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# Competition vs. Cooperation: A Game-Theoretic Decision Analysis for MIMO HetNets

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**Abstract**—This paper addresses the problem of competition vs. cooperation in the downlink, between base stations (BSs), of a multiple input multiple output (MIMO) interference, heterogeneous wireless network (HetNet). We present a scenario where a macrocell base station (MBS) and a cochannel femtocell base station (FBS) each simultaneously serving their own user equipment (UE), has to choose to act as individual systems or to cooperate in coordinated multipoint transmission (CoMP). We employ both the theories of non-cooperative and cooperative games in a unified procedure to analyze the decision making process. The BSs of the competing system are assumed to operate at the maximum expected sum rate (MESR) correlated equilibrium (CE), which we compare against the value of CoMP to establish the stability of the coalition. We prove that there exists a threshold geographical separation,  $d_{th}$ , between the macrocell user equipment (MUE) and FBS, under which the region of coordination is non-empty. Theoretical results are verified through simulation.

## I. INTRODUCTION

Small cells are an attractive easily deployable solution to the increasing demand for capacity [1]. Underlay small cells improve the capacity of the network through frequency reuse and higher link gains due to shorter distances to the user equipment (UE). On the downside the unplanned deployment of small cells in the larger cell structure creates unforeseen interference conditions for both the macro and small cell networks. Such dynamic interference situations require novel solutions [2].

*Coordinated multipoint transmission* (CoMP) introduces dynamic interaction between multiple cells to increase network performance and reduce interference. In our research we consider the CoMP scheme of joint transmission (JT) [3]. We begin with the hypotheses that JT must be a rational decision, which is profitable for both macro- and femto-systems, since these systems may belong to independent operators/users. In human interactions, cooperation among a group is justifiable if all the members are better off in that group than if they were in any other group structure among themselves. This rational behavior is embedded in the solution concept of *core* in coalition formation games.

Past research of heterogeneous networks (HetNets) of macro- femtocells, has used both non-cooperative and cooperative games. In [4] a Stackelberg game is formulated where pricing is employed to move the equilibria towards a tolerable interference level for the *macrocell base station* (MBS). In [5] a potential game based analysis of *Nash equilibrium* (NE)

of power and subcarrier allocation, for a multicell interference environment is presented. In [6] power distribution over resource blocks of cognitive *femtocell base stations* (FBSs) is analyzed for their *correlated equilibrium* (CE). In [7], [8]  $\epsilon$ -correlated equilibrium solution is presented for underlayed femtocells to minimize interference to the macro-system. CE is the form of equilibrium used in this paper as well.

In [9], [10] coalition formation games with externalities are used to group the femtocells to mitigate collisions and reduce interference. In [11] a coalition game together with the solution concept of *recursive core* is used to model the cooperative interaction between *macrocell user equipment* (MUE) and *femtocell user equipment* (FUE). They conclude that forming of disjoint coalitions increases the rates of both MUE and FUE. In [12] a coalition formation game is employed to partition a dense network of femtocells to minimize interference where they introduce a polynomial time algorithm for group formation. In [13] both *transferable utility* (TU) and *non-transferable utility* (NTU) coalition formation games are used for cooperation of receivers and transmitters in an interference environment.

Our work is set apart from the above related research, since we bring together both the theories of non-cooperative and coalition formation games to model the femto-macro interaction in CoMP. A similar analysis but, for non-CoMP case, is presented in [14]. We use the terms non-cooperative and cooperative according to their use in the game theory literature whereas the terms coordination, CoMP, and JT are used synonymously.

The rest of the paper consists of the system model in Section II, game-theoretic formulation and solution in Section III, simulation results in Section IV and conclusion with summary in Section V.

## II. SYSTEM MODEL

We consider the downlink transmission of a two tier HetNet, which consists of a single MBS  $m$  and a single FBS  $f$ , separated by a distance  $d > 0$ . Each *base station* (BS) has an active *user equipment* (UE). It is possible that the BSs serve more than one user but the assumption is that at any given instant each BS transmits to only one selected user. The two BSs each possesses  $T$  number of transmit antennas while each UE possesses  $R$  number of receive antennas. Fig. 1 depicts the system model. The origin of the plane is at MBS.

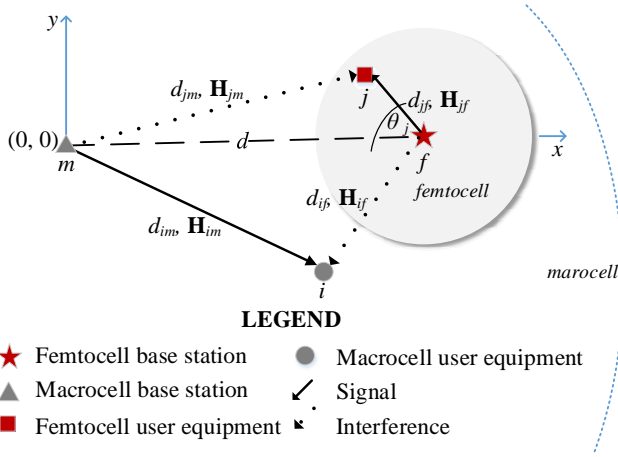


Figure 1. System model. The attributes on arrows indicate the effective distance between elements and their respective channel matrices.

We define two modes of operation, namely uncoordinated and coordinated. In uncoordinated mode the two BSs act as separate transmitters where MBS serves MUE while FBS serves FUE. On the contrary if the two cells conform to the coordinated mode, then the two BSs cooperate through CoMP.

We adopt a channel model with signal attenuation by distance [15]. Channel gain matrix is multiplied by a magnitude, which is a path loss function of distance between BS and UE. To simplify the analysis, we assume the separation between antennas are smaller compared to the distance between BSs and UE, moreover we assume approximately equal distances to scatters between a transmitter-receiver pair, hence we consider a single effective distance between a BS and a UE. The received baseband equivalent signal  $y_i$  (resp.  $y_j$ ) at MUE  $i$  (resp. FUE  $j$ ) for uncoordinated transmission are

$$y_i \triangleq d_{im}^{-\alpha} \mathbf{H}_{im} \mathbf{V}_m \mathbf{s}_i + d_{if}^{-\alpha} \mathbf{H}_{if} \mathbf{V}_f \mathbf{s}_j + \mathbf{n}_i, \quad (1)$$

$$y_j \triangleq d_{jf}^{-\alpha} \mathbf{H}_{jf} \mathbf{V}_f \mathbf{s}_j + d_{jm}^{-\alpha} \mathbf{H}_{jm} \mathbf{V}_m \mathbf{s}_i + \mathbf{n}_j, \quad (2)$$

where  $d_{im}, d_{if}, d_{jf}, d_{jm} \geq 0$  are the effective distances between the respective indexed elements,  $\mathbf{H}_{im}$  is the  $R \times T$  complex valued channel gain matrix from MBS  $m$  to MUE  $i$  and  $\mathbf{H}_{if}, \mathbf{H}_{jf}, \mathbf{H}_{jm}$  are interpreted analogously. The matrix  $\mathbf{V}_m$  (resp.  $\mathbf{V}_f$ ) is the precoder at MBS (resp. FBS). The transmit symbol vector of unit power at MBS  $m$  (resp. at FBS  $m$ ) to MUE  $i$  (resp. to FUE  $j$ ) is denoted by  $\mathbf{s}_i$  (resp.  $\mathbf{s}_j$ ). The exponent  $(-\alpha)$ , where  $\alpha$  is a positive real valued scalar, accounts for path loss, and  $\mathbf{n}_i, \mathbf{n}_j$  are circular symmetric, uncorrelated additive with Gaussian noise (AWGN) vectors.

The achievable rate, treating interference as noise, of the macro system,  $R_{uc}^m$ , is given by (3).

$$R_{uc}^m \triangleq \log \det \left( \mathbf{I}_R + \frac{d_{im}^{-2\alpha} \mathbf{H}_{im} \mathbf{V}_m \mathbf{V}_m^H \mathbf{H}_{im}^H}{d_{if}^{-2\alpha} \mathbf{H}_{if} \mathbf{V}_f \mathbf{V}_f^H \mathbf{H}_{if}^H + \sigma^2 \mathbf{I}_R} \right), \quad (3)$$

$$R_{uc}^f \triangleq \log \det \left( \mathbf{I}_R + \frac{d_{jf}^{-2\alpha} \mathbf{H}_{jf} \mathbf{V}_f \mathbf{V}_f^H \mathbf{H}_{jf}^H}{d_{jm}^{-2\alpha} \mathbf{H}_{jm} \mathbf{V}_m \mathbf{V}_m^H \mathbf{H}_{jm}^H + \sigma^2 \mathbf{I}_R} \right). \quad (4)$$

Above  $\sigma^2$  is variance of circular symmetric noise and  $\mathbf{I}_R$  is the  $R \times R$  identity matrix. For a matrix  $\mathbf{X}$  in the complex field,  $\mathbf{X}^H$  denotes the Hermitian transpose. Analogously we define the achievable rate,  $R_{uc}^f$ , of the femto-system (4).

Now suppose that the two BSs coordinate through JT. The coordination is such that, FBS must transmit to both UE their respective symbols. It is possible to extend this model to include the case where both BSs transmit to both UE. We only consider MUE receiving JT since FUE are mostly home/office users who are less mobile and they have higher downlink gains whereas MUE may be highly mobile and operate under high signal fading and interference. The received signals at MUE and FUE in coordinated transmission are then given by (5) and (6) respectively. Note matrix augmentation in (5).

$$y_i \triangleq \begin{bmatrix} d_{im}^{-\alpha} \mathbf{H}_{im} & d_{if}^{-\alpha} \mathbf{H}_{if} \end{bmatrix} \begin{bmatrix} \mathbf{V}_m \\ \mathbf{V}_{if} \end{bmatrix} \begin{bmatrix} \mathbf{s}_i \\ \mathbf{s}_j \end{bmatrix} + \mathbf{n}_i, \quad (5)$$

$$y_j \triangleq d_{jf}^{-\alpha} \mathbf{H}_{jf} \mathbf{V}_{jf} \mathbf{s}_j + d_{jm}^{-\alpha} \mathbf{H}_{jm} \mathbf{V}_m \mathbf{s}_i + \mathbf{n}_j, \quad (6)$$

$$R_c^f \triangleq \log \det \left( \mathbf{I}_R + \frac{d_{jf}^{-2\alpha} \mathbf{H}_{jf} \mathbf{V}_{jf} \mathbf{V}_{jf}^H \mathbf{H}_{jf}^H}{d_{jm}^{-2\alpha} \mathbf{H}_{jm} \mathbf{V}_m \mathbf{V}_m^H \mathbf{H}_{jm}^H + \sigma^2 \mathbf{I}_R} \right). \quad (7)$$

Above  $\mathbf{V}_{if}$  (resp.  $\mathbf{V}_{jf}$ ) is the precoder matrix at FBS for MUE (resp. FUE),  $\mathbf{V}_m$  is the MBS precoder. The above signal model assumes that the precoders  $\mathbf{V}_{if}$  and  $\mathbf{V}_{jf}$  are such that there is no interuser interference from FBS to the two UE. To that end methods such as block diagonalization or frequency domain techniques can be employed at FBS. Then the achievable rate of FUE  $R_c^f$ , and MUE  $R_c^m$ , for the coordinated transmission scheme are given by (7) and (8) respectively. In what follows we consider the precoders to be defined in a finite set. The finite model not only affords us a finite action space game, but also reflects the digital systems in practical implementations, which can only provide finite discrete levels.

### III. CORE SOLUTION

Now we construct two non-cooperative games one for the uncoordinated system,  $G_1$  and one for the coordinated system,  $G_2$ . We establish the relation between  $G_1$  and  $G_2$  in the ensuing development. Both games have identical set of players  $\mathcal{N} \triangleq \{\text{MBS}, \text{FBS}\}$ , i.e., the two BSs. The action spaces of the players are their discrete precoders. In the uncoordinated case (resp. coordinated case) the sets of precoders of MBS and FBS are denoted by  $\mathcal{A}_{muc}$  and  $\mathcal{A}_{fuc}$  (resp.  $\mathcal{A}_{mc}$  and  $\mathcal{A}_{fc}$ ) respectively. We assume  $\mathbf{0} \notin \mathcal{A}_{fc}$  (does not contain zero precoder), which avoids the trivial case of non-JT. We define the product sets of the action spaces  $\mathcal{A}_{uc} \triangleq \mathcal{A}_{muc} \times \mathcal{A}_{fuc}$  and  $\mathcal{A}_c \triangleq \mathcal{A}_{mc} \times \mathcal{A}_{fc}$ . The utility functions of the two players in the uncoordinated case (resp. coordinated case) are  $R_{uc}^m$  and  $R_{uc}^f$  (resp.  $R_c^m$  and  $R_c^f$ ). The joint action of  $G_1$  is  $\mathbf{V} \triangleq (\mathbf{V}_m, \mathbf{V}_f) \in \mathcal{A}_{uc}$  where  $\mathbf{V}_m \in \mathcal{A}_{muc}$  and  $\mathbf{V}_f \in \mathcal{A}_{fuc}$  and the joint action of  $G_2$  is  $\mathbf{V} \in \mathcal{A}_c$  such that  $\mathbf{V} \triangleq (\mathbf{V}_m, \mathbf{V}_f)$  where  $\mathbf{V}_m \in \mathcal{A}_{mc}$  and  $\mathbf{V}_f \in \mathcal{A}_{fc}$ . Note that FBS's action  $\mathbf{V}_f \in \mathcal{A}_{fc}$ , consists of two precoders  $\mathbf{V}_f \triangleq (\mathbf{V}_{if}, \mathbf{V}_{jf})$ . The MBS (resp. FBS) has identical maximum transmit

$$R_c^m \triangleq \log \det \left( \mathbf{I}_R + \frac{\begin{bmatrix} d_{im}^{-\alpha} \mathbf{H}_{im} & d_{if}^{-\alpha} \mathbf{H}_{if} \end{bmatrix} \begin{bmatrix} \mathbf{V}_m \\ \mathbf{V}_{if} \end{bmatrix} \begin{bmatrix} \mathbf{V}_m \\ \mathbf{V}_{if} \end{bmatrix}^H \begin{bmatrix} d_{im}^{-\alpha} \mathbf{H}_{im} & d_{if}^{-\alpha} \mathbf{H}_{if} \end{bmatrix}^H}{\sigma^2 \mathbf{I}_R} \right), \quad (8)$$

$$0 \prec \mathbf{I}_R + d_{im}^{\prime-2\alpha} \left( d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C} \right)^{-\frac{1}{2}} \mathbf{A} \left( d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C} \right)^{-\frac{1}{2}} \preceq \mathbf{I}_R + d_{im}^{-2\alpha} \left( d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C} \right)^{-\frac{1}{2}} \mathbf{A} \left( d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C} \right)^{-\frac{1}{2}}, \quad (9)$$

$$0 \prec \mathbf{I}_R + d_{im}^{-2\alpha} \mathbf{A}^{\frac{1}{2}} \left( d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C} \right)^{-1} \mathbf{A}^{\frac{1}{2}} \preceq \mathbf{I}_R + d_{im}^{-2\alpha} \mathbf{A}^{\frac{1}{2}} \left( d_{if}^{\prime-2\alpha} \mathbf{B} + \mathbf{C} \right)^{-1} \mathbf{A}^{\frac{1}{2}}. \quad (10)$$

power in both uncoordinated and coordinated cases, i.e., for FBS,  $\max_{\mathbf{V} \in \mathcal{A}_{fuc}} \{ \text{Trace}(\mathbf{V}^H \mathbf{V}) \} = \max_{\mathbf{V}_f \in \mathcal{A}_{fc}} \{ \text{Trace}(\mathbf{V}_{if} \mathbf{V}_{if}^H) + \text{Trace}(\mathbf{V}_{jf} \mathbf{V}_{jf}^H) \}$  and analogously for MBS.

Now we possess all the ingredients necessary to define the non-cooperative games,  $G_1$  and  $G_2$ . The uncoordinated game is give by the tuple  $G_1 \triangleq \langle \mathcal{N}, \mathcal{A}_{uc}, \{R_{uc}^m, R_{uc}^f\} \rangle$ . The game when the two systems are in coordination is  $G_2 \triangleq \langle \mathcal{N}, \mathcal{A}_c, \{R_c^m, R_c^f\} \rangle$ .

Let us set aside the above defined two games for a moment, we come back to them shortly. To analyze the coordinated system we must utilize coalitional games from the cooperative game theory. The most widely used solution concept in coalitional games is the *core*. In order for the two BSs to coordinate the core of the coalition game must be nonempty. A nonempty core implies that the grand coalition, which includes all the players, has a value, which is divisible among the players so that no other partition of subsets of players can give a better value to any of the players. The analysis of the core requires that the cooperative game has TU, which means that the sum utility of the coalition (the two cells in this case) is able to be shared between the members [16]. But we observe from the system model that the sum rate of the coordinated system is not arbitrarily transferable between the two players. Therefore we follow a usual trick employed in such situations, introduce a monetary transfer i.e., payment, between the macro and femto systems. It is imperative to understand that such a monetary transfer is not merely a tool to make the problem amenable to core analysis, but also has an important engineering aspect: coordination between the systems require sharing power with external users and communication of symbol information and channel state information (CSI) between the BSs. Such transactions have to be compensated in any practical system in order to provide an incentive to take part in CoMP. After introducing the payment  $c$ , the utility of MBS,  $U_c^m$ , and FBS,  $U_c^f$ , is given by (11). The payment is of units of rate, which can be interpreted in monetary terms as applicable.

$$U_c^m \triangleq R_c^m - c, \quad U_c^f \triangleq R_c^f + c. \quad (11)$$

A coalitional game in characteristic form requires a set of players and a value function [17]. In our model the set of players is  $\mathcal{N}$ , which has three nonempty subsets.

To define the value function we revisit the games  $G_1$  and  $G_2$ .

There are multiple definitions of equilibria for non-cooperative games. In this research we are interested in CE, which is a generalization of NE [17].

**Definition 1.** CE of the game  $G_1$  is a probability distribution  $\tilde{p}_{uc}(\cdot)$  on the joint action space  $\mathcal{A}_{uc}$  such that  $\forall \mathbf{V} \in \mathcal{A}_{uc}, \forall \mathbf{V}'_m \in \mathcal{A}_{muc},$  and  $\forall \mathbf{V}'_f \in \mathcal{A}_{fuc}$

$$\sum_{\mathbf{V}: \mathbf{V}_f \in \mathcal{A}_{fuc}} \tilde{p}_{uc}(\mathbf{V}) R_{uc}^m(\mathbf{V}) \geq \sum_{\mathbf{V}: \mathbf{V}_f \in \mathcal{A}_{fuc}} \tilde{p}_{uc}(\mathbf{V}) R_{uc}^m(\mathbf{V}'_m, \mathbf{V}_f), \quad (12)$$

$$\sum_{\mathbf{V}: \mathbf{V}_m \in \mathcal{A}_{muc}} \tilde{p}_{uc}(\mathbf{V}) R_{uc}^f(\mathbf{V}) \geq \sum_{\mathbf{V}: \mathbf{V}_m \in \mathcal{A}_{muc}} \tilde{p}_{uc}(\mathbf{V}) R_{uc}^f(\mathbf{V}'_f, \mathbf{V}_m). \quad (13)$$

Similarly we define the CE of the game  $G_2$ , the probability distribution  $\tilde{p}_c(\cdot)$  on the action space  $\mathcal{A}_c$ , which satisfies  $\forall \mathbf{V} \in \mathcal{A}_c, \forall \mathbf{V}'_m \in \mathcal{A}_{mc},$  and  $\forall \mathbf{V}'_f \in \mathcal{A}_{fc}$

$$\sum_{\mathbf{V}: \mathbf{V}_f \in \mathcal{A}_{fc}} \tilde{p}_c(\mathbf{V}) R_c^m(\mathbf{V}) \geq \sum_{\mathbf{V}: \mathbf{V}_f \in \mathcal{A}_{fc}} \tilde{p}_c(\mathbf{V}) R_c^m(\mathbf{V}'_m, \mathbf{V}_f), \quad (14)$$

$$\sum_{\mathbf{V}: \mathbf{V}_m \in \mathcal{A}_{mc}} \tilde{p}_c(\mathbf{V}) R_c^f(\mathbf{V}) \geq \sum_{\mathbf{V}: \mathbf{V}_m \in \mathcal{A}_{mc}} \tilde{p}_c(\mathbf{V}) R_c^f(\mathbf{V}'_f, \mathbf{V}_m). \quad (15)$$

While a finite game is guaranteed to have at least one CE [17], in most cases we find that there are an infinite set of CE. Out of this set of CE we choose the equilibrium, which maximizes the expected sum rate. The *maximum expected sum rate correlated equilibrium* (MESR-CE) of game  $G_1$  is the probability distribution obtained through solving the following linear system;

$$\begin{aligned} & \underset{P_{uc}}{\text{maximize}} \quad \sum_{\mathbf{V} \in \mathcal{A}_{uc}} p_{uc}(\mathbf{V}) (R_{uc}^m(\mathbf{V}) + R_{uc}^f(\mathbf{V})), \\ & \text{subject to (12), (13),} \\ & \quad \sum_{\mathbf{V} \in \mathcal{A}_{uc}} p_{uc}(\mathbf{V}) = 1, \\ & \quad p_{uc}(\mathbf{V}) \geq 0, \forall \mathbf{V} \in \mathcal{A}_{uc}, \end{aligned} \quad (16)$$

where  $p_{uc}(\mathbf{V})$  is the probability of joint action  $\mathbf{V} \in \mathcal{A}_{uc}$  and  $P_{uc} \triangleq (p_{uc}(\mathbf{V}))_{\mathbf{V} \in \mathcal{A}_{uc}}$ . The expected rate of each player at CE of  $G_1$  is

$$R_{\text{uc,cor}}^m \triangleq \sum_{\mathbf{V} \in \mathcal{A}_{\text{uc}}} \tilde{p}_{\text{uc}}(\mathbf{V}) R_{\text{uc}}^m(\mathbf{V}), \quad (17)$$

$$R_{\text{uc,cor}}^f \triangleq \sum_{\mathbf{V} \in \mathcal{A}_{\text{uc}}} \tilde{p}_{\text{uc}}(\mathbf{V}) R_{\text{uc}}^f(\mathbf{V}), \quad (18)$$

where  $\tilde{p}_{\text{uc}}(\cdot)$  is the MESR-CE solution of the linear program (16).

Analogously we obtain the MESR-CE of game  $G_2$  as the solution to the following linear system;

$$\begin{aligned} R_{\text{c,cor}} &\triangleq \underset{\mathbf{p}_c}{\text{maximize}} \sum_{\mathbf{V} \in \mathcal{A}_c} p_c(\mathbf{V}) (R_c^m(\mathbf{V}) + R_c^f(\mathbf{V})), \\ &\text{subject to (14), (15),} \\ &\sum_{\mathbf{V} \in \mathcal{A}_c} p_c(\mathbf{V}) = 1, \\ &p_c(\mathbf{V}) \geq 0, \forall \mathbf{V} \in \mathcal{A}_c, \end{aligned} \quad (19)$$

where  $\mathbf{p}_c \triangleq (p_c(\mathbf{V}))_{\mathbf{V} \in \mathcal{A}_c}$ . Let  $\tilde{p}_c(\cdot)$  be the MESR-CE distribution of game  $G_2$ . The expected rate of each player at CE of  $G_2$  is

$$R_{\text{c,cor}}^m \triangleq \sum_{\mathbf{V} \in \mathcal{A}_c} \tilde{p}_c(\mathbf{V}) R_c^m(\mathbf{V}), \quad (20)$$

$$R_{\text{c,cor}}^f \triangleq \sum_{\mathbf{V} \in \mathcal{A}_c} \tilde{p}_c(\mathbf{V}) R_c^f(\mathbf{V}). \quad (21)$$

Now we define the value function  $v(\cdot)$  of the coalition game as follows;

$$v(S) \triangleq \begin{cases} R_{\text{uc,cor}}^m & S = \{\text{MBS}\}, \\ R_{\text{uc,cor}}^f & S = \{\text{FBS}\}, \\ R_{\text{c,cor}} & S = \mathcal{N}. \end{cases} \quad (22)$$

Here we recap the development so far of this section: in the above definition of the value function  $v(S)$ ,  $R_{\text{uc,cor}}^m$  in (17) (resp.  $R_{\text{uc,cor}}^f$  in (18)) is the expected rate obtained by the macro system (resp. femto system) while playing the MESR-CE in  $G_1$ . On the other hand the value of the grand coalition,  $R_{\text{c,cor}}$  in (19), is the MESR of the two BSs while playing the MESR-CE in  $G_2$ . Then the coalitional game in characteristic form is defined by the tuple  $G_3 \triangleq (\mathcal{N}, v(\cdot))$ .

**Definition 2.** The core is the set of allocations such that no subgroup within the coalition can do better by leaving [17].

In our game the set of allocations are  $U_c^m$  and  $U_c^f$  in (11), such that  $U_c^m + U_c^f = R_{\text{c,cor}}$ .

#### A. Region of Coordination

We seek the geographical region where CoMP is preferable. As MUE moves closer to FBS, signal level drops and interference level rises, hence we expect cooperation with FBS to be preferable to MBS. Since the sum rate can be apportioned between the two systems through the monetary transfer  $c$ , we expect to find a  $c$ , at which the core is non empty. We call the

region where the core is non empty, the *region of coordination* or *CoMP region*.

**Proposition 1.**  $v(\mathcal{N}) \geq v(\text{MBS}) + v(\text{FBS})$  if and only if there exists a payment  $c$  such that  $U_c^m \geq R_{\text{uc,cor}}^m$  and  $U_c^f \geq R_{\text{uc,cor}}^f$ .

*Proof:* We provide a constructive proof. By (11) and while  $G_2$  system is in CE the utilities are  $U_c^m = R_{\text{c,cor}}^m - c$  and  $U_c^f = R_{\text{c,cor}}^f + c$ . Let us consider the LHS of iff, which is equivalent to  $R_{\text{c,cor}} \geq R_{\text{uc,cor}}^m + R_{\text{uc,cor}}^f$ , which implies either  $R_{\text{c,cor}}^m \geq R_{\text{uc,cor}}^m$  or  $R_{\text{c,cor}}^f \geq R_{\text{uc,cor}}^f$  or both. Let us take the case where  $R_{\text{c,cor}}^m \geq R_{\text{uc,cor}}^m$  and  $R_{\text{c,cor}}^f \leq R_{\text{uc,cor}}^f$ , all other cases can be similarly proven. Then there exists a positive constant  $c$  such that  $(R_{\text{c,cor}}^m - c) = U_c^m \geq R_{\text{uc,cor}}^m$  and  $(R_{\text{c,cor}}^f + c) = U_c^f \geq R_{\text{uc,cor}}^f$  since  $(R_{\text{c,cor}}^m - c) + (R_{\text{c,cor}}^f + c) \geq R_{\text{uc,cor}}^m + R_{\text{uc,cor}}^f$ . The converse (RHS  $\implies$  LHS) is proven simply by summing the two inequalities  $U_c^m \geq R_{\text{uc,cor}}^m$  and  $U_c^f \geq R_{\text{uc,cor}}^f$ . This completes the proof. ■

Proposition 1 claims that  $R_{\text{c,cor}} \geq R_{\text{uc,cor}}^m + R_{\text{uc,cor}}^f$  is a necessary and sufficient condition for the core of  $G_3$  to be nonempty.

In order to establish the final result we need the following propositions.

**Proposition 2.**  $R_{\text{uc}}^m$  is monotonically decreasing in  $d_{im}$  and monotonically increasing in  $d_{if}$ .

*Proof:* The proof depends on Loewner ordering of positive semidefinite (PSD) matrices ([18] 7.7). For two PSD matrices  $\mathbf{A}, \mathbf{B}$ , we write  $\mathbf{A} \succeq \mathbf{B}$  (resp.  $\mathbf{A} \succ \mathbf{B}$ ) if  $\mathbf{A} - \mathbf{B} \succeq 0$  is PSD (resp.  $\mathbf{A} - \mathbf{B} \succ 0$  positive definite (PD)). Let  $\mathbf{A} \triangleq \mathbf{H}_{im} \mathbf{V}_m \mathbf{V}_m^H \mathbf{H}_{im}^H$ ,  $\mathbf{B} \triangleq \mathbf{H}_{if} \mathbf{V}_f \mathbf{V}_f^H \mathbf{H}_{if}^H$  and  $\mathbf{C} \triangleq \sigma^2 \mathbf{I}_R$ .  $\mathbf{A}, \mathbf{B}$  are PSD and  $\mathbf{C}$  is PD, also  $d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C}$  is PD. Then the capacity of maro-system (3) can be reformulated as

$$R_{\text{uc}}^m = \log \det \left( \mathbf{I}_R + d_{im}^{-2\alpha} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-\frac{1}{2}} \mathbf{A} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-\frac{1}{2}} \right).$$

Let  $0 < d_{im} < d_{im}'$ , so (9) (see page above) holds, therefore the determinant of (24) is no less than the determinant of (23), which implies, if  $\mathbf{A}$  is invertible, that the determinant is monotonically decreasing in  $d_{im}$ .

$$\mathbf{I}_R + d_{im}'^{-2\alpha} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-\frac{1}{2}} \mathbf{A} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-\frac{1}{2}}. \quad (23)$$

$$\mathbf{I}_R + d_{im}^{-2\alpha} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-\frac{1}{2}} \mathbf{A} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-\frac{1}{2}}. \quad (24)$$

Next we reformulate (3),

$$R_{\text{uc}}^m = \log \det \left( \mathbf{I}_R + d_{im}^{-2\alpha} \mathbf{A}^{\frac{1}{2}} (d_{if}^{-2\alpha} \mathbf{B} + \mathbf{C})^{-1} \mathbf{A}^{\frac{1}{2}} \right),$$

and let  $0 < d_{if} < d_{if}'$ . Then (10) holds and by a similar argument to above we have that the determinant is increasing in  $d_{if}$ . This completes the proof. ■

Let

$$\gamma^m(d_{if}, \mathbf{V}_{if}) \triangleq \log \det \left( \mathbf{I}_R + \frac{d_{if}^{-\alpha} \mathbf{H}_{if} \mathbf{V}_{if} \mathbf{V}_{if}^H d_{if}^{-\alpha} \mathbf{H}_{if}^H}{\sigma^2 \mathbf{I}_R} \right).$$

**Proposition 3.**  $\gamma^m(\cdot)$  is monotonically decreasing in  $d_{if}$ .

*Proof:* Let  $\mathbf{X} \triangleq \mathbf{H}_{if} \mathbf{V}_{if}$ . Note that  $\mathbf{X}\mathbf{X}^H$  is positive semi-definite. Therefore all eigenvalues  $\lambda_1, \dots, \lambda_R$  of  $\mathbf{X}\mathbf{X}^H$  are non-negative and we have

$$\det \left( \mathbf{I}_R + \frac{d_{if}^{-2\alpha}}{\sigma^2} \mathbf{X}\mathbf{X}^H \right) = \prod_{i=1}^R \left( 1 + \frac{d_{if}^{-2\alpha}}{\sigma^2} \lambda_i \right),$$

which is decreasing in  $d_{if}$ . This completes the proof. ■

**Assumption 1.** The precoders  $\mathbf{V}_m \in \mathcal{A}_{mc}$  and  $\mathbf{V}_{if} \in \mathcal{A}_{fc}$  of  $\mathcal{A}_c$  are such that  $\gamma^m(\cdot, \mathbf{V}_{if}) \leq R_c^m(\cdot, \mathbf{V}_m, \mathbf{V}_{if})$ .

Assumption 1, on space  $\mathcal{A}_c$  claims that JT can not provide a lesser rate to MUE than when only receiving from FBS with identical precoder. It is reasonable, since if not there is no reason for JT and MUE can be served by MBS or FBS alone. Designing such precoders is beyond our scope and we refer the reader to [19]. Now we proceed to the main result.

**Theorem 1.** For some  $d > 0$  under Assumption 1,  $\exists d_{th}$  such that for  $d_{if} \leq d_{th}$ , the region of cooperation is nonempty<sup>1</sup>.

*Proof:* Let  $\mathbf{V} = (\mathbf{V}_{if}, \mathbf{V}_{jf}) \in \mathcal{A}_c$ ,  $\mathbf{V}' \in \mathcal{A}_{uc}$  be any two actions from the respective spaces and let the location of the FUE be fixed relative to the FBS at  $(\bar{d}_{jf}, \bar{\theta}_j)$ . Then  $R_c^f(\mathbf{V})$  and  $R_{uc}^f(\mathbf{V}')$  are constants irrespective of location of MUE. Now consider that MUE moves along a trajectory with decreasing  $d_{if}$  and increasing  $d_{im}$ . By Proposition 2,  $R_{uc}^m(\mathbf{V}')$  is decreasing. As  $d_{if} \rightarrow 0$ , by Proposition 3,  $\gamma^m(d_{if}, \mathbf{V}_{if}) \rightarrow \infty$  and by Assumption 1  $R_c^m(\mathbf{V}) \geq \gamma^m(d_{if}, \mathbf{V}_{if})$ . Therefore there must exist  $d_{if} \leq d_{th}$ , such that  $\gamma^m(d_{if}) + R_c^f(\mathbf{V}) \geq R_{uc}^m(\mathbf{V}') + R_{uc}^f(\mathbf{V})$ . Since the action choice was arbitrary  $\exists d_{th}$  such that,  $\min_{\mathbf{V} \in \mathcal{A}_c} (R_c^m(\mathbf{V}) + R_c^f(\mathbf{V})) \geq$

$$\max_{\mathbf{V}' \in \mathcal{A}_{uc}} (R_{uc}^m(\mathbf{V}') + R_{uc}^f(\mathbf{V}')).$$

Therefore  $\forall$  probability distributions  $\tilde{p}_{uc}$  and  $\tilde{p}_c$ , we have  $\sum_{\mathbf{V} \in \mathcal{A}_c} \tilde{p}_c(\mathbf{V}) (R_c^m(\mathbf{V}) + R_c^f(\mathbf{V})) \geq \sum_{\mathbf{V}' \in \mathcal{A}_{uc}} \tilde{p}_{uc}(\mathbf{V}') (R_{uc}^m(\mathbf{V}') + R_{uc}^f(\mathbf{V}'))$ . This completes the proof. ■

Theorem 1 together with Proposition 1 suggests the existence of a region around the FBS where the core is nonempty. Thus we establish the rationality of CoMP scheme JT.

#### IV. NUMERICAL RESULTS

The distances are measured in meters (m), we locate MBS at (0, 0), FBS at (1000, 0), and FUE (990, 0). Unless otherwise stated, the default maximum transmit power of MBS is 5 W and of FBS is 1 W. The two BSs each has 4 antennas and each UE has 2 antennas. In the coordinated mode of transmission, by default FBS distributes the power evenly among FUE and MUE. The AWGN power is set at  $10^{-4}$  W. In the Fig. 2 MUE moves from far negative  $x$  region towards the FBS in linear trajectories. One such trajectory is shown in the

<sup>1</sup>In order to avoid  $|\cdot|$  for each distance we assume all distances are absolute values throughout the paper. Also recall  $\mathbf{0} \notin \mathcal{A}_{fc}$ . See Fig. 1 and Section II for quick reference of distance and angles notation.

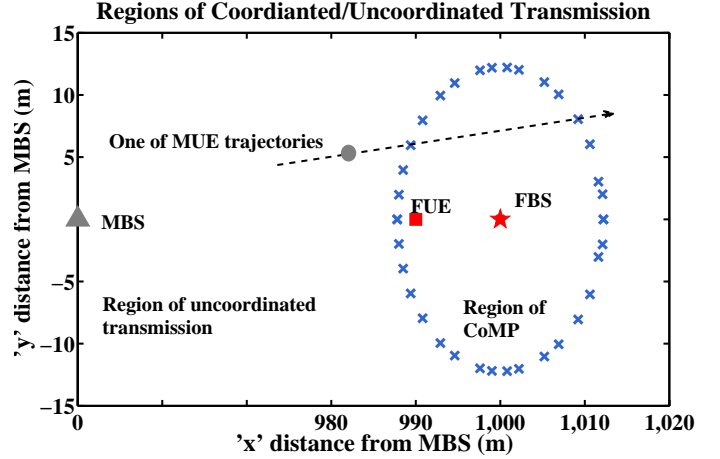


Figure 2. Region of coordination.

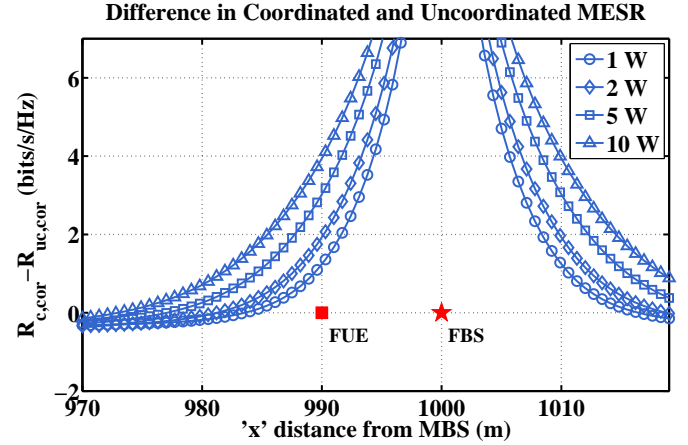


Figure 3. Dependence of CoMP region on FBS transmit power.

figure. We mark the region where coordination is preferred over uncoordinated transmission. The symmetry in the region is due to the use of symmetric channel matrices on either side of the FBS in simulation.

In the rest of the figures the trajectory of the MUE is on the  $x$  axis ( $y$  coordinate is 0). Fig. 3 denotes the expansion of the CoMP region as the FBS transmit power increases. We also see from the figure that  $R_{c,cor}$  far exceeds  $R_{uc,cor}$  as MUE approaches FBS inside the region of coordination. In Fig. 4 we show on the plan of  $(x, y)$  the value of the coalition over the value of uncoordinated system.

In Fig. 5 we demonstrate that as the amount of power allocated to MUE increases the diameter of the coordination region shrinks. The term diameter is loosely used to mean the distance between the entry point and exit point of CoMP region when the MUE's trajectory is on  $x$  axis ( $y$  coordinate 0). Consider the two MUE power ratios of  $a$  and  $c$  such that  $c > a$ . Then the explanation for the phenomenon seen in Fig. 5 is that while operating at ratio  $c$  if the FBS switches to CoMP at the coordination boundary of the ratio  $a$  then the reduction of FUE rate is higher than the increase in MUE rate as still



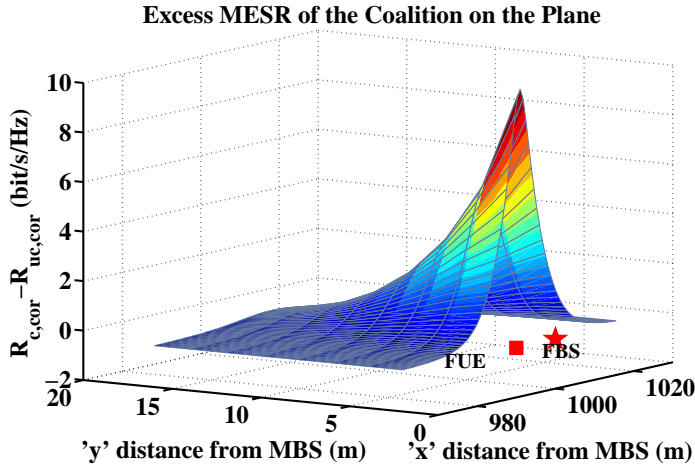


Figure 4. Geometric distribution of value of coalition,  $v(\mathcal{N})$ , over uncoordinated system.

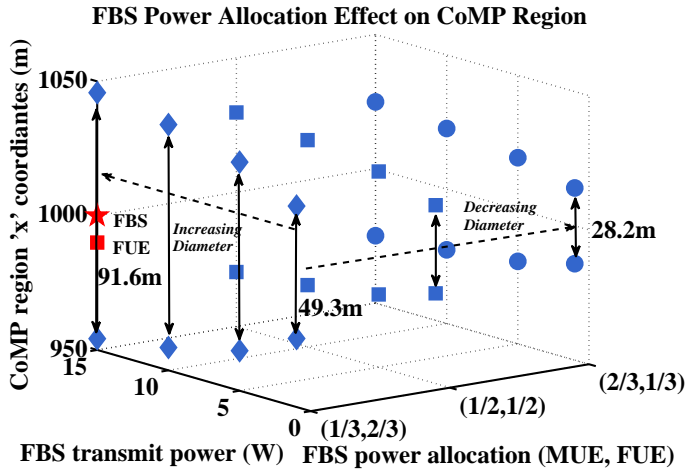


Figure 5. The variation of the diameter of the CoMP region with FBS power allocation ratio  $(a, b)$ . Here  $(a, b)$  implies  $a$  and  $b$  fractions of power are assigned to MUE and FUE respectively.

MUE is further away from FBS than FUE, thus discouraging the formation of the coalition till MUE moves closer to FBS.

## V. CONCLUSION

In this paper we have considered the downlink of a HetNet consisting of a macro- and a femtocell. We have devised two non-cooperative games. The first game,  $G_1$ , had the two cells in competition. In the second game,  $G_2$ , the cells were in coordination (CoMP). In each game we have let cells operate in the respective maximum expected sum rate-correlated equilibria (MESR-CE). Then we defined a third game,  $G_3$ , which is a coalition game in characteristic form with transferable utility. In  $G_3$  the value of the coalition was allowed to be arbitrarily transferred between the two cells via a payment. We have used solution mechanism of *core*, in coalitional games, to analyze  $G_3$ . Then we have proved the existence of a region where the core of the game  $G_3$  is nonempty, which demonstrates that CoMP is a rational decision in some region and the CoMP

decision has been reduced to identifying threshold  $d_{th}$ . CoMP decision mechanisms for more complex channel models can be considered in future work.

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