Design of Code Division Multiple Access Filters Using Global Optimization Techniques

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Design of code division multiple access filters based on sampled fiber Bragg grating by using global optimization algorithms

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Abstract In this article, we focus on the design of code division multiple access filters (used in data transmission) composed of a particular optical fiber called sampled fiber Bragg grating (SFBG). More precisely, we consider an inverse problem that consists in determining the effective refractive index profile of an SFBG that produces a given reflected spectrum. In order to solve this problem, we use an original multi-layers semi-deterministic global optimization method based on the search of suitable initial conditions for a given optimization algorithm. The results obtained with our optimization algorithms are compared, in term of complexity and final design, with those given by an hybrid genetic algorithm (the method generally considered in the literature for designing SFBGs).

Keywords Global optimization · Genetic algorithms · Descent algorithms · Optical code division multiple access · Sampled fiber Bragg grating design

1 Introduction

The use of optical fibers offering huge bandwidth in the telecommunication sector has known important developments in the last decade (Skaar and Risvik 1998; Yang et al. 1999) with applications such as cable television or cell phones (Takeshi and Makoto

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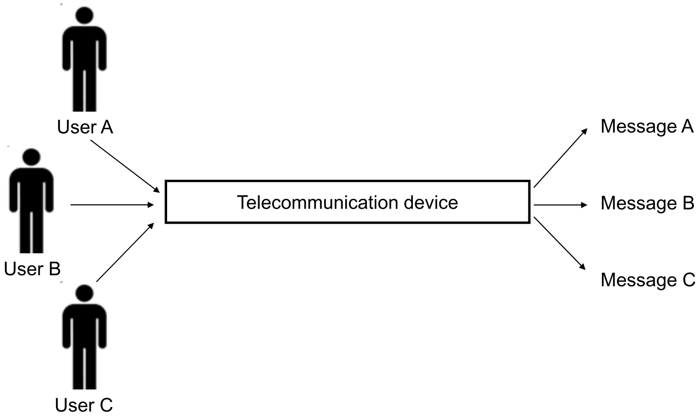


Fig. 1 Multiple access technique principle: Users A, B and C send at the same time a message in a communication device. The multiple access technique allows to properly deliver each message

2007; Yang 2008). The advantages of optical fibers are numerous as, for instance, they offer: large communication capacity, low transmission loss, low cost and easy hybridization with other optical devices (Yin and Richardson 2009). However, to be fully efficient, the fibers should allow multiple access. This means allowing various persons (called *users*) to send and receive messages in the fiber at the same time (see Fig. 1).

There exist three main schemes of multiple access: Time Division Multiple Access (TDMA) (Kyeo-Eun et al. 2005) which allows to consider a great number of users but requires fast synchronization, Wavelength-Division Multiplexing (WDM) (Bock and Prat 2005) which sometimes requires precise adjustments, and Code Division Multiple Access (**CDMA**) (Viterbi 1995) which primarily allows a great flexibility in multiple accesses. This last technique, which consists in allocating individualized codes to represent the messages of each user, exhibits various advantages such as high fidelity and high resistance to signal perturbations, secured communications and low power consumption (Jung et al. 1993).

Focusing on CDMA, an efficient and low-cost way to implement this approach is to consider an optical spectral representation of the user codes (Kavehrad and Zaccarin 1995; Lam and Yablonovitch 1995). This implementation is called optical CDMA (OCDMA). Those codes are then generated by optical guides, and in particular by sampled fiber Bragg gratings (**SFBG**) (Fang et al. 2003; Yin and Richardson 2009). SFBGs are optical fibers based on a periodic perturbation of their refractive effective index, obtained by exposing it to UV radiations (Chuang et al. 2004; Erdogan 1997; Malo et al. 1995). The objective of SFBGs is to reflect predetermined wavelengths and to let other wavelengths pass (Chow et al. 1996; Wei et al. 2000). In a OCDMA device we are interested in designing SFBGs that reflect part of the wavelengths corresponding to the code representing a particular user. This objective can be mathematically reformulated as an inverse problem that consists in determining the effective refractive index profile of an SFBG knowing a priori its reflected spectrum. To solve this inverse problem we consider global optimization algorithms

(Skaar and Risvik 1998). Currently, one of the most used optimization methods in this domain are genetic algorithms (GA) (Etezad et al. 2011; Cheng et al. 2008; Tremblay et al. 2005).

In this paper, we focus on the design of a CDMA filter, composed of an SFBG, used to reflect the whole spectral CDMA code of a user. As previously said, we could solve this problem by considering a GA. However, we would like to see if the target can be achieved with a lower computational effort by using an original multi-layers semi-deterministic global optimization method (Ivorra 2006; Ivorra et al. 2007, 2009). This approach is based on the reformulation of the optimization problem as a sub-optimization problem that consists in finding a suitable initial condition for a given optimization algorithm called core optimization algorithm (**COA**). This new problem is then solved using a multi-layers algorithm based on line search methods. Here, we use two particular implementations of our approach by considering the steepest descent (Luenberger 1984) and genetic algorithms (**GA**) (Goldberg 1989; Forrest 1993) as COAs. In order to check the efficiency of our algorithms on the considered SFBG design problem, we compare their results with those given by a traditional GA.

The paper is organized as follows. Section 2 briefly describes the CDMA filter based on SFBG, its mathematical modeling and the associated design problem. Section 3 presents the optimization methods used to solve previous problem. Finally, in Sect. 4, we consider a particular case and compare the results obtained by the different optimization algorithms in term of precision and computational time.

2 CDMA filter design problem

In this section, we first introduce some basic principles of CDMA and the SFBG to be designed. Then, we recall some previous works on SFBG optimization. Next, we present the mathematical model that allows to compute the reflected spectrum of an SFBG and we reformulate the design problem as an inverse problem.

2.1 CDMA principle and implementation

In the basic technique of CDMA, the bits ‘1’ or ‘0’ of a binary message, send by a user, are replaced at the level of the transmitter by two codes attributed to this user (Viterbi 1995). For instance, we can consider CDMA binary codes of length $N_{\text{code}} \in \mathbb{N}$. The code for the bit ‘1’ of a particular user ‘A’ is denoted by $c_1^A \in \{0, 1\}^{N_{\text{code}}}$ and its complement, denoted by $c_0^A = -(c_1^A - 1)$, is used for ‘0’ (e.g., if $N_{\text{code}} = 8$, two possible codes are $c_1^A =$ ‘10110011’ and $c_0^A =$ ‘01001100’). During this work, we only focus on this binary CDMA coding method used, for example, when considering the so-called Gold codes (El-khamy and Balamesh 1987).

In order to implement this approach, Kavehrad et al. (Kavehrad and Zaccarin 1995) and later Lam (Lam and Yablonovitch 1995) suggested to use spectra composed of a set of wavelengths $\Lambda = (\lambda_i)_{i=1}^{N_{\text{code}}}$ to transmit CDMA binary codes of length N_{code} trough optical devices (such as optical fibers). In that case, we represent a particular CDMA code c by a spectrum Λ_c whose reflectivity at wavelength

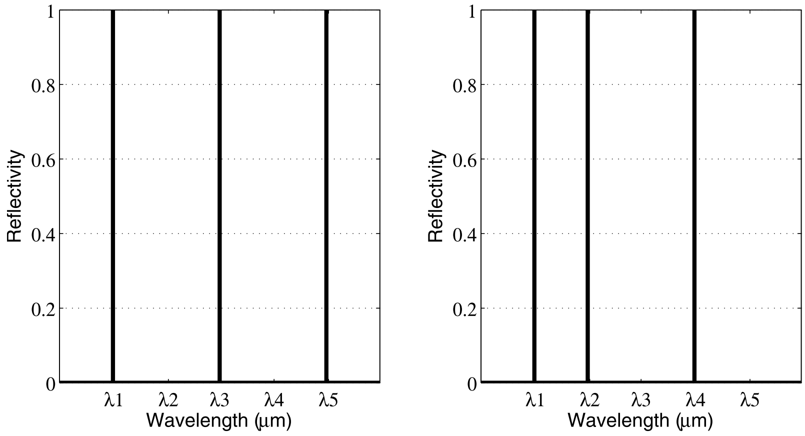


Fig. 2 Example of spectra representing the CDMA binary codes '10101' (*left*) and '11010' (*right*), when $N_{\mathrm{code}} = 5$

λ_i is equal to $c(i)$, for $i = 1, \ldots, N_{\mathrm{code}}$, and zero elsewhere. For instance, assuming $N_{\mathrm{code}} = 5$, the spectrum showed in Fig. 2(left) represents the code '10101' whereas the one depicted by Fig. 2(right) corresponds to the code '11010'.

One way to generate such a spectrum is to consider CDMA filters composed of SFBGs (Fang et al. 2003; Yin and Richardson 2009). As said in Sect. 1, SFBGs are optical fibers with a periodic perturbed effective refractive index that reflect predetermined wavelengths and let other wavelengths pass. When applied to CDMA filters, their objective (obtained by combining the effect of various SFBGs) is to reflect the spectrum Λ_c corresponding to the CDMA code c of a particular user.

In this work, we are interested in designing, by considering an optimization approach, an SFBG that reflects the whole spectrum associated to a CDMA code. The advantages of such an approach in comparison to other ones (for instance, combining various SFBGs) is the reduction of time and cost of CDMA filter fabrication (Pille 2005).

2.2 Previous works on SFBG optimization

Nowadays, the range of industrial applications of SFBGs is wide (Hill and Meltz 1997). For instance, they can be used as: dispersion compensation (Williams et al. 1994), sensors (Kersey 1996), network monitoring (Chan et al. 1997) or wavelength selective devices (as in this article).

Considering this last domain, the SFBG can be employed as a pass-band filter (Tremblay et al. 2005) or a multichannel filter (Chow et al. 1996; Wei et al. 2000). There is therefore an important demand for optimization methods generating SFBGs giving a desired reflectivity spectrum (Rothenberg et al. 2002). Furthermore, those method need to perform global optimization as it is observed that the considered functionals usually have multiple minima (Skaar and Risvik 1998).

In the literature, various optimization algorithms have been applied to solve such problems, considering some SFBG characteristics (explained in Sect. 2.3) as the optimization parameters. For example, Navruz et al. use a Simulated Annealing approach to optimize the phase profile of a Multichannel SFBG (Navruz and Guler 2007), whereas Ngo et al. consider a Tabu Search algorithm to design the index modulation function of a pass-band SFBG (Ngo et al. 2007). However, one of the most used method to solve SFBG design problems are GAs as in the works presented in Cheng et al. (2008), Etezad et al. (2011), Skaar and Risvik (1998) and Tremblay et al. (2005), where various SFBG parameters (such as the fiber length or the refractive index modulation length) are optimized to obtain pass-band filters.

Regarding this previous literature, the optimization algorithms are only presented as a tool for SFBG design without deeper details (e.g., the algorithm parameters, the number of cost function evaluations or the numerical precision of the solution are not given). Moreover, in each article the optimization parameters and the cost function are different. Thus, comparing the results and efficiency of an optimization approach with the ones given in those articles is difficult. For this reason, when we started to study, in collaboration with the company “Alcatel” (http://www.alcatel-lucent.com) and the “Département Photoniques et Ondes” of the Université Montpellier 2 (http://www.ies.univ-montp2.fr), SBFG design problems by using our own optimization method (presented in Sect. 3), we decided to compare the obtained results with those provided by our own GA (as it is the most used method in this domain). We point out that we have previously applied this methodology to the design of optical fibers simpler (regarding the fiber structure and reflected spectra) than the ones considered in this work (Ivorra 2006; Ivorra et al. 2006b). We obtained better results with our algorithms than with those given by GAs (in terms of number of functional evaluations and result precision). The work presented here is a continuation of that previous paper.

2.3 SFBG reflected spectrum evaluation model

In order to design the SFBG described in Sect. 2.1, we need to introduce the mathematical model that allows us to compute its reflected spectrum.

We consider an SFBG of length L (mm). We assume that for any wavelength λ (μm), in a considered transmission band $[\lambda_{\min}, \lambda_{\max}]$, sent in the SFBG, part of this wavelength is transmitted through the fiber with a varying amplitude $T(z,\lambda)$ at the position $z \in [0, L]$ in the fiber and other part is reflected with a varying amplitude $R(z,\lambda)$. Furthermore, for each $\lambda \in [\lambda_{\min}, \lambda_{\max}]$ and each $z \in [0, L]$, we have that $T(z,\lambda)$ and $R(z,\lambda)$ are coupled through the following equations (Erdogan 1997; Ivorra et al. 2006b):

$$\begin{cases} \dfrac{\mathrm{d}T}{\mathrm{d}z}(z,\lambda) = i\left(2\pi\dfrac{n_{\mathrm{eff}}+\delta n_{\mathrm{eff}}(z)}{\lambda} - \dfrac{\pi}{\Theta}\right)T(z,\lambda) + i\dfrac{\pi\,\delta n_{\mathrm{eff}}(z)}{\lambda}R(z,\lambda), \\ \dfrac{\mathrm{d}R}{\mathrm{d}z}(z,\lambda) = -i\left(2\pi\dfrac{n_{\mathrm{eff}}+\delta n_{\mathrm{eff}}(z)}{\lambda} - \dfrac{\pi}{\Theta}\right)R(z,\lambda) - i\dfrac{\pi\,\delta n_{\mathrm{eff}}(z)}{\lambda}T(z,\lambda), \\ T(0,\lambda) = 1, \\ R(L,\lambda) = 0, \end{cases} \tag{1}$$

where n_{eff} is the unperturbed effective refractive index; Θ is the nominal grating period (μm); and $\delta n_{\mathrm{eff}}(z)$ is the slowly varying index amplitude change over the grating (also called, in the following, *apodization*) which is periodic.

System (1) is solved numerically by using a simplified transfer matrix method (Baxter 1982).

Finally, the *reflected spectrum*, denoted by r, of the considered SFBG is defined as:

$$r(\lambda) = \left|R(0,\lambda)\right|^2. \tag{2}$$

2.4 Inverse problem formulation

We consider that the code c_1^A, introduced in Sect. 2.1, is represented by the following set of wavelengths:

$$\Lambda_A^1 = \{\lambda_i \mid i = 1, \ldots, N,\ N \le N_{\mathrm{code}},\ \lambda_i \in \Lambda\}. \tag{3}$$

Due to the fact that the considered SFBG introduced in Sect. 2.3 can only generate symmetric reflected spectra (Gemzický and Müllerová 2008), we want to design an SFBG that reflects Λ_A^1 and also the symmetrical wavelengths of Λ_A^1 centered around a predefined wavelength λ_c. This new set of wavelengths is denoted by Λ_{A,λ_c}^1.

This problem can be reformulated considering that each SFBG can be characterized by its apodization profile $\delta n_{\mathrm{eff}}(z)$. By denoting Ω_{apo} the search space of all admissible apodization profiles, we define a cost function h_0, to be minimized on Ω_{apo}, by:

$$h_0(x) = \int_{[\lambda_{\min},\lambda_{\max}]} \left(r_x(\lambda) - r_{\mathrm{t}}(\lambda)\right)^2 \mathrm{d}\lambda, \tag{4}$$

where $r_x(.)$ is the reflected spectrum (2) of the SFBG with an apodization profile associated to $x \in \Omega_{\mathrm{apo}}$ and r_{t} is the target reflected spectrum given by:

$$r_{\mathrm{t}}(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \Lambda_{A,\lambda_c}^1, \\ 0 & \text{elsewhere.} \end{cases} \tag{5}$$

We must include some restrictions on Ω_{apo} in order to find an SFBG with an apodization profile with suitable characteristics for practical realization. Indeed, complex profiles would require high-level and expensive mastering of the writing process (Chuang et al. 2004). In particular, we are interested by admissible profiles which have a low number of π-phase shifts (sign changes), are smooth and have a maximum index variation $\overline{n}_{\max}$ of less than 5×10^{-4} (Pille 2005). Thus, apodization profiles are generated by spline interpolation through a reduced number of N_S points equally distributed along the first half of their periodic pattern and completed by parity and periodicity (Ivorra et al. 2006b). N_S is chosen high enough to ensure enough peaks in the reflected spectra but small enough for the profile to remain admissible (Ivorra 2006).

Thus, the corresponding search space of the optimization problem is:

$$\Omega_{N_S} = [-\overline{n}_{\max}, \overline{n}_{\max}]^{N_S}. \tag{6}$$

The discrete version of the cost function (4) on Ω_{N_S} is defined by:

$$h_{0,N_c}(x) = \sum_{i=1}^{N_c-1} \frac{(\lambda_{i+1}-\lambda_i)}{2}\Big[\big(r_x(\lambda_{i+1})-r_{\mathrm{t}}(\lambda_{i+1})\big)^2+\big(r_x(\lambda_i)-r_{\mathrm{t}}(\lambda_i)\big)^2\Big]. \quad (7)$$

In Eq. (7), the reflected spectrum r_x of the SFBG with an apodization profile associated to $x \in \Omega_{N_S}$ is evaluated on N_c wavelengths equally distributed on the transmission band $[\lambda_{\min}, \lambda_{\max}]$.

Therefore, the SFBG design problem can be reformulated as the following optimization problem:

$$\begin{cases} \text{Find } x_m \in \Omega_{N_S} \text{ such that} \\ h_{0,N_c}(x_m) = \min\limits_{x\in\Omega_{N_S}} h_{0,N_c}(x). \end{cases} \quad (8)$$

In Sect. 3, we introduce various optimization algorithms used to solve Problem (8).

3 Global optimization methods

We consider the following minimization problem:

$$\begin{cases} \text{Find } x_m \in \Omega \text{ such that} \\ h_0(x_m) = \min\limits_{x\in\Omega} h_0(x), \end{cases} \quad (9)$$

where $h_0 : \Omega \to \mathbb{R}$ is the cost function and x is the optimization parameter belonging to a search space $\Omega \subset \mathbb{R}^N$, with $N \in \mathbb{N}$.

In Sects. 3.1 and 3.2, we present different optimization methods that can be applied to solve numerically optimization problems of the form (9), such as Problem (8).

3.1 Genetic algorithms

Genetic algorithms, denoted as **GA**, approximate the solution of (9) through stochastic processes based on an analogy with the Darwinian evolution of species (Forrest 1993).

A first family, called '*population*', $X^0 = \{x_j^0 \in \Omega, j = 1, \dots, N_p\}$ of $N_p \in \mathbb{N}$ possible solutions of the optimization problem, called '*individuals*', is randomly generated in Ω.

Starting from this population, we build recursively $N_g \in \mathbb{N}$ new populations $X^{i+1} = \{x_j^{i+1} \in \Omega, j = 1, \dots, N_p\}$ with $i = 0, \dots, N_g - 1$, called '*generations*', via three main steps:

Step 1 **Selection:** Each individual, x_j^i, $j = 1, \dots, N_p$ is ranked with respect to its cost function value $h_0(x_j^i)$ (i.e., the lower is $h_0(x_j^i)$ the higher is its ranking). Then, N_p individuals are randomly selected to become '*parents*', with a probability depending on the previous ranking (individuals with better ranking have higher chances to be selected) and with eventual repetitions.

Step 2 **Crossover:** This step leads to a data exchange between two parents and the apparition of two new individuals called '*children*'. We determine, with a fixed probability $p_c \in [0, 1]$, if two consecutive parents should exchange data (the created children are projected in Ω) or if they are directly copied into the new population.

Step 3 **Mutation:** This step leads to new parameter values for some individuals of the population. For each individual, we determine with a fixed probability $p_m \in [0, 1]$ if it is randomly perturbed (the perturbed individual is projected in Ω) or not.

With these three basic evolution processes, it is generally observed that the best obtained individual is getting closer after each generation to the optimal solution of the problem (Goldberg 1989).

At the end of the algorithm, after N_g iterations, the GA returns an output denoted by $A_0(X^0; N_p, N_g, p_m, p_c) = \text{argmin}\{h_0(x^i_j)/x^i_j \in X^i, \ i = 1, \dots, N_g, \ j = 1, \dots, N_p\}$.

These algorithms do not require sensitivity computation, perform global and multi-objective optimization and are easy to parallelize. However, their drawbacks remain their computational complexity, their slow convergence and their lack of accuracy. Since a fine convergence is difficult to achieve with GAs, it is recommended when it is possible, to complete the GA iterations by a descent method (Muyl et al. 2004).

A complete description of the GA considered during tis work can be found in the following literature (Ivorra 2006; Ivorra et al. 2009).

3.2 Multi-layers semi-deterministic global optimization method

3.2.1 General description of the method

We consider an optimization algorithm $A_0 : V \to \Omega$, called core optimization algorithm (COA), to solve (9).

We assume the existence of a suitable initial condition $v \in V$ such that the output returned by $A_0(v)$ approaches a solution of (9). In this case, solving numerically (9) with the considered COA can be formulated as:

$$\begin{cases} \text{Find } v \in V \text{ such that} \\ v \in \underset{w \in V}{\text{argmin}}\, h_0\big(A_0(w)\big). \end{cases} \tag{10}$$

In order to solve (10), we propose to use a I-layer semi-deterministic algorithm $A_I : V \to V$, with $I \in \mathbb{N}$, based on line search methods (see, for instance, Mohammadi and Hervé Saiac 2003) called here, for the sake of simplicity, 'Semi-Deterministic Algorithm' (**SDA**) and built recursively as following:

For $i = 1, 2, \dots, I$, we introduce $h_i : V \to \mathbb{R}$ by

$$h_i(v) = h_{i-1}\big(A_{i-1}(v)\big), \tag{11}$$

and we consider the problem:

$$\begin{cases} \text{Find } v \in V \text{ such that} \\ v \in \operatorname*{argmin}_{w \in V} h_i(w). \end{cases} \tag{12}$$

Problem (12) is equivalent to (10) and is solved by using the algorithm $A_i : V \to V$ that, for each $v_1 \in V$, returns an output given by

Step 1 Choose v_2 randomly in V.
Step 2 Find $v \in \operatorname{argmin}_{w \in \mathcal{O}_i(v_1,v_2)} h_i(w)$, where $\mathcal{O}_i(v_1, v_2) = \{v_1 + t(v_2 - v_1), t \in \mathbb{R}\} \cap V$, using a line search method.
Step 3 Return v.

The line search minimization algorithm in Step 2 is defined by the user.

When $I > 1$, due to the fact that line search directions $\mathcal{O}_i(v_1, v_2)$ in A_i, for $i = 1, \ldots, I-1$, are constructed randomly, the algorithm A_I perform a multi-directional search of the solution of (10).

In Sects. 3.2.2 and 3.2.3, we present two particular implementations of the SDA, considering steepest descent and genetic algorithms as COAs in the case where h_0 is a non negative function (such as the function (7) considered in this work).

A more detailed description (such as, the choice of the parameters, validation on benchmark cases, etc.) of this method and those two particular implementations can be found in (Ivorra 2006; Ivorra et al. 2007). Furthermore, various applications to industrial problems are presented in (Debiane et al. 2006; Carrasco et al. 2012; Gomez et al. 2011; Hertzog et al. 2006; Isebe et al. 2008; Ivorra et al. 2006a, 2006b, 2009).

A Matlab© version of this method is included in the free optimization package *Global Optimization Platform* which can be downloaded at

`http://www.mat.ucm.es/momat/software.htm`

3.2.2 SDA implementation with steepest descent COAs

We consider COAs that come from the discretization of the following initial value problem (Mohammadi and Hervé Saiac 2003):

$$\begin{cases} M\big(x(t), t\big)\dfrac{\mathrm{d}x}{\mathrm{d}t}(t) = -d\big(x(t)\big), \quad t \geq 0, \\ x(0) = x_0, \end{cases} \tag{13}$$

where t is a fictitious time, $M : \Omega \times \mathbb{R} \to M_{N\times N}$ (where $M_{N\times N}$ denotes the set of matrix $N \times N$) and $d : \Omega \to \mathbb{R}^N$ is a function giving a descent direction. For example, assuming $h_0 \in C^1(\Omega, \mathbb{R})$, if $d = \nabla h_0$ and $M(x,t) = Id$ (the identity operator) for all $(x,t) \in \Omega \times \mathbb{R}$ we recover the steepest descent method.

According to previous notations, we use $V = \Omega$ and denote by $A_0(x_0) := A_0(x_0; t_0, \epsilon)$ the solution returned by the COA starting from the initial point $x_0 \in \Omega$ after $t_0 \in \mathbb{N}$ iterations and considering a stopping criterion defined by $\epsilon \in \mathbb{R}$. In this case, Problem (10) can be rewritten as:

$$\begin{cases} \text{Find } v \in \Omega \text{ such that} \\ v \in \underset{w \in \Omega}{\operatorname{argmin}}\, h_0\Big(A_0(w)\Big) \end{cases} \tag{14}$$

We consider a particular implementation of the algorithms A_i, $i = 1, \dots, I$, introduced previously, to solve (14). For $i = 1, \dots, I$, $A_i(v_1)$ is applied with a secant method (a low-cost method well adapted to find the zeros of a function Mohammadi and Hervé Saiac 2003) in order to perform the line search. It reads:

Step 1 Choose $v_2 \in \Omega$ randomly.
Step 2 For l from 1 to $t_{l_i} \in \mathbb{N}$ execute:

Step 2.1 If $h_i(v_l) = h_i(v_{l+1})$ go to **Step 3**.
Step 2.2 Set $v_{l+2} = \operatorname{proj}_\Omega(v_{l+1} - h_i(v_{l+1})\frac{v_{l+1}-v_l}{h_i(v_{l+1})-h_i(v_l)})$ where $\operatorname{proj}_\Omega : \mathbb{R}^N \to \Omega$ is a projection algorithm over Ω defined by the user.

Step 3 Return the output: $\operatorname{argmin}\{h_i(v_m), m = 1, \dots, t_{l_i}\}$.

In the previous algorithm, the value of t_{l_i}, for $i = 1, 2, \dots$, depends on the desired computational complexity.

This algorithm is denoted by **SDDA**.

3.2.3 SDA implementation with genetic COAs

When a GA, described in Sect. 3.1, is used as the COA, Problem (10) can be rewritten as:

$$\begin{cases} \text{Find } X^0 \in V = \Omega^{N_p} \text{ such that} \\ X^0 \in \underset{w \in \Omega^{N_p}}{\operatorname{argmin}}\, h_0\big(A_0(w)\big) \end{cases} \tag{15}$$

where $A_0(X^0) := A_0(X^0; N_p, N_g, p_m, p_c)$ with N_p, N_g, p_m, p_c parameters of the GA that here are considered fixed.

The solution of (15) may be determined, for instance, by using the SDA implementation presented in Sect. 3.2.1. However, a first numerical study (see Ivorra 2006 for more details) shows that the following variation of previous algorithms A_i (with $i = 1, \dots, I$) is better adapted to the GA case. Let $X_1^0 = \{x_{1,j}^0 \in \Omega, j = 1, \dots, N_p\}$. Then $A_i(X_1^0)$ reads:

Step 1 For l going from 1 to $t_{l_i} \in \mathbb{N}$ execute:

Step 1.1 Set $o_l = \operatorname{argmin}\{h_0(x) : x \in A_{i-1}(X_l^0)\}$.
Step 1.2 We construct $X_{l+1}^0 = \{x_{l+1,j}^0 \in \Omega, j = 1, \dots, N_p\}$ as following: $\forall j \in \{1, \dots, N_p\}$, if $h_0(o_l) = h_0(x_{l,j}^0)$ set $x_{l+1,j}^0 = x_{l,j}^0$ else set $x_{l+1,j}^0 = \operatorname{proj}_{\Omega^{N_p}}(x_{l,j}^0 - h_0(o_l)\frac{o_l - x_{l,j}^0}{h_0(o_l)-h_0(x_{l,j}^0)})$ where $\operatorname{proj}_{\Omega^{N_p}} : \mathbb{R}^{N\times N_p} \to \Omega^{N_p}$ is a projection algorithm over Ω^{N_p} defined by the user.

Step 2 Return the output: $\operatorname{argmin}\{h_i(X_m^0), m = 1, \ldots, t_{l_i}\}$.

As previously, the value of t_{l_i}, for $i = 1, 2, \ldots$, depends on the desired computational complexity.

This version of the algorithm intends to optimize, individual by individual, the initial population of A_{i-1}. For each individual in the initial population:

– If there is a significant evolution of the cost function value between this individual and the best element found by A_{i-1}, the secant method used in Step 1.2 generates, in the optimized initial population, a new individual closer to this best element.
– If not, the secant method allows to create a new individual far from the current solution given by A_{i-1}.

Numerical experiments show that this coupling reduces the computational complexity of GAs (Gomez et al. 2011; Ivorra 2006; Ivorra et al. 2006b, 2006a, 2009). In particular, this allows to consider smaller N_p and N_g numbers, compared with the case of applying GA alone. This algorithm is denoted by (**SDGA**).

4 Numerical test

In this section, in order to illustrate the efficiency (i.e., computational time and result precision) of our optimization method to solve the SFBG design in comparison to the GA approach, we apply the optimization algorithms presented in Sect. 3 to solve a particular implementation of Problem (8) based on a realistic case. First, in Sects. 4.1 and 4.2, we introduce the parameters associated to the considered SFBG and optimization algorithms, respectively. Finally, in Sect. 4.3 we present and analyze the obtained results.

In this section, we present the solutions found for one particular numerical case and we consider that they are representative of the general behavior of our algorithms on this kind of problem. Indeed, other numerical examples, presented in part in (Ivorra 2006), have produced similar results.

4.1 SFBG characteristics

We consider a CDMA binary codification of length $N_{\text{code}} = 8$ represented by the set of wavelengths $\Lambda = \{\lambda_1 = 1.5465\ \mu\text{m},\ \lambda_2 = 1.5473\ \mu\text{m},\ \lambda_3 = 1.5481\ \mu\text{m},\ \lambda_4 = 1.5489\ \mu\text{m},\ \lambda_5 = 1.5497\ \mu\text{m},\ \lambda_6 = 1.5505\ \mu\text{m},\ \lambda_7 = 1.5513\ \mu\text{m}, \lambda_8 = 1.5521\ \mu\text{m}\}$.

In this case, the particular code c_1^A = ‘10110011’ is characterized by $\Lambda_A^1 = [\lambda_1, \lambda_3, \lambda_4, \lambda_7, \lambda_8]$. We are interested in designing an SFBG that reflects Λ^1_{A,λ_c} with $\lambda_c = 1.5525\ \mu\text{m}$. This spectrum is depicted by Fig. 3.

The SFBG characteristics are set to $n_{\text{eff}} = 1.45$, $L = 100$ mm, $\Theta = 0.53\ \mu\text{m}$ and the period of δ_{neff} is 1.039 mm. Those values correspond to real optical fiber data provided by our collaborators from “Alcatel” and the “Département Photoniques et Ondes” (Pille 2005). Furthermore, the SFBG apodization profiles are generated by $N_S = 9$ interpolation points with $\overline{n}_{\max} = 5 \times 10^{-4}$ and the functional (7) is evaluated considering $N_c = 1200$ wavelengths in the transmission band [1.545 μm, 1.56 μm]. This set of parameters give good results (apodization profiles easy to implement)

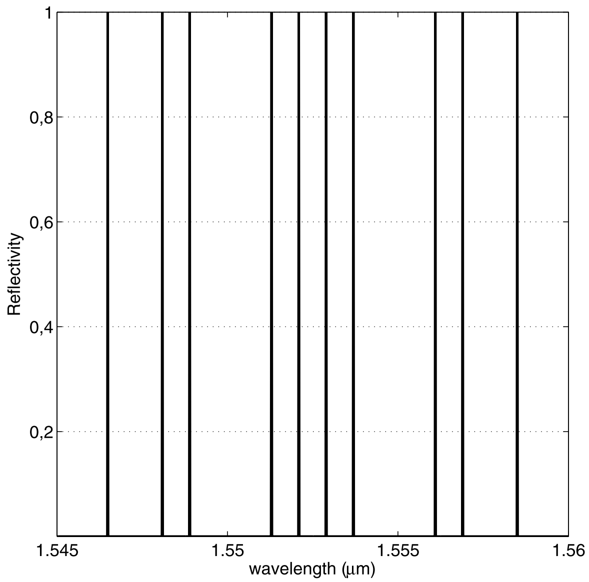


Fig. 3 Target reflected spectrum of the SFBG presented in Sect. 4.1

considering the problem of designing a multichannel optical filter of 16 peaks (Ivorra et al. 2006b).

4.2 Parameters of the optimization algorithms

In order to solve problem (8), considering the SFBG characteristics given in Sect. 4.1, we use the optimization methods presented in Sect. 3 (i.e., GA, SDDA and SDGA) with the following parameters:

- For SDDA: We use a two-layer algorithm (i.e., $I = 2$) with $t_0 = 10$, $t_{l_1} = 5$, $t_{l_2} = 5$ and $\epsilon = 0$. The initial point v_1 for A_2 is generated randomly in Ω_{N_S}. We consider $t_0 = 10$ iterations of the steepest descent algorithm (Mohammadi and Hervé Saiac 2003), which is used as the COA. The gradient of h_{0,N_c} used in A_0 is approximated considering a finite difference method.
- For SDGA: we use a one-layer algorithm (i.e., $I = 1$) with $t_{l_1} = 25$. The parameters considered for the GA, which is used as the COA, are the following:
  - The generation number and population size are set to $N_g = 10$ and $N_p = 10$, respectively.
  - The selection is a roulette wheel type (Goldberg 1989) proportional to the rank of the individuals in the population.
  - The crossover is barycentric in each coordinate with a probability of $p_c = 0.45$.
  - The mutation process is non-uniform with a probability of $p_m = 0.35$.
  - A one-elitism principle, that consists in keeping the current best individual in the next generation, has also been imposed.
  - 10 iterations of the steepest descent method are performed at the end of the SDGA starting from the obtained solution.

Table 1 Numerical results obtained considering (left) SDDA, SDGA and GA optimization methods, (center) value of h_{0,N_c} of the best element found by the optimization algorithm, (right) Number of evaluation of the function h_{0,N_c} needed by the optimization algorithm

Optimization method	Final cost function value	Evaluation number
SDDA	2.09	3000
SDGA	2.31	2700
GA	2.37	5600

– For GA: We use the same stochastic processes than SDGA but with a different set of parameters: $N_g = 30$, $N_p = 180$, $p_c = 0.35$, $p_m = 0.15$. 10 iterations of the steepest descent method are performed at the end of the GA starting from the obtained solution.

SDDA, SDGA and GA applied with those sets of parameters have been validated on various benchmark test cases (Ivorra 2006; Ivorra et al. 2007) and industrial applications (Debiane et al. 2006; Carrasco et al. 2012; Gomez et al. 2011; Isebe et al. 2008; Ivorra et al. 2006a, 2009), in particular on the design of pass-band and multi-channel optical filters (Ivorra et al. 2006b).

4.3 Results and discussion

Figure 4 shows the apodization profiles obtained with SDDA, SDGA and GA and Fig. 5 shows their associated reflected spectra. The convergence histories of each optimization process are presented in Fig. 6. Results reported in this section are summarized in Table 1.

For SDDA, the final value of the cost function h_{0,N_c} is equal to 2.09. The total number of functional evaluations is about 3000. SDDA optimization takes approximatively 10 hours real time in a 3.4 GHz PC computer with 1 Gb Memory.

For SDGA, the final cost function h_{0,N_c} is equal to 2.31. The total number of functional evaluations is about 2700. SDGA optimization takes 9 hours.

For GA, the final cost function h_{0,N_c} is equal to 2.38. The total number of functional evaluations is about 5600. GA optimization takes 18 hours 40 minutes.

The three optimized apodization profiles have different shapes and are situated in distinct attraction basins of the function h_{0,N_c}. This points out the fact that h_{0,N_c} is highly non convex and the optimization problem (8) difficult to solve. This is confirmed by observing the convergence history of the SDDA, which shows that, during the optimization process, the steepest descent algorithm has visited various attraction basins and found different local minima.

From a numerical points of view, both SDDA and SDGA have found better results than GA and are less time consuming.

From an implementation point of view, all optimized apodization profiles present interesting characteristics:

– As we can observe on Figs. 3 and 5, the reflected spectra associated to the optimized profiles correspond to good approximations of the target reflected spectrum.

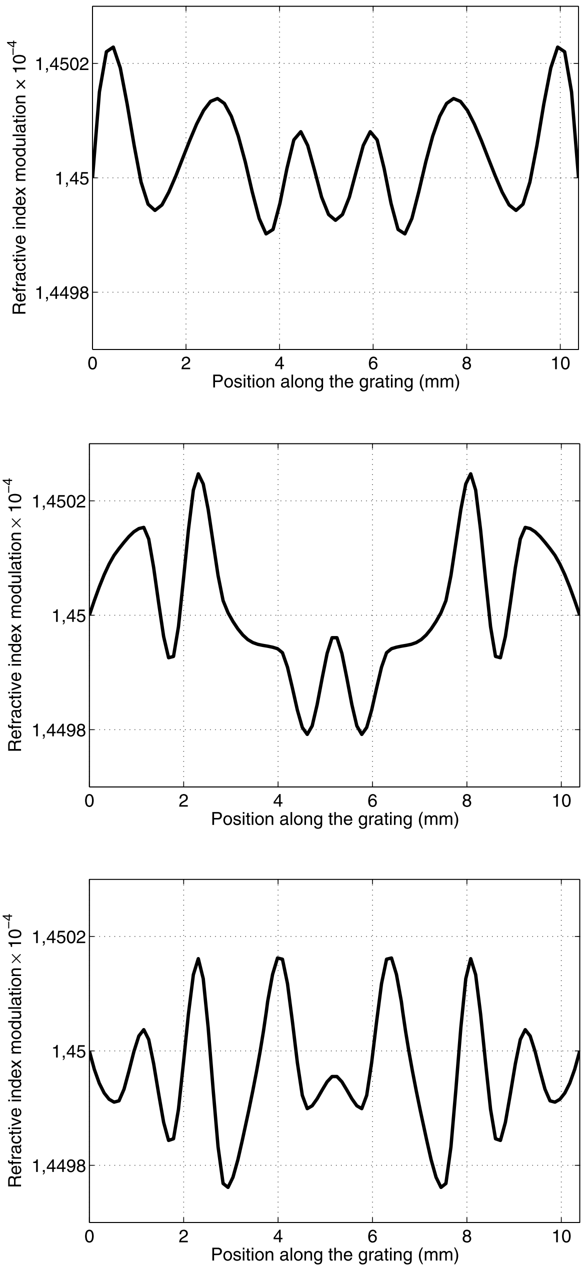


Fig. 4 Optimized apodization profiles obtained by SDDA (*up*), SDGA (*middle*) and GA (*bottom*) optimization methods when solving the SFBG design problem presented in Sect. 4

– The optimized apodization profiles (see Fig. 4) are suitable for practical implementation. Indeed, the number of necessary π-phase shifts is 5 (a number easy to implement), the index modulation of the profile is uniformly distributed along the pattern and the maximum index variation of the profile is inferior to 3×10^{-4}, which is a reasonable level (Ivorra et al. 2006b; Pille 2005).

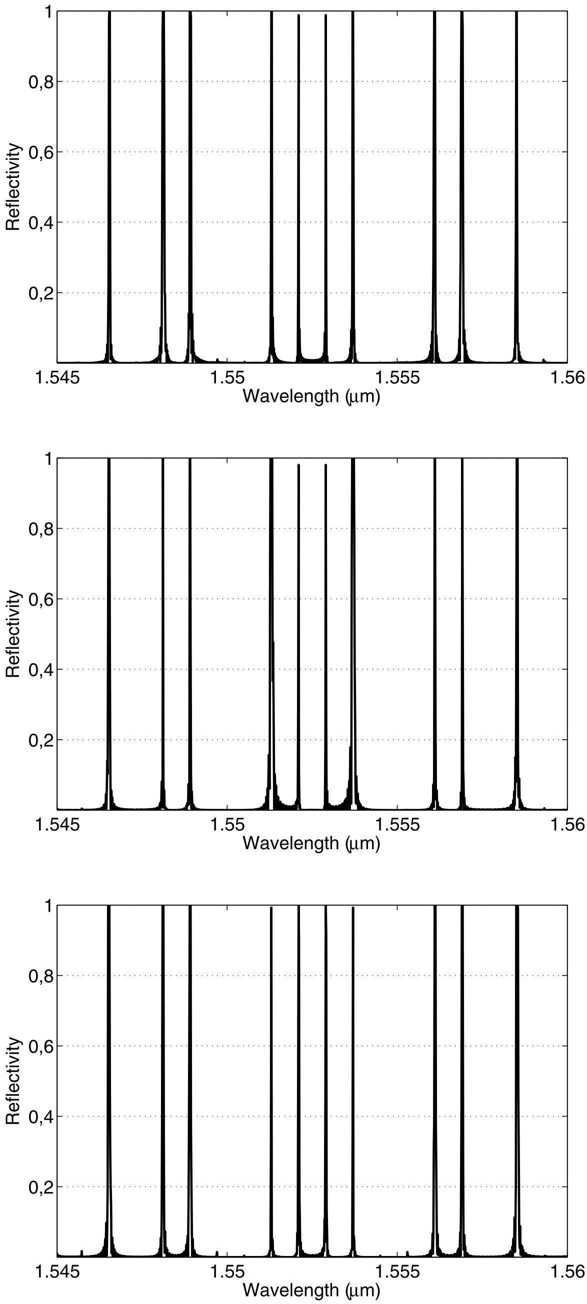
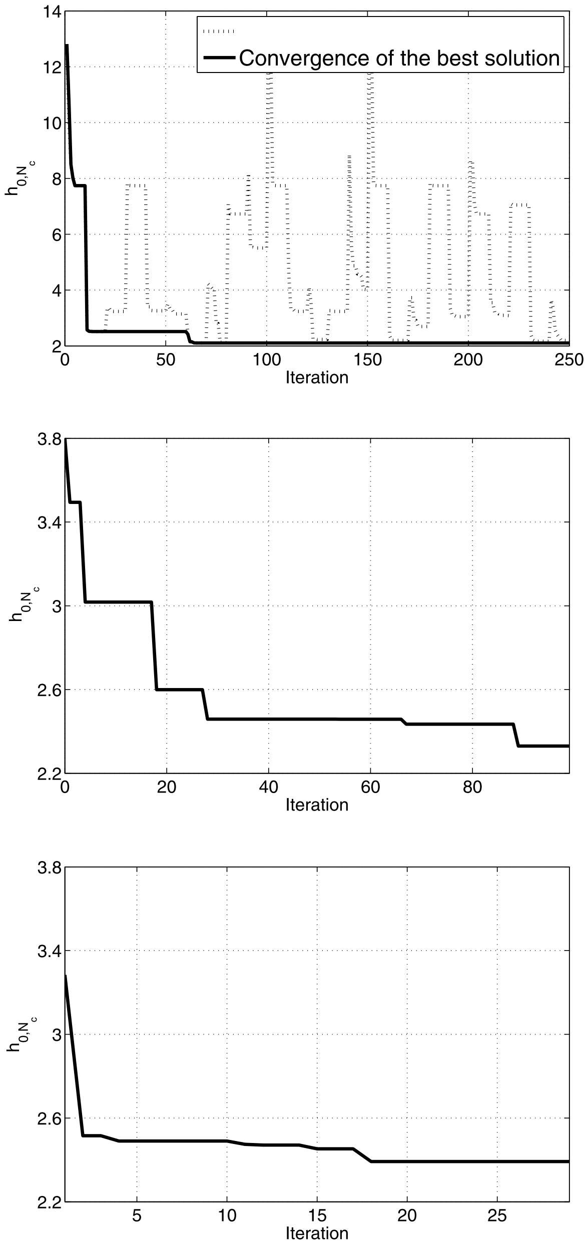


Fig. 5 Reflected spectrum associated with the optimized apodization profiles obtained by the SDDA (*up*), SDGA (*middle*) and GA (*bottom*) optimization methods when solving the SFBG design problem presented in Sect. 4

– A stability analysis on the reflected spectra, when applying a random perturbation of 10 % on the optimized apodization profiles, show that all optimized results have a small variation of ≈4.3 % on their reflected spectrum (considering a L^2 norm).

Fig. 6 Convergence histories obtained when solving the SFBG design problem presented in Sect. 4: (*up*) best element convergence history vs. iterations (—) and global convergence history vs. iterations (. . .) for SDDA; (*middle*) best element convergence history vs. iterations for SDGA; and (*bottom*) GA



This is important because, due to technical limitations, small perturbations could appear in the apodization profile during the writing process.

5 Conclusion

A particular code division multiple access filter based on sampled fiber Bragg grating has been designed using three particular optimization algorithms: two original semi-deterministic algorithms (SDDA and SDGA) and a genetic algorithm (GA) (considered as a classical method to design SFBG). The apodization profiles produced by those optimization approaches exhibit good characteristics for practical implementation because they have no step variation, low maximum index modulation values and small numbers of π-phase shifts. Also, their associated reflected spectrum are weakly sensitive to perturbations. However, SDDA and SDGA have produced better solutions (in term of cost function value) and need less computational time (approximatively a half) than GA alone.

A next step, could be the study of the effect of combined apodization and phase profiles optimization (Rothenberg et al. 2002) in order to avoid the symmetry in spectra mentioned previously. In fact, during this work, we have been interested only by apodization optimization to keep a grating easy to implement by any optical laboratory (Chuang et al. 2004; Malo et al. 1995). Indeed, phase variation requires more complex and expansive materials.

We finally recall that a Matlab© version of the algorithms presented in this paper are included in the free optimization package *Global Optimization Platform*, which can be downloaded

`http://www.mat.ucm.es/momat/software.htm`

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