessary. This paper gives an overview of the whole process to model the case of a two body wave energy converter including the determination of the hydrodynamic parameters using a semi- analytical approach.

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