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# Convex sums of Farlie-Gumbel-Morgenstern distributions

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## Abstract

A copula is a function that completely describes the dependence structure between the marginal distributions. One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family. We establish a stability property of the FGM copula with respect to convex sums.

A copula is a function that completely describes the dependence structure. It contains all the information to link the marginal distributions to their joint distribution. To obtain a valid multivariate distribution function, it suffices to combine several marginal distribution functions with any copula function. Thus, for the purposes of statistical modeling, it is desirable to have a large collection of copulas at one's disposal. Many examples of copulas can be found in the literature, most are members of families with one or more real parameters.

One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family defined as

$$C_{\theta}^{FGM}(u, v) = uv + \theta uv(1 - u)(1 - v), \quad u, v \in [0, 1], \quad (1)$$

where  $\theta \in [-1, 1]$ . The density function of FGM copulas is given by

$$\frac{\partial^2 C_{\theta}^{FGM}(u, v)}{\partial u \partial v} = \theta (2u - 1)(2v - 1) + 1, \quad (2)$$

for any  $u, v \in [0, 1]$ .

Members of the FGM family are symmetric, i.e.,  $C_{\theta}^{FGM}(u, v) = C_{\theta}^{FGM}(v, u)$  for all  $(u, v)$  in  $[0, 1]^2$  and have the lower and upper tail dependence coefficients equal to 0.

The copula given in (1) is PQD for  $\theta \in (0, 1]$  and NQD for  $\theta \in [-1, 0)$ . In practical applications this copula has been shown to be somewhat limited, for copula dependence parameter  $\theta \in [-1, 1]$ , Spearman's correlation  $\rho \in [-1/3, 1/3]$  and Kendall's  $\tau \in [-2/9, 2/9]$ , for more details on copulas see, for example, [1].

To overcome this limited dependence, several authors proposed extensions of this family.

- Theoretical issues: [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]
- Applications: [21, 22, 23, 24, 25, 26]

Our main result is the following:

**Theorem.** *Let  $C_\theta$  be a FGM copula with associated parameter  $\theta$ , considered as an observation of a random variable  $\Theta$  with distribution  $\Lambda$ . If we set*

$$C'(u, v) = \int C_\theta(u, v) d\Lambda(\theta)$$

*then,  $C'$  is a FGM copula with associated parameter  $\int \theta d\Lambda(\theta)$ .*

*Proof.* The convex sum operation is defined for instance in [1], page 64. The new copula  $C'$  can be computed as:

$$\begin{aligned} c(u, v) &= \int uv + \theta uv(1-u)(1-v) d\Lambda(\theta) \\ &= uv + \int \theta d\Lambda(\theta) uv(1-u)(1-v) \end{aligned}$$

and the conclusion follows from (1).

As a consequence, it follows that the dependence coefficient of  $C'$  can be interpreted as the mathematical expectation of distribution  $\Lambda$ .

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