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# Roll Angle Observer-Based Controller Design for Powered Two-Wheelers

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## ABSTRACT

The present work deals with the trajectory tracking for an unmanned motorcycle using observer-based-controller design. The nonlinear parameter varying system used for the development is converted to a Takagi-Sugeno (TS) fuzzy system. The design of the observer-based controller is based on the Lyapunov approach and the Linear Matrix Inequality (LMI) formulation. Sufficient conditions are derived to prove the Input to State Stability

(ISS) property and trajectory error convergence of the closed-loop system. We suggest ensuring a best profile tracking of the PTW in terms of roll preview (trajectory) for large range of longitudinal velocities. To improve the time response of the controller, a predictive generator is inserted. Finally, simulation results are provided to confirm the suitability of the proposed technique.

**Keywords:** Powered Two-Wheelers, Observer-Based Controller, TS-Fuzzy Systems, Predictive Control.

## 1 INTRODUCTION

Powered Two-Wheelers (PTW) are well known to be extremely hard to control. They exhibit unstable dynamics, under-actuation, counter-steering (non-minimum phase behavior) and speed dependent weave and wobble modes [1]. Moreover, some important signals are not always available such as the roll angle and the steering torque. According to these features, the control of PTW is still a challenging task. To our knowledge, the study proposed in [2] were probably the first to stabilize a motorcycle using nonlinear equations

of motion derived from a highly simplified model. Some control techniques are given in [3, 4, 5] for highly simplified motorcycle models or for specific velocities. In [6, 7, 8], control techniques are considered with more complex and realistic models but with the assumption of availability of all state vector and for the former by using gain scheduling techniques without any proof of the stability in all riding conditions. Furthermore, sparsely works deal with the estimation of the PTW vehicles dynamics. The estimate of the roll angle is proposed in [9, 10]. For the former a low-cost sensor configuration is given. In [10], an observer is given but with the neglect of the steering dynamics. The observation of steering angle is proposed in [11] with scheduling gains while the estimation convergence guarantee is missing. More recently, a higher order sliding mode observer is proposed to estimate the lateral dynamics and the steering torque in [12], but for only a specific forward velocity and its efficiency is not ensured for large variations of velocity. In [13], a nonlinear unknown input observer is proposed. The observer's convergence is studied using Lyapunov theory and Input-to-State Practical Stability property. Our interest, in this context, is the development of a trajectory tracking control strategy in order to automate an unmanned motorcycle. This motorcycle will be used, in future works, as a prototype for the validation of active or semi-active safety systems. In many works, the trajectory tracking of PTW is converted to a two-layer controller [14]: an external controller based on a simplified model which generates reference signals (i.e. roll angle and longitudinal velocity), and an internal controller based on a more accurate model to track the reference signals. In our work, we are interested by the lateral dynamics internal controller and it is assumed that we have an optimal roll angle reference (the longitudinal controller is not of our inter-

est in this work). Moreover and to perform the time response of the proposed controller, the roll reference generator is modified by adding a predictive bloc. This allows us to anticipate the dynamics of the closed-loop system and offers a more realistic description of the rider behavior. The paper is organized as follows: in section II, we present the motorcycle model used in this work. The observer and the controller are developed in section III. The predictive approach is presented in section IV. The simulation results of this work are given in section V. We finish by the conclusion and the appendix.

## 2 MOTORCYCLE DYNAMICS

### 2.1 Nonlinear model

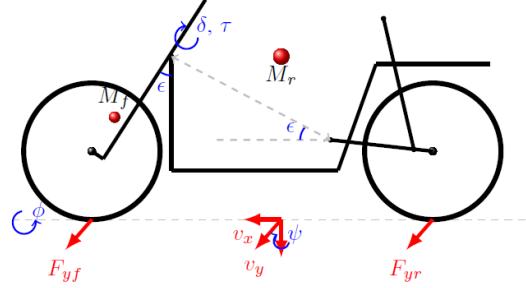
In the literature, many works addressed the modeling of single track vehicles. The first study as reported by Whipple goes back to 1899 [15]. In 1971, a reference study was undertaken by Sharp [16]. He studied the motorcycle's stability and sensitivity to geometric and tire parameters. In the last years, more accurate nonlinear models were given in [17, 18].

In general, two important dynamical characteristics can be observed, namely, in-plane modes and out-of-plane modes. The in-plane modes includes longitudinal, vertical and pitch motions. The principal out-of-plane modes are the weave and the wobble. They include lateral, roll, yaw and steering motions. In this paper, the out-of-plane dynamics are represented by Sharp's 1971 model. The motorcycle is considered as a set of two rigid frames joined at the steering axis with freedom, restrained by a linear steering damper (fig. 1). The front frame  $M_f$  consists of the front tire, fork, handlebar and fittings. The rear frame  $M_r$  consists of the main structure, the engine gearbox assembly, the petrol tank, seat, rear forks, rear wheel, etc. Equations of motion are expressed by the following state-space representation. (See appendix for variable definition).

$$\begin{cases} \dot{x}_v(t) = A_v(v_x, x)x_v(t) + B_v(t)\tau(t) \\ y(t) = Cx_v(t) + Dw(t) \end{cases} \quad (1)$$

where  $x_v = [\phi, \delta, v_y, \psi, \dot{\phi}, \dot{\delta}, F_{yf}, F_{yr}]$  is the state vector,  $\tau$  is the control torque applied to

the handlebar,  $A_v$  is the state matrix and it is function of the roll angle and the longitudinal



**Figure 1.** Geometrical representation of the Sharp's motorcycle model

velocity  $v_x, y$  is the measurement vector subject to measurement noises  $w$ . It contains steer angle, steer rate, roll rate and yaw rate.

### 2.2 Equivalent TS-fuzzy model

From the nonlinear model given in (1), the following nonlinearities are considered:

$$z_1 = v_x, \quad z_2 = \frac{\sin \phi}{\phi}$$

where  $v_x$  is considered as an external parameter time-varying. It varies from  $v_{min}$  to  $v_{max}$ . The roll angle  $\phi$  is considered varying from  $-\phi_{max}$  to  $\phi_{max}$ .

In this study, some assumptions were made and they are justified. The steering angle is supposed to have small values (it does not exceed  $10^\circ$ ). Also, the lateral pneumatic forces are modeled by linear models. Of course, the pneumatic forces have saturation limits [19], but we have restrict the lateral forces to such a way that they not exceed this limits with additional constraints on the control law (details are given in section 3).

Without loss of generality, the motorcycle model can be rewritten as:

$$\dot{x}_v(t) = \sum_{i=1}^4 \mu_i(v_x, \phi) A_{vi}x_v(t) + B_v(t)\tau(t) \quad (2)$$

One of the objectives of this work is to force the roll angle to track a desired reference roll angle. In order to guarantee this objective, we add a new state variable:

$$e_\phi(t) = \int_0^t (\phi(s) - \phi_r(s)) ds \quad (3)$$

where  $\phi_r(t)$  is the reference roll angle to track.

Indeed, we obtain an augmented state space model as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^4 \mu_i(v_x, \phi) A_i x(t) + B_1 \phi_{ref}(t) \\ \quad + B_2(t) \tau(t) \\ y(t) = C_2 x(t) + D w(t) \\ h_1 = C_{11} x(t) \\ h_2 = C_{12} x(t) \end{cases} \quad (4)$$

The two additional vectors  $h_1$  and  $h_2$  are added as sensitivity signals in order to be insensitive to noises in the interesting frequency bandwidth. This point is discussed in more details in [21].

### 3 OBSERVER-BASED CONTROLLER DESIGN

The originality of our work is the estimation of the state vector and at the same time the proof of stability of the controlled vehicle with the estimated states for a wide range of forward velocities. The design is made on two steps. Firstly, the state vector is estimated even with noised measurements. After and based on the measured states, a controller is designed to make the vehicle stable and to track the reference roll angle.

The proposed TS-observer and TS-controller follow PDC concept and are given by:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^4 \mu_i(v_x, \hat{\phi}) (A_i \hat{x}(t) \\ + L_i (y(t) - \hat{y}(t))) + B_1 \phi_{ref}(t) + B_2(t) \tau(t) \\ \hat{y}(t) = C_2 \hat{x}(t) \\ \tau(t) = - \sum_{i=1}^4 \mu_i(v_x, \hat{\phi}) K_i \hat{x}(t) \end{cases} \quad (5)$$

Thus, we propose the following theorem. Its proof is based on the Input to State Stability (ISS) [20] and common Lyapunov functions for the controller and the observer. Using Yalmip toolbox of Matlab, observer and controller gains are calculated in such a way to satisfy the obtained Linear Matrix Inequalities

(LMI's) and to minimize the convergence bound.

Before, we suppose that the following hypotheses are assumed to be satisfied:

#### Hypothesis

- The pairs  $(A_i, C_2)$  are observable,
- The pairs  $(A_i, B_2)$  are controllable,
- The measurement noise signals are centered and bounded,
- The reference roll angle is bounded,

#### Theorem

Under the hypothesis given below and given positive scalars:  $\zeta_1, \zeta_2 \in [0, 1]$ , if there exist symmetric and positive definite matrices  $P_1$  and  $P_2$ , gain matrices  $Y_i$  and  $Z_i$  and positive scalars  $\epsilon_1, \epsilon_2, \epsilon_3$  and  $\epsilon_4$  solutions to the two following optimization problems:

$$\min_{P_1, Z_i, \epsilon_1, \epsilon_2} \zeta_1 \epsilon_1 + (1 - \zeta_1) \epsilon_2 \quad (6)$$

s.t.

$$\begin{pmatrix} \Omega_i & P_1 & -Z_i D & C_{11}^T \\ P_1 & -\epsilon_1 I & 0 & 0 \\ -(Z_i D)^T & 0 & -\epsilon_2 I & 0 \\ C_{11} & 0 & 0 & -I \end{pmatrix} < 0, i = 1, \dots, 4 \quad (7)$$

and

$$\min_{P_2, Y_i, \epsilon_3, \epsilon_4} \zeta_2 \epsilon_3 + (1 - \zeta_2) \epsilon_4 \quad (8)$$

s.t.

$$\begin{pmatrix} \Pi_i & I & B_1 & P_2 C_{12}^T \\ I & -\epsilon_3 I & 0 & 0 \\ B_1^T & 0 & -\epsilon_4 I & 0 \\ C_{12} P_2 & 0 & 0 & -I \end{pmatrix} < 0, i = 1, \dots, 4 \quad (9)$$

where:  $\Omega_i = A_i^T P_1 + P_1 A_i - Z_i C_2 - C_2^T Z_i^T$ ,  $\Pi_i = A_i P_2 + P_2 A_i - B_2 Y_i - Y_i^T B_2^T$  and  $\zeta_1, \zeta_2$  are weighting scalars.

then, the whole observer-based controller system is ISS. The gains of the observer are computed from  $L_i = P_1^{-1} Z_i$  and those of controller from  $K_i = Y_i P_2^{-1}$ . The attenuation level of the transfers from  $r(t)$  and from  $w(t)$  to state vector and to state estimation error is bounded and depends proportionately to scalars  $\epsilon_i$  ( $i = 1, \dots, 4$ ).

## Proof

The proof is based on the ISS property. The details of the proof are given in [21].

### 3.1 Additional constraints

To be consistent with some real constraints affecting the nonlinear dynamics of the motor-cycle and the observer-based controller design, we suggest fulfilling additional constraints. The lateral forces must not exceed their saturation limits. This condition can be fulfilled by additional LMI's given by equations (10) and (11) (see [22] for more details).

$$\begin{bmatrix} 1 & x(0) \\ x(0) & P_2 \end{bmatrix} \geq 0 \quad (10)$$

$$\begin{bmatrix} P_2 & P_2 C_f^T \\ C_f P_2 & Y \end{bmatrix} \geq 0 \quad (11)$$

where:  $C_f x(t) = [F_{yf} \ F_{yr}]^T$  and  $Y$  is a diagonal matrix containing the saturation limits of the lateral forces. The poles of the observer must also be faster than those of the controller. This condition can be fulfilled by additional constraints LMIs on the observer and the controller gains (see [23] for more details). The observer's pole assignment is performed in  $\{z_{obs} | \mathcal{R}(z_{obs}) < -3, |z_{obs}| < 1000\}$  and the controller's poles assignment is performed in  $\{z_{con} | \mathcal{R}(z_{con}) < -1, |z_{con}| < 1000\}$ .

## 4 PREDECTIVE APPROACH

To perform the time response of the controller, we add another layer between the reference generator and the controller. The block diagram of the closed-loop system with the predictive part is shown in figure (fig. 2).

Firstly, we need to identify the closed-loop system by a linear one. Thanks to the controller, the closed-loop system is stable and its poles are clustered in a known region of the complex plane. Thus, we propose to identify the closed-loop system by a second order linearized model. This is done using the System Identification Toolbox of Matlab. The system is considered as a black box with the reference

roll angle as the input signal and the estimated roll angle as the output one. Now, and based on the linearized model, a predictive generator is designed using the Model Predictive Control (MPC) toolbox of Matlab. The MPC toolbox allows us to customize input constraints i.e. reference roll angle in our case. Thus, we design the prediction layer in such a way that the desired roll angle does not exceed  $60^\circ$ . The design of the predictive generator does not affect the stability of the closed-loop system. Its only purpose is to modify the reference roll angle in such a way to anticipate the closed-loop system dynamics and to make the system response faster.

## 5 SIMULATION RESULTS

The observer estimates the lateral dynamics using only the measured states  $\psi$  and  $\phi$  given by the inertial unit and  $\delta$  and  $\dot{\delta}$  obtained by a suitable encoder. The observer-based controller is designed for longitudinal velocities going from 5 to 30m/s and a maximal roll angle of  $45^\circ$ . The initial conditions of the system are  $x(0) = [0.1 \ 0 \ 0.1 \ 0 \ 0 \ 0 \ 10 \ 10]^T$  and those of the observer are  $\hat{x}(0) = [0 \ 0 \ 0 \ 0 \ 0]^T$ . The simulation conditions are given for the corresponding longitudinal velocity (fig. 3) which corresponds to a double lane change maneuver. The vehicle decelerates from 20m/s to 12m/s before reaching the first corner; it remains at a constant speed when turning and accelerates after the second corner.

The results of simulation without the predictive part are given in fig. (4, 5). We see that all the state variables are well estimated except the lateral velocity, but the boundedness of the error estimation is always guaranteed. Moreover, the estimated roll angle converges to the real one and the vehicle track the desired roll angle.

In the same simulation conditions and with the predictive generator, the results of simulation are given in fig. (6,7). We see that the vehicle reacts faster in this case. Of course, the price to pay for fast responses of the vehicle is the high values of the steer torque, so we have a compromise between time response and applied torque.

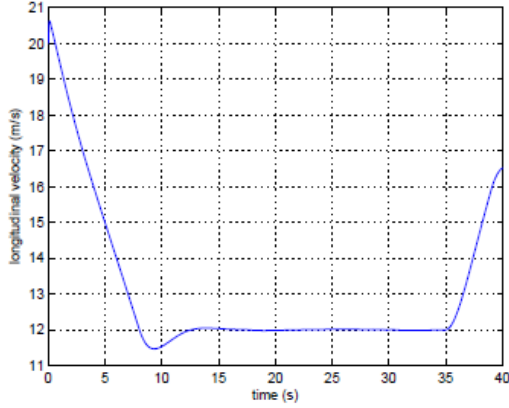
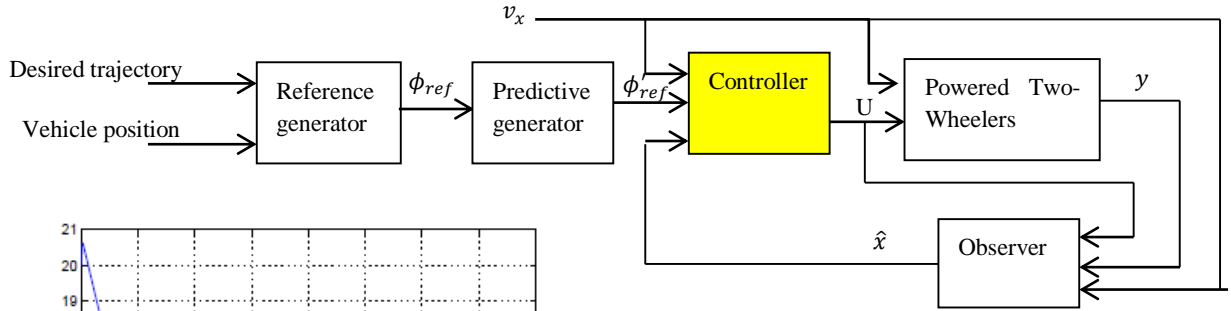


Figure 3. Longitudinal velocity profile

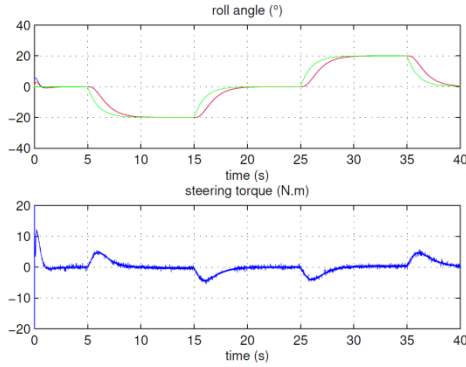


Figure 4. Above: real roll angle (blue) esti-

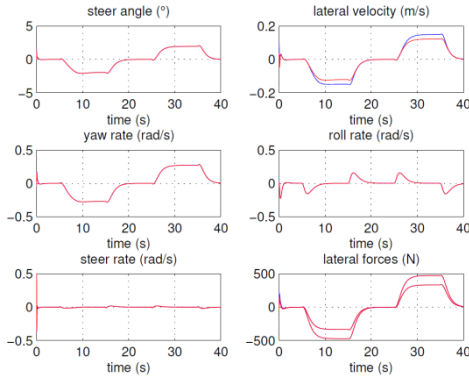


Figure 5. Actual states (blue) and estimated states (red)

to estimate the lateral dynamics of the PTW and to ensure a best profile tracking in terms of roll preview for a large range of longitudinal velocities. The convergence study of the coupled observer-controller is studied using Lyapunov theory. Moreover and from an engineering point of view, to our knowledge, designing a controller or an observer-based controller and proving its stability for a large range of forward velocities has never been

observer-based controller block diagram

addressed for motorcycles. Another specificity of this work is the inclusion of the predictive generator to make the system faster. The generation of the predictive generator requires only the identification toolbox and the MPC toolbox of Matlab. Simulation results are provided to illustrate the effectiveness of the proposed observer-based controller without and with the predictive generator.

## APPENDIX

Motorcycle (numerical values and components of the matrices  $M, E$  from [16])

$v_x, v_y$	longitudinal and lateral velocities
$\phi, \psi, \delta$	roll, yaw and steering rotations
$F_{yf}, F_{yr}$	front and rear side slip force
$\tau$	Rider's torque
$A_v$	state matrix $A_v = M^{-1}E$
$B_v$	input matrix $B_v = M^{-1}[0, 0, 0, 0, 1, 0, 0]^T$
$M$	motorcycle mass matrix
$E$	motorcycle generalized effort vector

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