



HAL
open science

G rard Rauzy (1938– 2010)

Pierre Liardet

► **To cite this version:**

Pierre Liardet. G rard Rauzy (1938– 2010). Uniform Distribution Theory, 2012, 7 (1), pp.1 - 9.
hal-00871121

HAL Id: hal-00871121

<https://hal.science/hal-00871121>

Submitted on 11 Oct 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destin e au d p t et   la diffusion de documents scientifiques de niveau recherche, publi s ou non,  manant des  tablissements d'enseignement et de recherche fran ais ou  trangers, des laboratoires publics ou priv s.

GÉRARD RAUZY (1938 – 2010)

The French mathematician Gérard Rauzy was born in Paris on May 29, 1938. He passed away on May 4, 2010, in Marseilles. At the time of his death, he was *Professeur Émérite* of the *Université Aix-Marseille II*.

Gérard Rauzy began his primary education in Paris. Then his family moved to Marseilles, and the young boy was admitted *en cinquième* to the famous Lycée Thiers with two years of advanced placement. He was brilliant in mathematics, physics, and the natural sciences — in fact, in all subjects except history and geography, where he obtained *catastrophic* marks. He was just 16 years old when he received his Baccalauréat degree *Math'elem* and began higher education in the *Classes Préparatoires* of Lycée Thiers. He passed the most selective competitive exam of the *École Normale Supérieure de Paris, rue d'Ulm* (promotion 1957). The young *normalien* devoted his third year to prepare and successfully pass the *Agrégation de Mathématiques* (1960). Later he received his master's degree for *Mathématiques Approfondies* in number theory, with a remarkable dissertation on diophantine approximations [1] under the direction of C. Pisot and R. Salem. At the same time, he attended the *Séminaire de théorie des nombres* of Paris, created in 1959 by Charles Pisot, Hubert Delange and Georges Poitou. His first talk [2] in this seminar prepared the way for a second one devoted to *L-series* and Dirichlet's theorem on the density of prime numbers in arithmetic progressions [3]. He continued to participate actively in seminar sessions, giving talks on diophantine approximation [4], diophantine equations [5] and transcendence [6].

In 1963, G. Rauzy started research on sequences of integers satisfying linear recurrences, and studied the periodicity of these sequences modulo an integer [7]. Then he studied the following general problem: assume that a property $P(n)$, verified for all natural numbers, implies another property. Now, does this latter property still hold if $P(n)$ is assumed to hold only for the integers n lying in a suitable subset J of \mathbf{N} , the set of natural numbers? To achieve this program, G. Rauzy introduced the notion of *frequency* $\alpha(J)$ for any subset J of \mathbf{N} . By definition, $\alpha(J)$ is the supremum (possibly infinite) of the set of real numbers $A \geq 1$ with the property that for all $x_0 > 0$, there exist $x \in \mathbf{N}$, $x > x_0$, such that all integers in the interval $[x, Ax)$ belong to J . He obtained an impressive

number of results [8, 9, 10, 11], which are collected in his *Thèse d'État* [12] written under the direction of C. Pisot and defended in 1965. Let us note two significant corollaries of this work:

- Generalization of a classical result of G. Pólya: assume that f is an entire function such that $\sup_{0 \leq \theta < 2\pi} \limsup_{r \rightarrow \infty} \frac{1}{r} \log |f(re^{i\theta})| < \log 2$. If $f(J) \subset \mathbf{N}$ for a subset J of \mathbf{N} with $\alpha(J) = \infty$, then f is a polynomial.
- Generalizations of results of C. Pisot and R. Salem: let θ be an algebraic number $\theta > 1$. There exist a subset J of \mathbf{N} with $\alpha(J) = \infty$ and a real number $\lambda \neq 0$ such that

$$\overline{\lim}_{n \in J} \|\lambda \theta^n\| = 0$$

(where $\|x\| = \min_{k \in \mathbf{Z}} |x - k|$) if and only if θ is a Pisot number or a Salem number. The case with only $\alpha(J) > 1$ but λ algebraic implies that θ is a Pisot number and $\lambda \in \mathbf{Q}(\theta)$.

Gérard Rauzy was associated professor at the University of Lille from 1965 to 1967. Then, he was recruited as a Professor by the *Faculté des Sciences de Marseille* and from 1971, moved to the *Université d'Aix-Marseille II*, a newly-formed university, where he played an important role in the creation of the *Centre International de Rencontres Mathématiques* (1981). Later on, he managed to obtain in 1992 the creation of the first *Unité Propre du CNRS* on Discrete Mathematics and he became the director. After 1995, this laboratory extended its research domains considerably and became the *Institut de Mathématiques de Luminy* (IML).

Arriving in Marseilles, in 1967, G. Rauzy at once gave a strong research impetus to the *Séminaire de Théorie des Nombres de Marseille* created by André and Christiane Blanchard. At this time, he was particularly interested in two new research subjects. The first one was in relation to some classes of meromorphic functions stable on some set of algebraic numbers [14, 15, 18]. More explicitly the problem is to define sets of algebraic numbers E such that a function f , meromorphic at infinity taking values $f(n)$ in E for sufficiently large integers n , is an algebraic function and, even stronger, belongs to some pre-assigned set of algebraic functions. In the case where f is already algebraic, a typical result⁽¹⁾ says that if $P(X, Y)$ is a polynomial in two variables with rational coefficients such that for any Pisot number α a root of the polynomial $P(\alpha, Y)$ is also a Pisot number, then either $P(X, \theta) = 0$ for some Pisot number θ , or there exists an integer $m \geq 1$ such that $P(X, Y)$ is divisible by $X^m - Y$. The second subject is related to uniform distribution theory, a research domain where Rauzy's contributions are particularly original and deep, involving ingenious

¹Talk in *Séminaire de théorie des nombres* de Bordeaux, 1968-69, unpublished.

constructions as well as various tools, opening new fields of investigations. Let us consider some of them.

We shall use some definitions and classical results from uniform distribution theory. The interested reader not familiar with this theory is invited to consult the three main monographs on the subject⁽²⁾. We also recommend reading the fascinating and original book of Rauzy [29] published in 1976, which contains both classical results and frameworks for research investigations in uniform distribution modulo one, and more.

In 1968, Michel Mendès France introduced the notion of *normal set*⁽³⁾ and proposed the problem of identifying such sets. Recall that a set E of real numbers is said to be normal if there exists a sequence of real numbers $\Lambda = (\lambda_k)_k$ such that x belongs to E if and only if the sequence $x\Lambda$ is uniformly distributed modulo one. The case where E is a subset of \mathbf{Z} was solved by François Dress and M. Mendès France⁽⁴⁾. Using an ingenious construction of sequences by blocks, G. Rauzy first showed that \mathbf{Q}^* is normal [17] and *dans la foulée* (see [16]) proved that normal sets E are characterized by the following two conditions:

- (i) $0 \notin E$ and $qE \subset E$ for all rational integers $q \neq 0$;
- (ii) there exists a sequence of continuous maps $f_n : \mathbf{R} \rightarrow \mathbf{R}$ such that $E = \{x \in \mathbf{R}; \lim_n f_n(x) = 0\}$.

The method of blocks is also the cornerstone in [27] used to construct a real-valued sequence $(u_n)_{n \geq 0}$ completely uniformly distributed modulo one with the surprising properties that u has low discrepancy; more precisely

$$\limsup_N ND_N^*(u) / \log N \leq 1 / \log 2$$

with $D_N^*(u) := \sup_{0 \leq a < 1} |\frac{1}{N} \text{card}\{0 \leq n < N; 0 \leq u_n < a\} - a|$. Going back to normal sets, it is easy to prove that \mathbf{R}^* is normal, for example, by taking sequences $(k^c)_k$ (c positive and not an integer), but it is an interesting problem to consider *natural* sequences that increase faster than any polynomial but slower than any exponential growth. In [19, 20, 21, 23, 24], G. Rauzy investigated sequences of the form $f(n)$ where f is an entire function that increases slowly. For example, he proved in [21] that if f is not a polynomial, such that $f(\mathbf{R}) \subset \mathbf{R}$ and $\limsup_{r \rightarrow \infty} \frac{\log \log M(r)}{\log \log r} < 5/4$ with $M(r) := \sup_{|z| \leq r} |f(z)|$, then the sequences $(\lambda f(k))_k$ ($\lambda \in \mathbf{R}^*$) are completely uniformly distributed. The proof

²*Uniform Distribution of Sequences* by L. Kuipers and H. Niederreiter, John Wiley and Sons Inc. (1974); *Sequences, Discrepancies and Applications* by M. Drmota and R. Tichy, Springer-Verlag Berlin (1997); *Distribution of Sequences: A Sampler* by O. Strauch and S. Porubsky, Peter Lang Pub. Inc. (2005).

³*Séminaire DPP, Théorie des Nombres*, t. 9, No 2 (1967–1968), exp. No. 16, p. 1–4.

⁴*Séminaire DPP, Théorie des Nombres*, t. 10, No. 2 (1968–1969), exp. No. 17 p. 1–5.

makes use of the Vinogradov method to estimate Weyl sums associated with suitable approximations of f by polynomials.

With the international conference in Marseilles on *Uniform distribution* [24], G. Rauzy began to study stability problems related to distribution of sequences in connection with the notion of statistical independence [22, 25, 26, 28]. Recall that two sequences $(u_n)_{n \geq 0}$ and $(v_n)_{n \geq 0}$ in a compact metrizable space X are said to be (statistically) *independent* if for all continuous maps $f : X \rightarrow \mathbf{R}$ and $g : X \rightarrow \mathbf{R}$, one has

$$\lim_N \left(\frac{1}{N} \sum_{n < N} f(u_n)g(u_n) - \left(\frac{1}{N} \sum_{n < N} f(u_n) \right) \left(\frac{1}{N} \sum_{n < N} g(u_n) \right) \right) = 0.$$

Now let us mention a nice result to emphasize the creative spirit of G. Rauzy in solving a question of M. Mendès France⁽⁵⁾: let r be an integer greater or equal to 2 and let $B(r)$ denote the set of real numbers that are normal in base r . Then, introducing the notions of *upper noise* of a real number $x = \sum_{n=0}^{\infty} c_n/r^{n+1}$ in base r as the quantity

$$\beta(x) = \limsup_{s \rightarrow \infty} \left(\limsup_{N \rightarrow \infty} \left(\inf_{\varphi \in E_s} \frac{1}{N} \sum_{n < N} \inf\{1, |c_n - \varphi(c_{n+1}, \dots, c_{n+s})|\} \right) \right)$$

where E_s is the set of maps from \mathbf{R}^s to \mathbf{R} , G. Rauzy showed in [28] that $\gamma \in \mathbf{R}$ verifies $\gamma + B(r) \subset B(r)$ if and only if the upper noise of γ is equal to 0. Such a number γ is also characterized in terms of dynamical systems. More explicitly, let K_c be the orbit closure of $c := (c_0, c_1, c_n, \dots)$ with respect to the shift on the usual compact metrizable space $\{0, 1, \dots, r-1\}^{\mathbf{N}}$. Then $\beta(\gamma) = 0$ means that the entropy of any shift-invariant measure with support in K_c is null. In other words, c is deterministic for the shift map. The notion of *lower noise* is also introduced and normal numbers in base r are exactly those having the maximal lower noise (which in this case is also equal to the upper noise, that is, $(r-1)/r$).

Starting from a particular substitution π on four letters (explicitly $\pi(1) = 142$, $\pi(2) = 1422$, $\pi(3) = 143342$ and $\pi(4) = 14342$) in a seminar talk [30], G. Rauzy set up the main tools and guidelines for investigating symbolic sequences arising from pertinent substitutions σ on a finite alphabet, or from dynamical systems in number theory. This required interactions with combinatorial and graph analysis of the language of a fixed point $u = u_0 u_1 u_2 \dots$ of σ , distributions of frequencies of letters occurring in u , strict ergodicity of the symbolic dynamical system built from u and the shift transformation, identification of such a system with an interval-exchange transformation T or a similar geometrical transformation that also constructed u by coding the orbit of a point under T with respect to a specific partition, a renormalisation process from induced transformations, etc.

⁵Séminaire DPP, Théorie des nombres (1973–74), exp. No. 7, 6 p.

This research program, stimulated by the works of J.-P. Conze, T. Kamae, M. Keane, M. Mendès France, and W. Veech, led to a series of papers related to ergodic theory [27, 30, 31, 33, 35, 36, 44, 48, 52] or involving replicative symbolic sequences and their complexity [37, 38, 40, 41, 43, 45, 50, 51, 52]. See also [47].

A subset A of a compact metrizable space X is said to be a *bounded remainder set* for a given X -valued sequence $(u_n)_{n \geq 0}$ if $\sup_N \text{card}\{0 \leq n < N; u_n \in A\} - aN < +\infty$ for some a . The problem to find a non-trivial bounded remainder set for a sequence such as $n \mapsto n\alpha$ in \mathbf{T}^d ($d \geq 2$) is studied in [44], but the key idea in use emerged already in the seminal paper [40], where a link is set up between the distribution of sequences $(n\eta)_n$ modulo \mathbf{Z}^2 where $\eta = (\eta_1, \eta_2)$ are vectors in \mathbf{R}^2 such that $\{1, \eta_1, \eta_2\}$ forms a base of the module of algebraic integers in the cubic field generated by the Tribonacci number θ (the Pisot number root of $X^3 - X^2 - X - 1 = 0$), and the fixed point ω of the substitution $1 \rightarrow 12, 2 \rightarrow 13$ and $3 \rightarrow 1$.

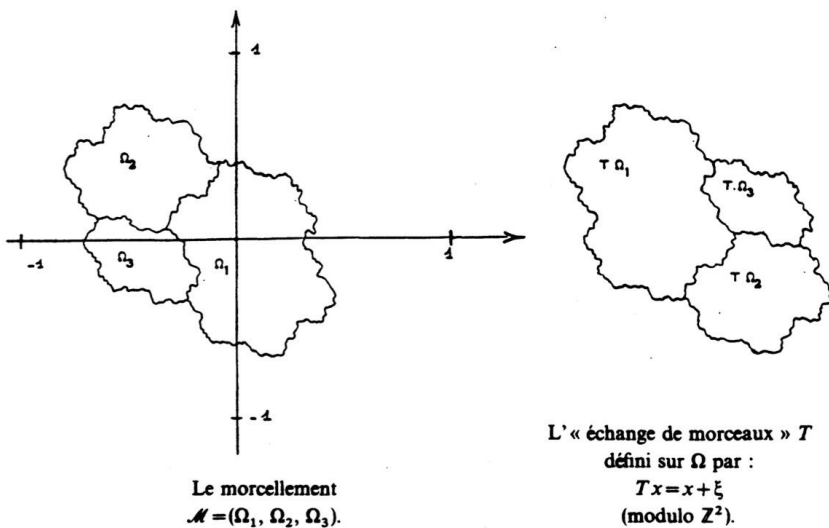


FIGURE 1. The Rauzy splitting (morcellement) associated with the Tribonacci number θ and the translation T by $\xi = (1/\theta, 1/\theta^2)$ modulo \mathbf{Z}^2 that realizes an exchange of the three pieces Ω_1, Ω_2 and Ω_3 up to their boundaries.

The well-known Rauzy fractals appeared for the first time in this paper. They are three pieces Ω_1, Ω_2 and Ω_3 that constitute a splitting \mathcal{M} of \mathbf{R}^2 modulo \mathbf{Z}^2 ,

that is to say they satisfy the following properties: (i) each Ω_i is open, bounded and connected; (ii) the sets $\Omega_i + \mathbf{Z}^2$ are mutually disjoint and their union is dense in \mathbf{R}^2 ; (iii) each Ω_i is disjoint from translation $\Omega_i + g$ by any vector $g \neq 0$ with integer entries. The main fact proved in [40] is that there exists such a splitting, that permits one to identify the translation $T : x \rightarrow x + \xi \pmod{\mathbf{Z}^2}$ (with $\xi = (1/\theta, 1/\theta^2)$) as an exchange of pieces built from the Ω_i (see Figure 1, selected from [40]). Also, by construction, the Ω_i are bounded remainder sets for $(T^n(0))_n$ and $\omega_n = k$ if and only if $T^n(0) \in \Omega_k$. In addition, the number of words of length n occurring in ω is $p(n) = 2n + 1$. This kind of language complexity for minimal symbolic sequences is investigated in [41, 52]. Predicting future developments, G. Rauzy wrote in the introduction of [40]: *le lecteur pourra constater que beaucoup de raisonnements s'étendent à des situations plus générales*. His remark could be applied equally well to many other results that he proved.

G. Rauzy was also captivated by problems in the spirit of P. Erdős. Paper [54] concerns sets of positive integers that contain no three terms in arithmetic progression and in [55] a large family of non-decreasing sequences $(a_n)_{n \geq 1}$ of positive integers is constructed such that the set of finite sums $\sum_{n \in F} \varepsilon_n a_n$ with $\varepsilon_n \in \{0, 1\}$ contains an infinite arithmetic progression.

We end this short overview by quoting in chronological order the names of Rauzy's students who defended their French Ph. D. thesis under his direction: P. Liardet, A. Thomas, A. Cissé, E. Pouspourikas, C. Mauduit, Th. Tapsoba, P. Martinez, S. Fabre, P. Gonzalez, M.-L. Santini, P. Alessandri, L. Vuillon, N. Tchekhovaya, A. Messaoudi, and V. Canterini. All of them contributed by extending in various directions the work of Gérard Rauzy.

We warmly thank Evelyne Rauzy for having communicated private information on the life of her brother, Jean-Paul Allouche for his encouragements and Jeffrey Shallit for his kind assistance in the writing.

REFERENCES

- [1] RAUZY, G.: *Approximation diophantienne des nombres algébriques*, Faculté des Sciences de l'Université de Paris. Mathématiques approfondies. Théorie des nombres. 1960/61 Secrétariat mathématique, Paris (1961), 60 p.
- [2] RAUZY, G.: *Caractères sur les groupes abéliens finis. Caractères sur les classes de restes*, Séminaire Delange-Pisot-Poitou. Théorie des nombres, **2** (1960-1961), Exposé No. 3, 7 p.
- [3] RAUZY, G.: *Les séries L et le théorème de Dirichlet*, Séminaire Delange-Pisot-Poitou. Théorie des nombres, **2** (1960-1961), Exposé No. 4, 9 p.
- [4] RAUZY, G.: *Approximations diophantiennes linéaires homogènes*, Séminaire Delange-Pisot-Poitou. Théorie des nombres, **3** (1961-1962), Exposé No. 1, 18 p.

GÉRARD RAUZY (1938 – 2010)

- [5] RAUZY, G.: *Équations de la forme $\lambda\alpha^x - \mu\beta^y = \nu$ aux inconnues x, y (entiers ≥ 0)*, Séminaire Delange-Pisot-Poitou. Théorie des nombres, **4** (1962-1963), Exposé No. 9, 13 p.
- [6] RAUZY, G.: *Points transcendants sur les variétés de groupe*, Séminaire Bourbaki, **16** (1963-1964), Exposé No. 276, 8 p.
- [7] RAUZY, G.: *Relations de récurrence modulo m* , Séminaire Delange-Pisot-Poitou. Théorie des nombres, **5** (1963-1964), Exposé No. 2, 10 p.
- [8] RAUZY, G.: *Fonctions entières prenant des valeurs entières*, Séminaire Delange-Pisot-Poitou. Théorie des nombres, **6** no. 1 (1964-1965), Exposé No. 8, 10 p.
- [9] RAUZY, G.: *Répartition modulo 1 pour des suites partielles d'entiers. Développements en série de Taylor donnés sur des suites partielles*, C. R. Acad. Sci. Paris **258** (1964), 4881–4884.
- [10] RAUZY, G.: *Fonctions entières prenant des valeurs entières sur des ensembles partiels d'entiers*, C. R. Acad. Sci. Paris, **259** (1964), 19–21.
- [11] RAUZY, G.: *Suites partiellement récurrentes. Ensembles partiels d'entiers*, Séminaire Dubreil. Algèbre et théorie des nombres, **18** no. 1 (1964-1965), Exposé No. 13, 11 p.
- [12] RAUZY, G.: *Suites partiellement récurrentes (applications à la répartition modulo 1 et aux propriétés arithmétiques des fonctions analytiques)*, Annales de l'institut Fourier, **16** no. 1 (1966), 159-234
- [13] RAUZY, G.: *Ensembles arithmétiquement denses*, C. R. Acad. Sci. Paris Sér. A-B **265** (1967), A37–A38.
- [14] RAUZY, G.: *Algèbricité des fonctions méromorphes prenant certaines valeurs algébriques*, Bull. Soc. Math. France, **96** (1968), 197-208.
- [15] RAUZY, G.: *Transformations rationnelles pour lesquelles l'ensemble des nombres de Pisot-Vijayaraghavan est stable*, C. R. Acad. Sci. Paris Sér. A-B **268** (1969), A305–A307.
- [16] RAUZY, G.: *Caractérisation des ensembles normaux*, Bull. Soc. Math. France, **98** (1970), 401-414.
- [17] RAUZY, G.: *Normalité de \mathbf{Q}^** , Acta Arith. **19** (1971), 43–47.
- [18] RAUZY, G.: *Ensembles de nombres algébriques et transformations rationnelles*, Colloque de Th. Nombres (1969, Bordeaux), Bull. Soc. math. France, Mémoire **25** (1971), 165–168.
- [19] RAUZY, G.: *Croissance et répartition modulo 1*, Séminaire de Théorie des Nombres 1971–1972, Univ. Bordeaux I, Talence (1972), Exp. No. 28, 9 p.
- [20] RAUZY, G.: *Fonctions entières et répartition modulo 1*, Bull. Soc. Math. France **100** (1972), 409-415.
- [21] RAUZY, G.: *Fonctions entières et répartition modulo un. II*, Bull. Soc. Math. France **101** (1973), 185-192.
- [22] RAUZY, G.: *Étude de quelques ensembles de fonctions définis par des propriétés de moyenne*, Séminaire de Théorie des Nombres, 1972–1973, Univ. Bordeaux I, Talence (1973), Exp. No. 20, 18 p.
- [23] RAUZY, G.: *Fonctions entières et répartition modulo 1*, Journées arithmétiques (1973, Grenoble), Bull. Soc. math. France, Mémoires **37** (1974), 137-138.
- [24] RAUZY, G.: *Répartition modulo 1*, Actes du Colloque de Marseille-Luminy, 4–7 Juin, 1974, G. Rauzy Edit. in: Lecture Notes in Mathematics Vol. **475**, Springer-Verlag, Berlin-New York (1975), iv+258 p.
- [25] RAUZY, G.: *Équirépartition et entropie*, in [24], 155–175.
- [26] RAUZY, G.: *Nombres normaux et processus déterministes*, Journées Arithmétiques de Bordeaux (1974, Bordeaux), Soc. math. France, Astérisque, **24-25** (1975), 263–265.

GÉRARD RAUZY (1938 – 2010)

- [27] RAUZY, G.: *Sur une suite liée à la discrédance de la suite $(n\alpha)_{n \in \mathbb{N}}$* , C. R. Acad. Sci. Paris Sér. A-B **282** (1976), A1323–A1325.
- [28] RAUZY, G.: *Nombres normaux et processus déterministes*, Acta Arith. **29**, no. 3 (1976), 211–225.
- [29] RAUZY, G.: *Propriétés statistiques de suites arithmétiques*, Le Mathématicien, No. 15. Collection SUP. Presses Universitaires de France, Paris (1976), 133 p.
- [30] RAUZY, G.: *Une généralisation du développement en fraction continue*, Séminaire Delange-Pisot-Poitou, Théorie des nombres, **18** no. 1 (1976-1977), Exposé No. 15, 16 p.
- [31] RAUZY, G.: *Répartition modulo 1*, Journées Arithmétiques de Caen (Univ. Caen, Caen, 1976), in Astérisque **41–42**, Soc. Math. France, Paris (1977), 81–101.
- [32] RAUZY, G.: *Les zéros entiers des fonctions entières de type exponentiel*, Séminaire de Théorie des Nombres 1976–1977, Univ. Bordeaux I, Talence (1977), Exp. No. 6, 10 p.
- [33] RAUZY, G.: *Répartition de suites et équations fonctionnelles associées*, Monatsh. Math. **83** (1977), no. 4, 315–329.
- [34] PARENT, D. P. (coauthors: BARSKY, D. – BERTRANDIAS, F. – CHRISTOL, G. – DECOMPS, A. – DELANGE, H. – DESHOUILLEERS, J.-M. – LAMÈCHE-GÉRARDIN, K. – LAGRANGE, J. – NICOLAS, J.-L. – PATHIAUX, M. – RAUZY, G. – WALD-SCHMIDT, M. (with a preface by Ch. Pisot and preliminary remarks by Jean-Louis Nicolas): *Exercices de théorie des nombres*, Gauthier-Villars, Paris (1978), v+307 p.
- [35] RAUZY, G.: *Échanges d'intervalles et transformations induites*, Acta Arith. **34** (1979), no. 4, 315–328.
- [36] KEANE, M. S. – RAUZY, G.: *Stricte ergodicité des échanges d'intervalles*, Math. Z. **174** (1980), no. 3, 203–212.
- [37] CHRISTOL, G. – KAMAE, T. – MENDÈS FRANCE, M. – RAUZY, G.: *Suites algébriques, automates et substitutions*, Bull. Soc. Math. France, **108** (1980), 401–419.
- [38] RAUZY, G.: *Mots circulaires et réseaux électriques*, Primaths, **4** (1981), 35–42.
- [39] RAUZY, G.: *Discrédance d'une suite complètement équirépartie*, Ann. Fac. Sci. Toulouse Math, 5^e série, **3**, no. 2 (1981), 105–112.
- [40] RAUZY, G.: *Nombres algébriques et substitutions*, Bull. Soc. Math. France, **110** (1982), 147–178.
- [41] RAUZY, G.: *Suites à termes dans un alphabet fini*, Séminaire de Théorie des Nombres, 1982–1983, Univ. Bordeaux I, Talence (1983), Exp. No. 25, 16 p.
- [42] DABOUSSI, H. – LIARDET, P. – RAUZY, G. éditeurs: *Colloque de théorie analytique et élémentaire des nombres*, Marseille, May 30–June 3 (1983) in Publications Mathématiques d'Orsay **86-1**. Université de Paris-Sud, Département de Mathématique, Orsay (1986).
- [43] RAUZY, G.: *Des mots en arithmétique*, Avignon Conference on language theory and algorithmic complexity (Avignon, 1983), in: Publ. Dép. Math. Nouvelle Sér. B, **84-6** (1984), Univ. Claude-Bernard, Lyon, 103–113.
- [44] RAUZY, G.: *Ensembles à restes bornés*, Séminaire de Théorie des Nombres, 1983–1984, Univ. Bordeaux I, Talence (1984), Exp. No. 24, 12 p.
- [45] RAUZY, G.: *Mots infinis en arithmétique*, Automata on infinite words (Le Mont-Doré, 1984), Lecture Notes in Comput. Sci. **194**, Springer, Berlin (1985), 165–171.
- [46] KANEMITSU, S. – NAGASAKA, K. – RAUZY, G. – SHIUE, J.-S.: *On Benford's law: the first digit problem*, Probability theory and mathematical statistics (Kyoto, 1986), in: Lecture Notes in Math. **1299** Springer, Berlin-New York (1988), 158–169.

GÉRARD RAUZY (1938 – 2010)

- [47] LIARDET, P. – RAUZY, G. (Guest-Editors): *Conference on Arithmetics and Coding Systems, Marseille – Luminy, France, 9-13 June 1987*, in Theoretical Comp. Sci., Fundamental Studies, Vol. 65, no. 2 (1989), ii + 150 p.
- [48] RAUZY, G.: *Rotations sur les groupes, nombres algébriques, et substitutions*, Séminaire de Théorie des Nombres 1987–1988, Univ. Bordeaux I, Talence (1988), Exp. No. 21, 12 p.
- [49] AMICE, Y. – BERTIN, M.-J. – BERTRANDIAS, F. – DECOMPS, A. – DRESS, F. – GRANDET, M. – MENDÈS FRANCE, M. – RAUZY, G.: *Charles Pisot*, Acta Arith. **51** (1988), no. 1, 1–4.
- [50] RAUZY, G.: *Numbers and automata*, Formal properties of finite automata and applications (Ramatuelle, 1988), in: Lecture Notes in Comput. Sci., **386**, Springer, Berlin (1989), 176–185.
- [51] RAUZY, G.: *Sequences defined by iterated morphisms*, in: Sequences (Naples/Positano, 1988), Springer, New York (1990), 275–286.
- [52] ARNOUX, P. – RAUZY, G.: *Représentation géométrique de suites de complexité $2n + 1$* , Bull. Soc. Math. France, **119** no. 2 (1991), 199–215.
- [53] RAUZY, G.: *Low complexity and geometry*, Dynamics of complex interacting systems (Santiago, 1994), in Nonlinear Phenom. Complex Systems, 2, Kluwer Acad. Publ., Dordrecht (1996) 147–177.
- [54] ERDÖS, P. – LEV, V. – RAUZY, G. – SÁNDOR, C. – SÁRKÖZY, A.: *Greedy algorithm, arithmetic progressions, subset sums and divisibility*, in: Paul Erdős memorial collection. Discrete Math. **200**, no. 1-3 (1999), 119–135.
- [55] HEGYVÁRI, N. – RAUZY, G.: *On the completeness of certain sequences*, Publ. Math. Debrecen **55** (1999), no. 3-4, 245–252.

Pierre Liardet
Université d’Aix-Marseille
CMI, 39 rue F. Jolio-Curie,
13453 Marseille cedex 13, France
E-mail: pierre.liardet@univ-amu.fr