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Emil Stoica

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A stability property of Farlie-Gumbel-Morgenstern distributions

Emil Stoica

Dragan European University of Lugo

Abstract

A copula is a function that completely describes the dependence structure between the marginal distributions. One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family. We establish a stability property of the FGM copula with respect to the \star operation.

A copula is a function that completely describes the dependence structure. It contains all the information to link the marginal distributions to their joint distribution. To obtain a valid multivariate distribution function, it suffices to combine several marginal distribution functions with any copula function. Thus, for the purposes of statistical modeling, it is desirable to have a large collection of copulas at one's disposal. Many examples of copulas can be found in the literature, most are members of families with one or more real parameters.

One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family defined as

$$C_{\theta}^{FGM}(u, v) = uv + \theta uv(1 - u)(1 - v), \quad u, v \in [0, 1], \quad (1)$$

where $\theta \in [-1, 1]$. The density function of FGM copulas is given by

$$\frac{\partial^2 C_{\theta}^{FGM}(u, v)}{\partial u \partial v} = \theta(2u - 1)(2v - 1) + 1, \quad (2)$$

for any $u, v \in [0, 1]$.

Members of the FGM family are symmetric, i.e., $C_{\theta}^{FGM}(u, v) = C_{\theta}^{FGM}(v, u)$ for all (u, v) in $[0, 1]^2$ and have the lower and upper tail dependence coefficients equal to 0.

The copula given in (1) is PQD for $\theta \in (0, 1]$ and NQD for $\theta \in [-1, 0)$. In practical applications this copula has been shown to be somewhat limited, for copula dependence parameter $\theta \in [-1, 1]$, Spearman's correlation $\rho \in [-1/3, 1/3]$ and Kendall's $\tau \in [-2/9, 2/9]$, for more details on copulas see, for example, [1].

To overcome this limited dependence, several authors proposed extensions of this family.

- Theoretical issues: [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]
- Applications: [20, 21, 22, 23, 24, 25]

Our main result is the following:

Theorem. *Let C_1 and C_2 be two FGM copulas with associated parameters θ_1 and θ_2 . Then, $C_1 \star C_2$ is a FGM copula with associated parameter $\theta_1\theta_2/3$.*

Proof. Let $C = C_1 \star C_2$ where the \star operation is defined for instance in [1]. The density associated to C can be computed as:

$$\begin{aligned}
c(u, v) &= \int_0^1 c_1(u, t)c_2(v, t)dt \\
&= \int_0^1 [\theta_1(2t-1)(2v-1)+1][\theta_2(2t-1)(2v-1)+1]dt \\
&= 1 + \theta_1(2v-1) \int_0^1 (2t-1) dt + \theta_2(2u-1) \int_0^1 (2t-1) dt \\
&\quad + \theta_1\theta_2(2u-1)(2v-1) \int_0^1 (2t-1)^2 dt \\
&= 1 + \frac{1}{3}\theta_1\theta_2(2u-1)(2v-1),
\end{aligned}$$

and the conclusion follows from (2).

As a consequence, it follows that the dependence coefficients of $C_1 \star C_2$ can be bounded as $\rho \in [-1/9, 1/9]$ and $\tau \in [-2/27, 2/27]$.

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