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Generation of spectrum-compatible accelerograms using information theory

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Abstract. The research addressed here is devoted to the generation of seismic accelerograms compatible with a given response spectrum and other associated properties. The time sampling of the stochastic accelerogram yields a time series represented by a random vector in high stochastic dimension. The probability density function (pdf) of this random vector is constructed using the Maximum Entropy (MaxEnt) principle under constraints defined by the available information. In this research, a new algorithm, adapted to the high stochastic dimension, is proposed to identify the Lagrange multipliers introduced in the MaxEnt principle to take into account the constraints. This novel algorithm is developed in the context of the methodology based on (1) the minimization of an appropriate convex functional and (2) the construction of the probability distribution defined as the invariant measure of an Itô Stochastic Differential Equation (belonging to the class of MCMC methods) in order to estimate the integrals in high dimension of the problem. The algorithm is validated through an application for which the available information is relative to the variance of each component of the random vector representing the accelerogram, statistics on the response spectrum such as the mean value and the envelopes, statistics on the Peak Ground Acceleration (PGA) and the velocity and displacement traces (behavior of the signals at the final time) .

Keywords: Seismic accelerogram; Maximum Entropy Principle; ISDE; PGA.

1 INTRODUCTION

This research is devoted to the generation of seismic accelerograms which are compatible with some design specifications such as the Velocity Response Spectrum, the Peak Ground Acceleration (PGA), etc. The Maximum Entropy (MaxEnt) principle (Kapur and Kevasan 1992) is a powerful method which allows us to construct a probability distribution of a random vector under some constraints defined by the available information. This method has recently been applied (Soize 2010) for the generation of spectrum-compatible accelerograms as trajectories of a non-Gaussian non-stationary centered random process represented by a high-dimension random vector for which the probability density function (pdf) is constructed using the MaxEnt principle under constraints relative to (1) the mean value, (2) the variance of the components and (3) the mean value of the Velocity Response Spectrum (VRS).

The objective of this paper is to take into account additional constraints which characterize the natural features of a seismic accelerogram. To achieve this objective, the methodology proposed by (Soize 2010) is extended to take into account constraints relative to statistics on (1) the end values for the velocity and the displacement, (2) the PGA, (3) the Peak Ground Velocity (PGV), (4) the envelop of the random VRS and (5) the Cumulative Absolute Velocity (CAV). The MaxEnt pdf is constructed and a generator of independent realizations adapted to the high-stochastic dimension of an accelerogram is proposed. Furthermore an adapted method for the identification of the Lagrange multipliers is developed.

In Section 2, the MaxEnt principle is used to construct the pdf of the acceleration random vector under constraints defined by the available information. In Section 3, the available information relative to seismic accelerograms is presented. Finally, Section 4 is devoted to an application of the methodology for which the target VRS is constructed following the Eurocode 8.

2 CONSTRUCTION OF THE PROBABILITY DISTRIBUTION

The MaxEnt principle is a powerful method to construct the probability distribution of a random vector associated with a sampled stochastic process under some constraints defined by the available information. The random acceleration of the soil is modeled by a non-Gaussian second-order centered stochastic process $\{A(t), t \in [0,T]\}$. A time sampling, with $t_j = j\,\Delta t$, $j = 1,\ldots,N$ and $T = N \times \Delta t$, of this stochastic process is introduced yielding a time series $\{A_1,\ldots,A_N\}$ with $A_j = A(t_j)$ and for which the random vector $\boldsymbol{A} = (A_1,\ldots,A_N)$ is associated with. Finally, we have to construct the probability distribution of random vector $\boldsymbol{A}$ such that

$$E\{\boldsymbol{g}(\boldsymbol{A})\} = \boldsymbol{f}\,, \tag{1}$$

in which $\boldsymbol{g}(\boldsymbol{A})$ is a given function and where $\boldsymbol{f}$ is a target vector. Equation (1) can be rewritten as

$$\int_{\mathbb{R}^N} \boldsymbol{g}(\boldsymbol{a})\, p_{\boldsymbol{A}}(\boldsymbol{a})\, \mathrm{d}\boldsymbol{a} = \boldsymbol{f}\,. \tag{2}$$

An additional constraint relative to the normalization of the pdf is introduced such that

$$\int_{\mathbb{R}^N} p_{\boldsymbol{A}}(\boldsymbol{a})\, \mathrm{d}\boldsymbol{a} = 1\,. \tag{3}$$

The entropy of pdf $p_{\boldsymbol{A}}$ is defined by

$$S(p_{\boldsymbol{A}}) = -\int_{\mathbb{R}^N} p_{\boldsymbol{A}}(\boldsymbol{a}) \log(p_{\boldsymbol{A}}(\boldsymbol{a}))\, \mathrm{d}\boldsymbol{a}\,, \tag{4}$$

Then the MaxEnt principle consists in constructing the pdf $p_{\boldsymbol{A}}$ as the unique pdf which maximizes the entropy. Then by introducing a Lagrange multiplier $\boldsymbol{\lambda}$ associated with Eq. (2), it can be shown that the MaxEnt solution, if it exists, is defined by

$$p_{\boldsymbol{A}}(\boldsymbol{a}) = c_0^{\mathrm{sol}} \exp(-\langle \boldsymbol{\lambda}^{\mathrm{sol}}, \boldsymbol{g}(\boldsymbol{a})\rangle)\,, \tag{5}$$

in which $c_0^{\mathrm{sol}}(\boldsymbol{\lambda}^{\mathrm{sol}})$ is a normalization constant and where $\langle .,. \rangle$ denotes the Euclidean inner product. The Lagrange multiplier $\boldsymbol{\lambda}^{\mathrm{sol}}$ is calculated by minimizing the following functional (Golan et al. (1996))

$$\Gamma(\boldsymbol{\lambda}) = \langle \boldsymbol{\lambda}, \boldsymbol{f}\rangle - \log(c_0(\boldsymbol{\lambda}))\,. \tag{6}$$

The optimal value is calculated iteratively using the Newton method

$$\boldsymbol{\lambda}^{\mathrm{i}+1} = \boldsymbol{\lambda}^{\mathrm{i}} - \alpha\, [H(\boldsymbol{\lambda}^{\mathrm{i}})]^{-1}\, \boldsymbol{\nabla}\Gamma(\boldsymbol{\lambda}^{\mathrm{i}})\,, \tag{7}$$

in which α is an under-relaxation parameter and where $\boldsymbol{\nabla}\Gamma(\boldsymbol{\lambda})$ and $[H(\boldsymbol{\lambda})]$ are respectively the gradient vector and the Hessian matrix of $\Gamma(\boldsymbol{\lambda})$ with respect to $\boldsymbol{\lambda}$, and are respectively written as

$$\boldsymbol{\nabla}\Gamma(\boldsymbol{\lambda}) = \boldsymbol{f} - E\{\boldsymbol{g}(\boldsymbol{A}_{\boldsymbol{\lambda}})\}\,, \tag{8}$$

$$[H(\boldsymbol{\lambda})] = E\{\boldsymbol{g}(\boldsymbol{A}_{\boldsymbol{\lambda}})\boldsymbol{g}(\boldsymbol{A}_{\boldsymbol{\lambda}})^T\} - E\{\boldsymbol{g}(\boldsymbol{A}_{\boldsymbol{\lambda}})\}\, E\{\boldsymbol{g}(\boldsymbol{A}_{\boldsymbol{\lambda}})\}^T\,. \tag{9}$$

The calculation of $\nabla\Gamma(\boldsymbol{\lambda})$ and $[H(\boldsymbol{\lambda})]$ requires the estimation of integrals in high dimension. In general, these integrals cannot explicitly be calculated and cannot be discretized. In this research, these integrals are estimated using the Monte Carlo simulation method for which independent realizations of the random vector $\boldsymbol{A}$ are generated using an algorithm belonging to the MCMC class which is adapted to the high dimension, as proposed in (Soize 2010). This algorithm consists in constructing the pdf of random vector $\boldsymbol{A}$ as the density of the invariant measure associated with the stationary solution of a second-order nonlinear Itô Stochastic differential equation (ISDE).

3 AVAILABLE INFORMATION RELATIVE TO SEISMIC ACCELEROGRAMS

3.1. Mean Value

The seismic accelerogram is modelled by a centered stochastic process. Therefore the vector $\boldsymbol{A}$ has to be centered. We then have the constraint

$$E\{\boldsymbol{A}\} = \boldsymbol{0}\,. \tag{10}$$

3.2. Variance of the components

This constraint allows the envelop of the accelerogram to be specified and therefore the strong motion duration to be fixed. Since random vector must be centered, it is equivalent to impose the variance of the components or their second-order moments. For $j = \{1, \ldots, N\}$, these constraints are defined by

$$E\{A_j^2\} = \sigma_j^2 < +\infty\,. \tag{11}$$

3.3. Mean value of the random VRS

For $0 < \omega_{\min} < \omega < \omega_{\max}$ and $0 < \xi_{\min} < \xi < \xi_{\max} < 1$, the random VRS $s(\omega, \xi; A)$ of stochastic process $\{A(t), t \in [0, T]\}$ is defined by (Clough and Penzien 1975)

$$s(\omega, \xi; A) = \omega \max_{t\in[0,T]} |y(t; \omega, \xi, A)|\,, \tag{12}$$

in which the stochastic process $\{y(t; \omega, \xi, A), t \in [0, T]\}$ is defined by

$$y(t; \omega, \xi, A) = \int_0^t h(t - \tau; \omega, \xi)\, A(\tau)\, d\tau\,, \tag{13}$$

where

$$h(t; \omega, \xi) = -\mathbb{1}_{[0,+\infty[}(t) \frac{1}{\omega\sqrt{1-\xi^2}}\, \exp\{-\xi\, \omega\, t\} \sin\{\omega\, \sqrt{1 - \xi^2}\, t\}\,, \tag{14}$$

in which the function $\mathbb{1}_{[0,+\infty[}(t)$ is equal to 1 if $t \in [0, +\infty[$ and is equal to 0 otherwise. Let $\{\omega_1, \ldots, \omega_{\kappa_\omega}\}$ be a sampling of interval $[\omega_{\min}, \omega_{\max}]$ (such that $\omega_{\kappa_\omega} < \pi/\Delta t$) and let $\{\xi_1, \ldots, \xi_{\kappa_\xi}\}$ be a sampling of interval $[\xi_{\min}, \xi_{\max}]$. Let be $\kappa = \kappa_\omega \times \kappa_\xi$. The discretization of Eqs. (12), (13) and (14) yields the random VRS vector $\boldsymbol{S} = \boldsymbol{s}(\boldsymbol{A})$ in which $\boldsymbol{s} = (s_1, \ldots, s_\kappa)$ is a nonlinear mapping such that

$$s_j(\boldsymbol{a}) = s(\omega, \xi, \boldsymbol{a}) \quad \text{for } (\omega, \xi)_j \text{ in } \{\omega_1, \ldots, \omega_{\kappa_\omega}\} \times \{\xi_1, \ldots, \xi_{\kappa_\xi}\}\,, \tag{15}$$

in which

$$s(\omega,\xi,\boldsymbol{a}) = \omega \max\{|y_1(\omega,\xi,\boldsymbol{a})|,\ldots,|y_N(\omega,\xi,\boldsymbol{a})|\} \quad , \tag{16}$$

with

$$y_i(\omega,\xi,\boldsymbol{a}) = \{[B(\omega,\xi)]\boldsymbol{a}\}_i\,, \tag{17}$$

and where $[B(\omega,\xi)]$ is a $(N\times N)$ real matrix defined by

$$[B(\omega,\xi)]_{ij} = -\frac{\Delta t}{\omega\sqrt{1-\xi^2}}\exp\{-(i-j)\xi\omega\,\Delta t\}\sin\{(i-j)\omega\sqrt{1-\xi^2}\,\Delta t\}\,. \tag{18}$$

The available information relative to the mean value of the random VRS is defined, for all j in $\{1,\ldots,\kappa\}$, by

$$E\{s_j(\boldsymbol{A})\} = \underline{s}_j\,, \tag{19}$$

where $\underline{\boldsymbol{s}} = (\underline{s}_1,\ldots,\underline{s}_\kappa)$ is the mean VRS which is chosen as the target.

3.4. Variability of the random VRS

The constraint defined in Section 3.3, which concerns the mean value of the random VRS, does not allow us to control the statistical fluctuations of the random VRS around its mean value. In this section, the variability of the random VRS is controlled by introducing a constraint relative to the probability that the random VRS belongs to a region delimited by two given envelops. The VRS upper envelope is defined by the vector $\boldsymbol{s}^{\rm up} = (s_1^{\rm up},\ldots,s_\kappa^{\rm up})$ and the VRS lower envelope is defined by the vector $\boldsymbol{s}^{\rm low} = (s_1^{\rm low},\ldots,s_\kappa^{\rm low})$. We then introduce the following constraint

$$P(\{s_1^{\rm low} < s_1(\boldsymbol{A}) < s_1^{\rm up},\ldots,s_\kappa^{\rm low} < s_\kappa(\boldsymbol{A}) < s_\kappa^{\rm up}\}) = p_0\,, \tag{20}$$

which can be rewritten as

$$E\{\textstyle\prod_{j=1}^{\kappa} \mathbb{1}_{[s_j^{\rm low},s_j^{\rm up}]}(s_j(\boldsymbol{A}))\} = p_0\,. \tag{21}$$

3.5. Variance of the end-velocity and the end-displacement

This constraint is introduced in order to control the end-velocity and the end-displacement which are assumed to be zero. In this paper, this correction is directly taken into account in the construction of the pdf. Let $V(t)$ and $D(t)$ be the velocity and the displacement stochastic processes indexed by $[0\,,T]$. Assuming that $V(0)=D(0)=0$ almost surely, it can easily be proven that

$$V(t) = \textstyle\int_0^t A(\tau)\,d\tau\,, \qquad D(t) = \int_0^t V(\tau)\,d\tau\,. \tag{22}$$

Performing an integration by parts in the right-hand side of Eq. (22) yields,

$$D(t) = \textstyle\int_0^t (t-\tau)A(\tau)\,d\tau\,. \tag{23}$$

Using the time sampling $t_j = j\,\Delta t$ for $j=1,\ldots,N$ and the corresponding sampling $A_j = A(t_j)$, the following discretization of Eq. (22) is then introduced,

$$I_n^{(1)}(\boldsymbol{A}) = V(t_n) \simeq \Delta t \textstyle\sum_{j=1}^n A_j\,, \tag{24}$$

$$I_n^{(2)}(\boldsymbol{A}) = D(t_n) \simeq (\Delta t)^2 \sum_{j=1}^{n} (n-j+1)\, A_j\,, \tag{25}$$

in which $\boldsymbol{A} = (A_1, \ldots, A_N)$. The zero end-velocity, $I_N^{(1)}(\boldsymbol{A}) = V(t_N)$ and the zero end-displacement, $I_N^{(2)}(\boldsymbol{A}) = D(t_N)$, are then specified in writing $I_N^{(1)}(\boldsymbol{A}) = 0$ and $I_N^{(2)}(\boldsymbol{A}) = 0$. These properties should be verified almost surely, which means that all the simulated trajectories of the acceleration stochastic process should verify this property. In this paper, these constraints are imposed in the mean-square sense and not almost surely. Since random vector $\boldsymbol{A}$ is centered, then random variables $I_N^{(1)}(\boldsymbol{A})$ and $I_N^{(2)}(\boldsymbol{A})$ are also centered. We then introduce the following constraint,

$$E\{(I_N^{(1)}(\boldsymbol{A}))^2\} = 0 \quad , \quad E\{(I_N^{(2)}(\boldsymbol{A}))^2\} = 0\,. \tag{26}$$

3.6. Mean value of the random PGA and mean value of the random PGV

The PGA characterizes the maximum amplitude of the accelerogram. The random PGA, relative to acceleration process $\{A(t), t \in [0,T]\}$, is defined by

$$\mathrm{PGA}(A) = \max_{t\in[0,T]} |A(t)|\,. \tag{27}$$

In the regulation codes, this value is used to construct the target VRS. Nevertheless, even if the mean VRS of the simulated accelerograms matches perfectly the target VRS, the mean PGA of the simulated accelerograms does not match the PGA which has been used to construct the target VRS. In this section, we propose to enforce this matching. Using the time sampling of Eq. (27), the following constraint is introduced,

$$E\{\max\{|A_1|, \ldots, |A_N|\}\} = \underline{\mathrm{PGA}}\,, \tag{28}$$

in which $\underline{\mathrm{PGA}}$ is the target value for the mean value of the PGA.
Concerning the random PGV, which is defined by $\mathrm{PGV}(A) = \max\{|V(t)|, t \in [0,T]\}$, its mean value is controlled by imposing the following constraint

$$E\{\max\{|I_1^{(1)}(\boldsymbol{A})|, \ldots, |I_N^{(1)}(\boldsymbol{A})|\}\} = \underline{\mathrm{PGV}}\,, \tag{29}$$

in which $\underline{\mathrm{PGV}}$ is the target value for the mean value of the PGV and where $I_j^{(1)}(\boldsymbol{A})$ is defined by Eq. (24).

3.7. Mean value of the random CAV

The random CAV is defined as the integral of the absolute value of the random acceleration over time range $[0,T]$,

$$\mathrm{CAV}(A) = \int_0^T |A(\tau)| d\tau\,. \tag{30}$$

The CAV is usually used for the risk assessment of nuclear power-plants. Using a discretization of Eq.(30) the corresponding constraint

$$E\{\{\Delta t \sum_{j=1}^{N} |u_j|\}\} = \underline{\mathrm{CAV}}\,, \tag{31}$$

in which $\underline{\mathrm{CAV}}$ is the target value for the mean value of the random CAV.

4 APPLICATION

The acceleration stochastic process is sampled such that the final time $T = 20$ s. The time step is $\Delta T = 0.0125$ s. We then have $N = 1600$ (we assume $A(0) = 0$ ms^{-2} almost surely). The available information is relative to the variance of the components of the random vector $\mathbf{A}$, the mean value of the VRS, the envelop of the VRS, the variance of the end value of the velocity and displacement random vectors (resulting from two successive numerical integrations of the acceleration random vector $\mathbf{A}$). The methodology for the construction of the MaxEnt pdf introduced in the previous section is applied.
Figure 1 compares the target standard deviation of the components with the estimated one.

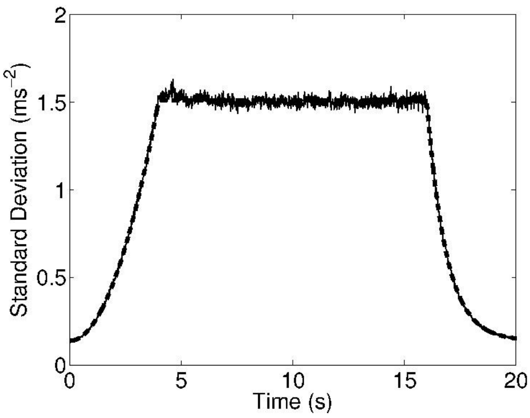

Figure 1. Standard deviation: Target (thick dashed line) and estimation (thin solid line).

The target VRS in constructed following the Eurocode 8. There are two constraints relative to the VRS. The first one concerns the mean value and the second one concerns the probability for the acceleration trajectories of being between two envelops (target VRS +/- 50%). The results are plotted in Fig .2.

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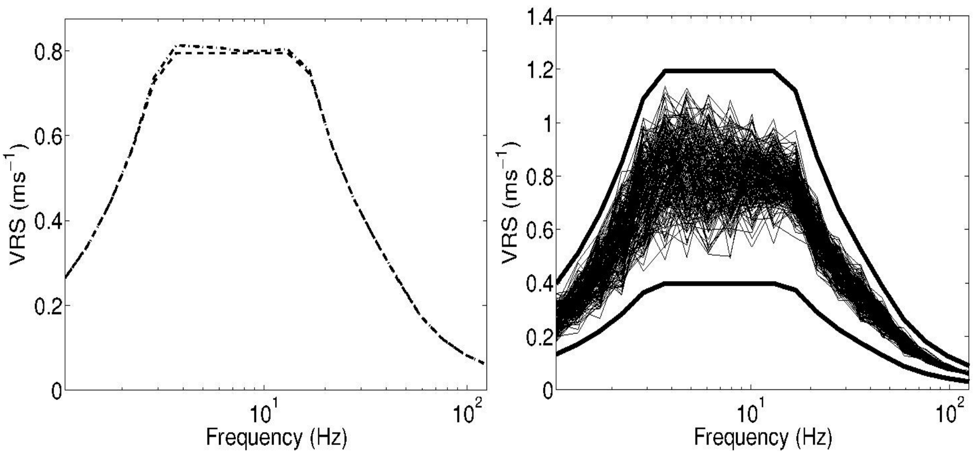

Figure 2. Random VRS. Left figure: mean value (the target) (dashed line) and estimation (mixed line). Right figure: 100 trajectories (thin lines), lower and upper envelops (thick lines).

For the velocity and displacement trajectories, the end values are controlled by constraining the variance of the end value of the velocity and displacement random vectors (obtained by two successive numerical integrations) to be zero. Figure 3 shows a simulated accelerogram and the corresponding velocity time series and displacement time series.

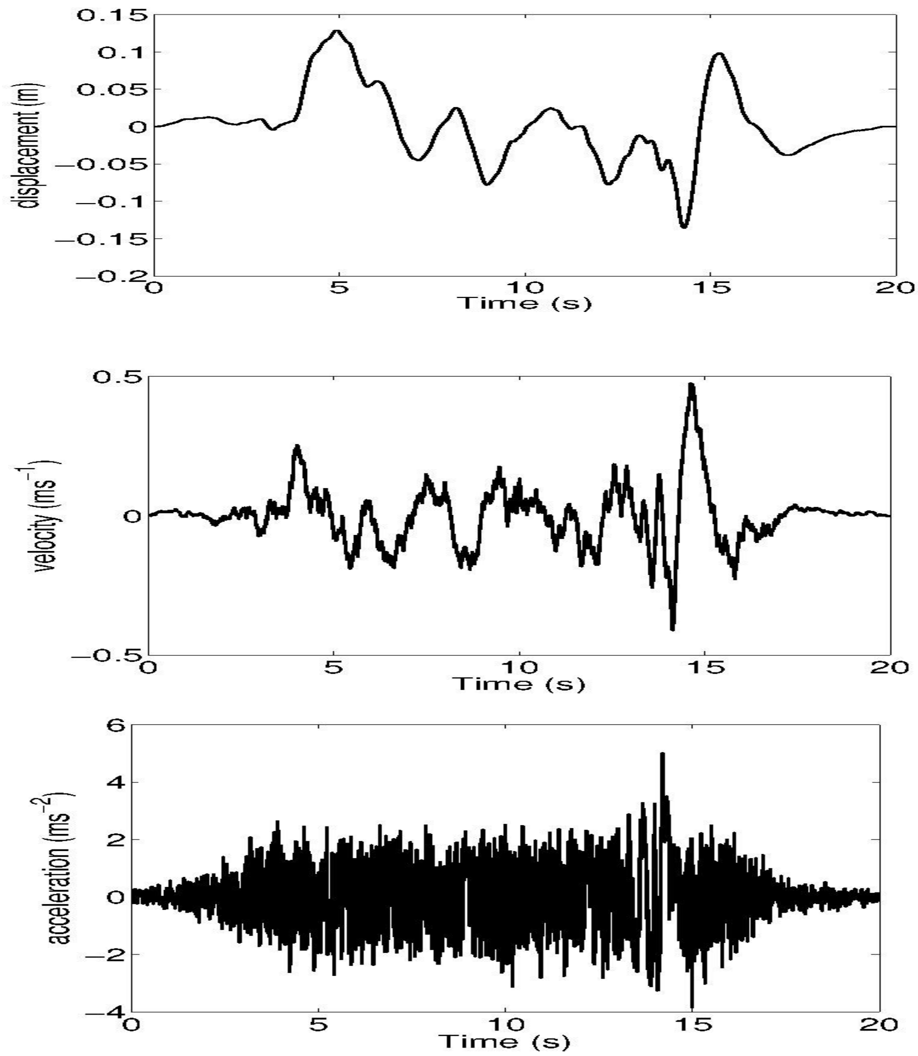

Figure 3. A realization of the random acceleration, of the random velocity and of the random displacement.

Finally the mean value for the PGA, the PGV and the CAV are also constrained. The results are reported on Table 1.

Table 1. For the PGA, the PGV and the CAV: comparison of the estimated mean value with the target value

Constraint	Target	Estimation
Mean PGA (ms^{-2})	5	5.08
Mean PGV (ms^{-1})	0.45	0.46
Mean CAV (ms^{-1})	20	19.99

Figures 1 to 3 and Table 1 show a good matching of the estimated values with the target values.

5 CONCLUSION

A new methodology has been presented for the generation of accelerograms compatible with a given VRS and other properties. If necessary, additional constraints could easily be taken into. The application shows a good matching between the estimated values and the target values.

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