special theory of relativity for photons

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By

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Abstract

The photon mass is ordinarily assumed to be exactly zero. However, this is merely a theoretical assumption; there is no experimental evidence to indicate that the photon mass is identically zero. In contrast, there are various experimental methods that have been used to set upper limits on the photon mass. If there is any deviation from zero, it must be very small. Nevertheless, even a small nonzero value would have many consequences in many theories in modern physics. It would mean that we could treat the photon as a particle that is approximately analogous to an electron. Photon mass would imply that the famous c is not a universal constant but instead depends on the photon energy, just as in the case of other particles within nonzero mass. In a related problem, we will study the Lorentz contraction of a rod using the Lorentz transformation equations. We will see how Lorentz transformations can demonstrate, remarkably, that under certain special conditions, length expansion is also possible! The aim of this study is combine all of these components – photon mass, length variation, and Doppler effect – to develop a complete special theory of relativity for the photon as a particle.

Introduction

The great success of Maxwellian electrodynamics and QED is based on the hypothesis that the photon should be a particle with zero mass, which has led to an almost total acceptance of the massless photon concept. The possibility of a nonzero photon mass has been studied by De Broglie, Vigier, Bass and Schrödinger [2][3][4] as well as Okun [13] [14] and others. The photon mass can be estimated using the uncertainty principle [10]

$$m_\gamma \sim \frac{\hbar}{(\Delta t)c^2} \sim 10^{-66} g \ ,$$

with the knowledge that the age of the universe is approximately 10^{10} years. Many laboratory experiments and astrophysical observations have been performed, using many methods, to check directly or indirectly whether the photon has mass. Table 1 shows several important limits on the photon mass [11]. The Particle Data Group lists the mass of the photon to be $< 6\times10^{-17} eV$ or $m_\gamma \leq 1\times10^{-49} g$ [11].

There are many consequences of nonzero photon mass: the speed of light would depend on its frequency, the usual Coulomb potential would become a Yukawa potential, Maxwell's equations would be replaced by Proca's equations, the black-body radiation formula would take on a new form, and many other theories would also be affected.

In addition, it seems that a nonzero photon mass would have an impact on the special theory of relativity, because the photon mass would affect the universal constant c. In fact, however, this is not necessarily true. We could simply consider that the velocity that is the key quantity in special relativity is not the velocity of light but rather a constant of nature, which is the maximum speed that any object could theoretically attain in space-time.

Although the mass of the photon is very small, any nonzero photon mass would have many consequences at a theoretical level. In this study, we will attempt to derive a dynamical relativistic energy equation for the photon as a particle. We then will see how Lorentz transformations can demonstrate, remarkably, that under certain special conditions, length expansion is also possible. All of these results together provide us with a bizarre new picture of the photon behavior.

Table 1

Author (date)	**Experimental approach**	**Upper limit on m_γ/ g**
Terrestrial results		
Goldhaber et al. (1971)	Speed of light	5.6×10^{-42}
Williams et al. (1971)	Test of Coulomb's law	1.6×10^{-47}
Chernikov et al. (1992)	Test of Ampere's law	8.4×10^{-46}
Lakes (1998)	Static torsion balance	2×10^{-50}
Luo et al. (2003)	Dynamic torsion balance	1.2×10^{-51}
Extraterrestrial results		
De Broglie (1940)	Dispersion of starlight	0.8×10^{-39}
Feinberg (1969)	Dispersion of starlight	10^{-44}
Schaefer (1999)	Dispersion of gamma-ray bursts	4.2×10^{-44}
Davis et al. (1975)	Analysis of Jupiter's magnetic field	8×10^{-49}
Fischbach et al. (1994)	Analysis of Earth's magnetic field	1.0×10^{-48}
Ryutov (1997)	Solar wind magnetic field and plasma	10^{-49}
Gintsburg (1964)	Altitude dependence of geomagnetic field	3×10^{-48}
Patel (1965)	Alfvén waves in Earth's magnetosphere	4×10^{-47}
Hollweg (1974)	Alfvén waves in the interplanetary medium	1.3×10^{-48}
Barnes et al. (1974)	Hydromagnetic waves	3×10^{-50}
DeBernadis et al. (1984)	Cosmic background radiation	3×10^{-51}
Williams et al. (1971)	Galactic magnetic field	3×10^{-56}
Chibisov (1976)	Stability of the galaxies	3×10^{-60}

Part I Photon mass

1. Maxwell and Proca equations

The photon mass is assumed to be exactly zero in the original Lagrangian density, which is given by [5]

$$\mathcal{L} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu} - \frac{1}{c}J_\mu A^\mu , \qquad (1.1)$$

where A^μ and J_μ are the 4- dimensional vector potential and the 4-dimentional vector current density, respectively. $F_{\mu\nu}$ is the antisymmetric field stress tensor and is defined as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \qquad (1.2)$$

From the Euler-Lagrange equation, the equations of motion for a massless field are

$$\partial_\nu F^{\mu\nu} = \frac{4\pi}{c}J^\mu . \qquad (1.3)$$

In the 1930's, Proca added a mass term to the Lagrangian density for the electromagnetic field, modifying the Lagrangian density (1.1) to [5]

$$\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \frac{\mu_\gamma^2}{8\pi} A_\mu A^\mu - \frac{1}{c} J_\mu A^\mu . \qquad (1.4)$$

The equations of motion that correspond to this Lagrangian density are Proca's equations:

$$\partial^\mu F_{\mu\nu} + \mu_\gamma^2 A_\nu = \frac{4\pi}{c} J_\nu . \qquad (1.5)$$

The wave equation can be obtained by substituting equation (1.2) into (1.5):

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \mu_\gamma^2\right) A_\mu = \frac{4\pi}{c} J_\mu , \qquad (1.6)$$

and in the absence of the source, equation (1.6) reduces to

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \mu_\gamma^2\right) A_\mu = 0 . \qquad (1.7)$$

This is the Klein-Gordon equation for the photon. Because the extra term could be interpreted as a characteristic length, the Compton wavelength of the photon is [11]

$$\mu_\gamma^{-1} = \frac{\hbar}{m_\gamma c} , \qquad (1.8)$$

where m_γ is the photon mass. Unfortunately, gauge invariance is lost when $\mu_\gamma^{-1} > 0$.

In the presence of a nonzero photon mass, Maxwell's equations will become Proca's equations [11]:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} - \mu_\gamma^2 \phi \qquad (1.9)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (1.10)$$

$$\nabla \cdot \vec{B} = 0 \qquad (1.11)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} - \mu_\gamma^2 \vec{A} . \qquad (1.12)$$

Here, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, ρ is the charge density, $\vec{J}$ is the current density, ϕ is the scalar potential, $\vec{A}$ is the vector potential, c is the speed of light, and μ_γ^2 is the Compton wavelength of the photon. It is clear that Proca's equations reduce to Maxwell's equations when $\mu_\gamma = 0$.

The solution of equation (1.6) is the electromagnetic field in free space,

$$A_\nu = e^{-i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r})} , \qquad (1.13)$$

with

$$k^2 = \frac{\omega^2}{c^2} - \mu_\gamma^2 , \qquad (1.14)$$

where $\boldsymbol{k}$ is the wave vector, and ω is the angular frequency. Then, the group velocity will be

$$v_g = \frac{d\omega}{dk} = c\sqrt{1 - \frac{\mu_\gamma^2 c^2}{\omega^2}} , \qquad (1.15)$$

or, using (1.8), we can write

$$v_g = c\sqrt{1-\frac{m_\gamma^2 c^4}{h^2\nu^2}}\,. \qquad (1.16)$$

This is the most important consequence of nonzero photon mass: the speed of light will depend on the frequency of the electromagnetic wave. It is clear that $v_g = c$ only when $m_\gamma \to 0$ or when the frequency approaches infinity, $\upsilon \to \infty$.

2. Relativistic total energy of the photon

We can derive equation (1.16) in another simple way. If we use the quantum energy formula for photons and the relativistic total energy of particles, we will find, directly, that

$$E = h\nu = \frac{m_\gamma c^2}{\sqrt{1-\frac{v_g^2}{c^2}}} \Rightarrow\; v_g = c\sqrt{1-\frac{m_\gamma^2 c^4}{h^2\nu^2}}\,. \qquad (1.17)$$

When $v_g = c$, it must be that $m_\gamma = 0$, and E will be unknown. Also, $v_g = 0$ only when

$$\frac{m_\gamma^2 c^4}{h^2\upsilon^2} = 1 \Rightarrow\; m_\gamma c^2 = h\nu_\circ\ . \qquad (1.18)$$

This means that the photon is in its rest frame. We know that there is no rest frame for the photon if the speed of light does not change. However, if c is variable with mass and frequency, we could imagine, theoretically, that the photon could have a rest frame. Then, $\nu_\circ$ would be the rest frequency, which would correspond, in some way, to the photon rest mass m_γ.

Now, we will combine the relativistic Doppler effect with equation (1.17). The relativistic Doppler effect for an observer receding from the light source is given by [9]

$$\nu = \nu_\circ\sqrt{\frac{1-{}^{v}/_{c}}{1+{}^{v}/_{c}}}\,. \qquad (1.19)$$

For an observer approaching the light source, it is given by [9]

$$\nu = \nu_\circ\sqrt{\frac{1+{}^{v}/_{c}}{1-{}^{v}/_{c}}}\,, \qquad (1.20)$$

where $\nu_\circ$ is the rest frequency, and v is the velocity of the observer. When we insert equations (1.19) and (1.20) into (1.17), we find, after some algebra, that

$$E = h\nu_\circ\sqrt{\frac{1-{}^{v}/_{c}}{1+{}^{v}/_{c}}}\,, \qquad (1.21)$$

and

$$E = h\nu_\circ\sqrt{\frac{1+{}^{v}/_{c}}{1-{}^{v}/_{c}}}\,. \qquad (1.22)$$

Using (1.18) instead, we find

$$E = m_\gamma c^2\sqrt{\frac{1-{}^{v}/_{c}}{1+{}^{v}/_{c}}}\,, \qquad (1.23)$$

and

$$E = m_\gamma c^2\sqrt{\frac{1+{}^{v}/_{c}}{1-{}^{v}/_{c}}}\,. \qquad (1.24)$$

Equation (1.23) shows that the energy of the single photon, treated as a particle, decreases as the observer's velocity increases, whereas equation (1.24) shows that the energy increases with an increase in the observer's velocity.

Equations (1.21) and (1.22) are merely the normal Doppler-shift effect, i.e., (1.19) and (1.20), multiplied by Planck's constant. However, the interpretation of the two sets of equations is entirely different. The Doppler effect treats light as a wave, but equations (1.23) and (1.24) apply to a single photon with a tiny mass m_γ. Thus, we must be careful here, because the photon has finite dimensions like other particles.

Part II Photon dimensions

1. Lorentz contraction and Lorentz expansion

Equation (1.23) indicates that space-time has a special character. We know that all kinematical and dynamical phenomena in the special theory of relativity arise as necessary consequences of the nature of space-time and the Lorentz transformations. Diagram 1 illustrates how the equations of the special theory of relativity have been carefully constructed in relation to each other.

$$lorentz\ transformations$$

$$\Downarrow$$

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}} \quad ; \quad l = l_\circ \sqrt{1 - \frac{v^2}{c^2}}$$

$$\Downarrow$$

$$u^i = \frac{v^i}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Downarrow$$

$$p = mu^i$$

$$\Downarrow$$

$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Diagram 1

where $d\tau$ is the proper time, and u^i is the proper velocity.

We have shown, in equation (1.23), that the energy of the photon, when treated as a tiny particle, decreases as the velocity of the light source increases. This phenomenon is familiar from the relativistic Doppler effect, but here, we focused our attention on a single photon, which is clear from the presence of Planck's constant in equations (1.21) and (1.22). Equation (1.23) poses an important and exciting question: are there kinematical and dynamical equations that are consistent with equation (1.23)? Fortunately, just as Lorentz transformations lead to length contraction, we will show that, under very special conditions, Lorentz transformations can also lead to both expansion and invariance in length!

Suppose that there is reference frame S with spatial coordinates x, y, z and time t. Let $S^{'}$ be another reference frame with coordinates $x^{'}, y^{'}, z^{'}$ and time $t^{'}$ that moves with speed v, relative to S in the positive direction along the x axis. The relation between the coordinates and time of an event in the inertial frame S and the coordinates and time of the same event as observed in the second inertial frame $S^{'}$ is given by the following Lorentz transformations [9]:

$$x^{'} = \beta(x - vt) \qquad (1.25)$$

$$y^{'} = y \qquad (1.26)$$

$$z^{'} = z \qquad (1.27)$$

$$t^{'} = \beta\left(t - \frac{vx}{c^2}\right), \qquad (1.28)$$

where $\beta = \sqrt{1 - \frac{v^2}{c^2}}$. If $S^{'}$ instead moves in the negative direction along the x axis, we have only to change the sign of the relative velocity v because of the symmetry of the equations. Therefore,

$$x = \beta\left(x^{'} + vt\right) \qquad (1.29)$$

$$y^{'} = y \qquad (1.30)$$

$$z^{'} = z \qquad (1.31)$$

$$t = \beta\left(t^{'} + \frac{vx}{c^2}\right). \qquad (1.32)$$

The famous consequences of the Lorentz transformations are length contraction and time dilation, as revealed by Einstein in his famous paper in 1905 [1]. However, the Lorentz transformations also lead to other results concerning the relativity of length. Under certain special conditions, Lorentz transformations also result in length invariance and length expansion! This result was demonstrated by Sadanand D. Agashe [6]. Therefore, we will follow his derivation to illustrate how Lorentz transformations can lead to length contraction and then how these transformations can also lead to length invariance and expansion. He wrote the following:

"In deducing the Lorentz contraction in such a scenario, most authors talk about a rigid rod lying at rest on the x' − axis of the moving system S'. The ends $P1$ and $P2$ of this rod can thus be thought of as a series of events: P 1 ≡ $\{(x_1', 0, 0, t')\}$ and P 2 ≡ $\{(x_2', 0, 0, \bar{t}')\}$, with $x_2^{'} - x_1^{'} > 0$, say, so that we can call l the constant (in S') length of the rod, and so we are justified in calling the rod "rigid" in S'. Next, one shows that although the rod is at rest in S', it is "observed" to be moving in S with speed v, of course. Further, as it moves in S, its length remains constant in S, and so it is rigid in S also. However, its length in S is different from its length l in S', and is, in fact, $\beta^{-1}l$, which is smaller than l. Hence the term "contraction". Indeed, using [(1.29)...(1.32)], at any time t of S, the coordinates of $P1$ in S are $(\beta(x_1^{'} + vt_1^{'}), 0, 0)$, and the coordinates of $P2$ in S are $(\beta(x_2^{'} + vt_2^{'}), 0, 0)$, where $t_1^{'}$ and $t_2^{'}$ in S' correspond to a common time t in S and so,

$$t = \beta\left(t_1^{'} + \frac{vx_1^{'}}{c^2}\right) = \beta\left(t_2^{'} + \frac{vx_2^{'}}{c^2}\right) \qquad (1.33)$$

The distance between $P1$ and $P2$ in S at time t, and, thus, the length of the rod in S at time t are given by,

$$\begin{aligned}\beta\left(x_2^{'} + vt_2^{'}\right) - \beta\left(x_1^{'} + vt_1^{'}\right) &= \beta\left(x_2^{'} - x_1^{'}\right) + \beta v\left(t_2^{'} - t_1^{'}\right) \\ &= \beta\left(x_2^{'} - x_1^{'}\right) - \beta v\frac{v}{c^2}\left(x_2^{'} - x_1^{'}\right) \\ &= \beta\left(1 - \frac{v^2}{c^2}\right)\left(x_2^{'} - x_1^{'}\right) \\ &= \frac{1}{\beta}l. \qquad (1.34)\end{aligned}$$

All this is very familiar and is written only to fix the notation and to avoid misunderstanding. Note that one could allow the rod to be anywhere in the space of S', provided it is parallel to the

x' $-$axis." [6]

He then derived length expansion and invariance, under the title "What happens to a rod moving with an arbitrary velocity?":

"Now, this business of considering the rod at rest in a moving frame of reference goes back to the early days of the special theory of relativity. One could have talked about a rod lying at rest in the first-mentioned frame, namely, S, and then considered its history as observed from the second-mentioned frame, namely, S', assumed to be moving uniformly relative to S. (This is, indeed, pointed out by many authors.) Leaving that aside, no authors seem to have considered a rod rigidly moving in S and its history in S'. This is what we will do, deriving some surprising consequences. (In one excellent textbook [9], in Exercise 3.11, p.135, the possibility of the times t and t' of a moving point being equal is explored.)

So, let a point $P1$ have a history- or motion! - in S, given by the series of events $\{(x_\circ + ut, 0, 0, t)\}$, thus $P\,1$ moves uniformly in S with speed $|u|$ in the direction of the positive $x-$ axis of S if $u > 0$ and in the opposite direction if $u < 0$. Let another point have the motion $\{(x_\circ + l + u\bar{t}, 0, 0, \bar{t})\}$, with $l > 0$. Thus $P2$ also moves in S with the same speed and in the same direction, and the distance between $P1$ and $P2$ remains constant in S. We could think of $P1$ and $P2$ as the ends of a rod moving in S, and that too, rigidly, since its length remains constant in S.

What are the motions of $P1$ and $P2$ in S'? Is the distance between them constant in S' too, so that the rod remains rigid in S'? Indeed the motions of $P1$ and $P2$ are uniform in S' too, since they are given by,

$$P1 : \left[\left(\beta(x_\circ + ut - vt), 0, 0, \beta\left(t - \frac{v(x_\circ + ut)}{c^2}\right)\right)\right]$$

$$P2 : \left[\left(\beta(x_\circ + l + u\bar{t} - v\bar{t}), 0, 0, \beta\left(\bar{t} - \frac{v(x_\circ + l + u\bar{t})}{c^2}\right)\right)\right].$$

Their common speed is given by,

$$\frac{u - v}{(1 - \frac{uv}{c^2})}. \qquad (1.35)$$

The S' $-$ distance between $P1$ and $P2$ at a time t' in S' is given by

$$\beta(x_\circ + l + u\bar{t} - v\bar{t}) - \beta(x_\circ + ut - vt), \qquad (1.36)$$

where t and $\bar{t}$ are related to t' by,

$$t' = \beta\left(t - \frac{v(x_\circ + ut)}{c^2}\right) = \beta\left(\bar{t} - \frac{v(x_\circ + l + u\bar{t})}{c^2}\right). \qquad (1.37)$$

The distance calculates out to be,

$$\frac{1}{\beta(1 - \frac{uv}{c^2})} l. \qquad (1.38)$$

Thus, the rod is observed to stay rigid in S' too. But is its length in S' necessarily smaller than its length l observed in S? Denoting the factor multiplying l in (1.38) by $k(u)$, the function k has the following values:

$$K\left(\frac{c^2}{v}\right) = \infty, \qquad (1.39)$$

$$k(c) = \frac{1}{\beta\left(1-\frac{v}{c}\right)} = \sqrt{\frac{1+{}^{v}/_{c}}{1-{}^{v}/_{c}}} > 1, \qquad (1.40)$$

$$k(v) = \frac{1}{\beta\left(1-\frac{v^2}{c^2}\right)} = \beta > 1, \qquad (1.41)$$

$$k(0) = \frac{1}{\beta} < 1, \qquad (1.42)$$

$$k(-c) = \frac{1}{\beta\left(1-\frac{v}{c}\right)} = \sqrt{\frac{1-{}^{v}/_{c}}{1+{}^{v}/_{c}}} < 1\,, \qquad (1.43)$$

$$k(-\infty) = 0, \qquad (1.44)$$

with

$$k(-c) < k(0) < 1 < k(v) < k(c).$$

The case $u = 0$ corresponds to the rod being at rest in S and its length in S' is observed to be smaller than its length in S, but if $u = v$, the rod is at rest in S', its length in S' is observed to be larger than its length in S. Further, there is a particular value of u, namely:

$$\bar{u} = \frac{c^2}{v}\left[1-\sqrt{1-{}^{v^2}/_{c^2}}\right], \qquad (1.45)$$

such that $k(\bar{u}) = 1$, and $0 < \bar{u} < v$. Thus, there is a speed $\bar{u}$ for which the rod is observed to be moving in both S and S', but its length is observed to be the same in both.
So, we can have not only a contraction but also an expansion and even invariance!" [6]

Agashe analyzed all of these results in detail, and he also carefully discussed Einstein's papers [6][7][8]. He considered that [6]

[...**the "change" in length is purely a kinematical fact, arising out of the manner in which the two systems S and S' and their coordinates and times are related, and we need not look for any dynamical reason for the change in either system**.]

In my opinion, all of these equations may be merely superfluous mathematics, with the exception of equation (1.41). We can relate this result, i.e., length expansion, to the idea of the decrease in the photon energy when the photon is treated as a particle, i.e., to equation (1.23). Via this connection, we can build a full model of the photon as a particle.

Imagine two reference frames: S moves with velocity u, and S' moves with velocity v with respect to S. As Agashe argued, if we consider a rod lying at rest in the frame S and then consider its history as observed from the second frame S', we will obtain many surprising consequences. The rod length may contract, expand, or even remain invariant. We will focus our attention on the phenomenon of expansion. As we have seen, length expansion occurs under very special conditions: when the velocity of frame S is equal to the velocity of frame S', i.e., when $u = v$. Suppose that frame S' moves at a speed very close to the speed of light, $v{\sim}c$, but not identical to the absolute universal constant c, and frame S moves with the same velocity, $u = v$. Imagine that frame S is a single photon with velocity u and has a tiny mass m_γ. Now, we will consider the history of this single photon, as observed in frame S$'$.

If the photon has a mass m_γ, then it must also have a dimension in space-time in some sense. According to equation (1.41), the observer in S' will see the photon become longer than its original length in the direction of the positive x axis. We can imagine the single photon as a pulse, with a wavelength λ (Fig. 1); when the photon expands, it becomes redder and redder, according to the relativistic Doppler-shift effect. Thus, equation (1.41) is well consistent with equation (1.23). As the velocity of frame $S^{\prime}$ increases, the length of the single photon in S will expand further, and the energy of that photon, treated as a particle, will also decrease further. Of course, we can also imagine the contrasting situation. When the velocity of the single photon exceeds the velocity of frame $S^{\prime}$, i.e., when $v < u$, the photon will contract and appear bluer as it approaches frame $S^{\prime}$ (Fig. 2).

2. What about the relativistic time?

We know that the length, or, more accurately, space, is working together with time to keep the speed of light a universal constant. In the special theory of relativity, length contraction and time dilation maintain the speed of light at a constant. They also conserve Lorentz invariance. Diagram 1 shows very clearly that the equations of the special theory of relativity depend on one another. Therefore, any change in one of them will lead to a change in all other equations. We derived the new equation of energy; the energy of photons, when they are treated as particles, can decrease. We then showed, as derived by Agashe, that the length of a rod can expand or remain invariant, and length expansion is consistent, in principle, with the equation of decreasing energy. Because of this consistency, the proper velocity, the relativistic momentum and all other dynamical quantity must exist. However, the most bizarrely affected quantity is the time. If equation (1.41) is correct and is applied to the length of a rod, time or the clocks used to measure it must be speeding up! We can easily demonstrate the truth of this statement. Consider again (fig. 1), and suppose that there is a particle the frame $S^{\prime}$ with a mean lifetime $d\tau$. If equation (1.41) is correct, the length of the rod in frame S will expand to

$$l = \frac{l_{\circ}}{\sqrt{1-\frac{v^2}{c^2}}}. \qquad (1.46)$$

In this case, however, to the observer in the particle frame, all space behind the observer will appear to expand according to equation (1.46), not just the rod in frame S. Thus, if the time interval between two events as measured in the particle frame $S^{\prime}$ is $d\tau$, it will be different in frame S. In this frame, the length will not change, so the particle will occupy a smaller distance than in frame $S^{\prime}$, but the two observers must agree on the time at which the particle disappears. For this to happen, there is only one possibility: time must speed up in frame $S^{\prime}$ relative to frame S! This means that the equation for the time interval must reverse in frame S to obtain the same result for the two reference frames. Therefore, for S, time must obey this equation:

$$\frac{dt}{d\tau} = \sqrt{1-\frac{v^2}{c^2}}. \qquad (1.47)$$

Of course, time also speeds up in frame S relative to frame $S^{\prime}$. The two equations (1.46) and (1.47) will leave the speed of light, universally, constant, and they satisfy Lorentz invariance and the laws of physics.

In conclusion, we can say that if photons have a mass, albeit a very tiny one, there will be many surprising consequences at the theoretical level. A nonzero photon mass would allow the photon to have a mean lifetime and to decay to lighter particles! If the photon really has a mass and a mean lifetime and if the equations (1.23), (1.46) and (1.47) are correct, using some experimental arrangement that simulates Fig 1, one could verify that time speeds up for the photon and that its energy decreases. Of course, it is very difficult to verify equation (1.46) because there is no direct evidence of length contraction, but we know that it must occur to be consistent with time dilation. The same logic applies for these new results: evidence of one of them is sufficient to know that the others must also be correct.

This model may apply only for photons because of the dual nature of the photon. Space-time, according to this model, does not exhibit only spatial contraction accompanied by time dilation but also spatial expansion accompanied by time speeding up. Space-time may be very elastic. It may be able to contract and expand in both length and time to leave the speed of light universally constant.

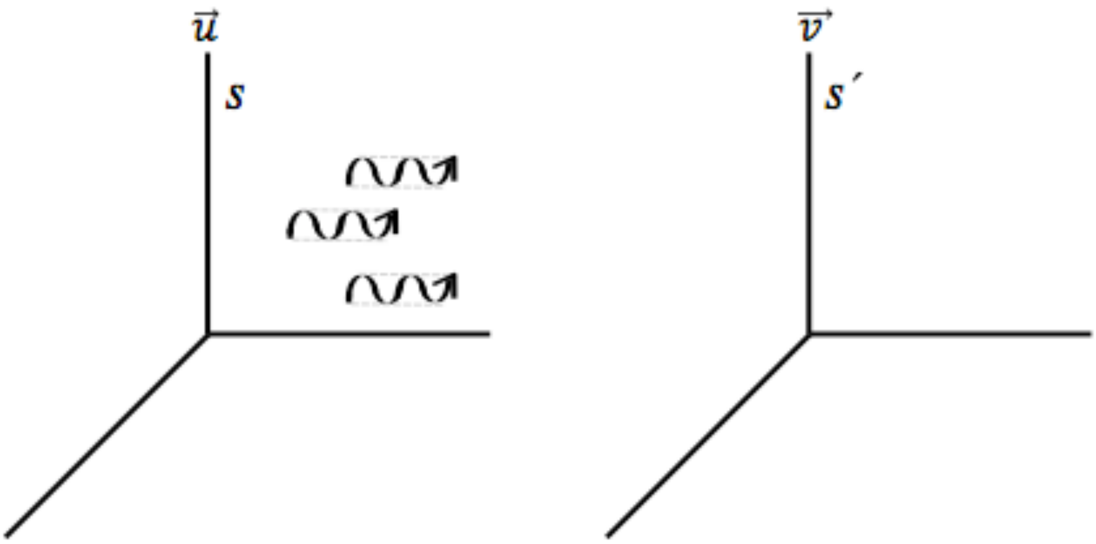

Fig. 1

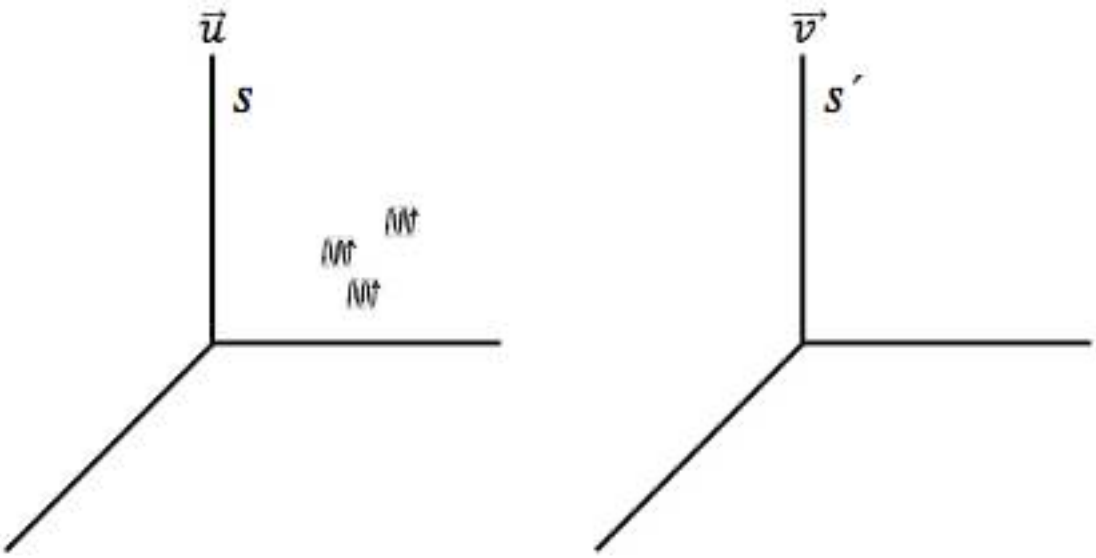

Fig. 2

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