Dynamic Model of Steer-by-wire System for Driver Handwheel Feedback

N. Ait-Oufroukh, K. Messaoudène, S. Mammar

IBISC - Informatique, Biologie Intégrative et Systèmes Complexes, Université d’Evry, France
(e-mail: naima.aitoufroukh, kamel.messaoudene, said.mammar}@ibisc.univ-evry.fr).

Abstract—Recent advances towards steer-by-wire technology have promised significant improvements in vehicle handling performance and safety. This technology inspired from aeronautics, consists in replacing the traditional mechanical linkages and/or hydraulic power assist with electrical equivalents. This paper describes the adopted approach for development of a new dynamic model of a Steer-by-wire system with hydraulic assistance in the case of stopped vehicle. The aim is to establish dynamic equations of the forces and torques exchanged between handwheel and the front wheels during steering operations. Some simulation results are provided. The proposed model will be experimented using a prototype equipped with handwheel feedback system.

Keywords—Steer-by-wire, Dynamic model, handwheel, tires friction

1. INTRODUCTION

The aim of electronics improvement is to enhance safety, reliability and driving comfort, while reducing the manufacturing costs. Advances in electronics have revolutionized many automotive engineering aspects, especially in the engine combustion management field and vehicle safety systems. The important benefit of applying electronic technology is to improve the performance of vehicle. However, only recently has the electronic begun to find its way into automotive steering systems in the form of electronically controlled variable assistance [1, 2].

The work described in this paper represents a part of development of new innovative brake handwheel concept, coupled to a Steer-by-wire system [3]. This device was intended initially for the paraplegic people, but it can be also used by the valid drivers considering these many advantages in safety and ergonomics. Two DC electrical engines are used in the designed prototype. First DC engine provides steering feedback on the handwheel by belt transmission. Braking action is obtained by exerting a pressure force on the handwheel equipped with a controllable braking feedback provided by second DC electrical engine with an active system by rack-pinion transmission. This solution allowed us to integrate dynamic brake assistance by using a command law which considers the displacement of the handwheel in braking action and its velocity.

The acceleration is simply obtained through an already available solution which consists of a circle behind the handwheel. The braking handwheel system developed in our laboratory is shown in Fig. 1. This concept allows the driver to brake and accelerate while keeping the hands on the handwheel in all driving situations. This also offers the advantage of a significant optimization in braking time and a more flexible and instinctive braking for a comfortable driving.

Fig. 1. Braking handwheel prototype

The use of X-by-wire technology in the automotive domain, offers many advantages in safety and driving assistance [4, 5, 6]. In the developed prototype, this technology permits a greater simplicity and precision of braking and acceleration actions by the driver during steering operations.

In this paper, we will be interested only on the modeling, simulation and experimental test of a Steer-by-wire system in the case of stopped vehicle.

In the first, we will describe the proposed dynamic model for Steer-by-wire system with hydraulic assistance. We will establish dynamic equations and commands laws for two DC electric engines included in the Steer-by-wire architecture. Simulation results will be given and commented in the end of this paper, by illustrating the handwheel feedback relevance obtained on the prototype used.
2. TIRE BEHAVIOR DURING STEERING IN STOPPED VEHICLE CASE

In the case of stopped vehicle, the front tires are sheared only by the resulting force coming from the driver torque on the handwheel and by the friction forces created in the pneumatic contact. When driver tries to steer the front wheels in stopped vehicle case, a road reaction force is applied in the tires and prevents front wheels to steering, by creating a sheared surface of an angle $\theta$ (see Fig. 2). The front wheels begin to steer only when the side force on the tires reaches its saturation zone. Thus, this force becomes insufficient for opposing to the driver force on the handwheel [7].

Contrary to the moving vehicle case where the aligning torque on the front wheels tends to bring back the handwheel on the zero neutral position. It is known that when the vehicle is stopped, the neutral position of the handwheel is not null. Thus, it is a friction torque which will be felt on the handwheel. The neutral position of the handwheel depends on the equilibrium position of the front wheels, which is generally different to zero when the vehicle is stopped. When driver decreases torque applied on the handwheel, this one turns over to its initial position if the reaction force on the front wheels did not reach the saturation limit. As soon as the saturation limit is exceeded, the handwheel turns over to a more important position from where the steering action [8].

In order to restore on the handwheel the dynamic behaviour described, we will modeling the friction forces $F_s$ on the tire-road contact by using the following equation [9]:

$$F_s = F_s \cdot \text{sign} (\theta)$$

With $\dot{\theta}$, the rotation velocity of the sheared surface on the front tires. $F_s$ the side force developed on the tire-road contact, due to the load $F_{nf}$ applied on the front pneumatic. $F_s$ is obtained by the following expression, with $\mu_f$ the adhesion coefficient of the front tires:

$$F_s = F_{nf} \cdot \mu_f$$

(2)

3. USED STEER-BY-WIRE ARCHITECTURE & EFFORTS INTERACTION

Steer-by-wire architecture used in the modelling preserves the hydraulic assistance during steering actions [10].

As shown in the Fig. 3, Steer-by-wire system is composed of two blocks: The first block provides to the driver a feedback on the handwheel, the second one ensures the steering act on the front wheels. A first electrical motor of steering is connected to the rack-and-pinion by reducer via the assistance valve. A second electrical motor of restitution, equipped with reducer, is connected to the handwheel [11].

The signification of all the parameters used in modelling is shown on Table I.

![Fig. 2. Shearing of the front tire in steering for stopped vehicle](image)

![Fig. 3. Steering system with hydraulic assistance](image)

| $J_h - B_h$ | Inertia of handwheel - Its viscous friction |
| $J_u - B_u$ | Inertia of steering engine - Its viscous friction |
| $J_{re} - B_{re}$ | Inertia of handwheel restitution engine - Its viscous friction |
| $J_u - B_u$ | Inertia of front wheels – Its viscous friction |
| $\theta_h - \theta_u$ | Handwheel angle – Front wheels angle |
| $\theta_{re} - \theta_{re}$ | Angle of handwheel restitution engine - Angle of steering engine |
| $\tau_d - \tau_{ss}$ | Driver torque – Torque due to sheared surface |
| $\tau_{re} - \tau_{re}$ | Torque of handwheel restitution engine – Torque of steering engine |
| $R_{re} - R_{re}$ | Reducer ratio of the restitution engine - reducer ratio of the steering engine |
The dynamics of the restitution block on the handwheel is described by the following equations.

Assessment of the efforts on the handwheel and the restitution motor gives:

\[
\begin{align*}
J_{\theta_h} \dot{\theta}_h &= -B_{\theta} \dot{\theta}_h + \tau_d - K_{bh} \delta_h \\
J_{\theta_e} \dot{\theta}_e &= -B_{\theta} \dot{\theta}_e + \tau_r + \frac{K_{bh}}{R_{re}} \delta_e
\end{align*}
\]

(3)

The state-representation of the restitution block on the handwheel is thus given by these two equations:

\[
\begin{align*}
J_{\theta_h} \ddot{\theta}_h + B_{\theta} \dot{\theta}_h + K_{sh} \theta_h &= \tau_d + \frac{K_{bh}}{R_{re}} \theta_e \\
J_{\theta_e} \ddot{\theta}_e + B_{\theta} \dot{\theta}_e + \frac{K_{bh}}{R_{re}} \theta_e &= \tau_r + \frac{K_{bh}}{R_{re}} \theta_h
\end{align*}
\]

(4)

The steering block dynamics on the front wheel is described by the following equations:

\[
J_{\theta_f} \ddot{\theta}_f = -B_{\theta} \dot{\theta}_f + \tau_f - \frac{K_{bf}}{R_{re}} \delta_{valve}
\]

(5)

With:

\[
\delta_{valve} = \frac{\theta_a - \theta_{valve}}{R_{re}}
\]

(6)

\[
R_{ratio} = \frac{d_{arm}}{R_p \cos \psi_f \cos \alpha_{pr}}
\]

(7)

The steering ratio \(R_{ratio}\) corresponds to the transmission ratio between handwheel and front wheels around its pivot axes. It is often approximated by \(d_{arm} / R_p\) knowing that the pivot angle \(\alpha_{pr}\) and track angle \(\alpha_r\) are relatively small [12].

Replacing the expression of torsion angle on the assistance valve \(\delta_{valve}\), one obtains:

\[
J_{\theta_f} \ddot{\theta}_f + B_{\theta} \dot{\theta}_f + \frac{K_{bf}}{R_{re}} \theta_f = \tau_f + \frac{d_{arm} K_{bh} \theta_a}{R_{re} R_p \cos \psi_f \cos \alpha_{pr}}
\]

(8)

The steering system used is combined with a hydraulic assistance; modelling thus is very inspired of the real conventional steering. The efforts Assessment at the level of front wheels steering around its pivot axis gives:

\[
J_u \ddot{\theta}_u = -B_u \dot{\theta}_u - \tau_u + R_{rutil} \left[ \tau_{trans} + \tau_{axis} \right]
\]

(8)

\(\tau_{trans}\) and \(\tau_{axis}\) are respectively the driver torque transmitted to the rack-and-pinion and the assistance torque provided by the hydraulic system. These torques are applied to the rack-and-pinion and obtained by the following expressions:

\[
\begin{align*}
\tau_{trans} &= K_{bh} \delta_{valve} \\
\tau_{axis} &= R_p f(\delta_{valve}) S
\end{align*}
\]

(9)

\(f(\delta_{valve})\) is the law of hydraulic assistance valve and \(S\) the section of the assistance actuator. The product \(f(\delta_{valve}) S\) represents thus the assistance force applied to the rack-and-pinion by the assistance actuator. Several dynamic models of the law valve function are proposed in [13] and [14] for static model. In our case, we linearized the law valve, around an operating point is adopted:

\[
f(\delta_{valve}) = K_u \delta_{valve}
\]

(10)

Where \(K_u\) is the slope of the function \(f(\delta_{valve})\) at the selected operating point.

The torque \(\tau_{ss}\) is created by the reaction force \((\theta_a - \theta_r) C_f\) in the sheared surface of the tire with an arm of distance \((t_u + t_p) \cos \phi_i \cos \alpha_{pr}\). This torque takes the form:

\[
\tau_{ss} = (\theta_a - \theta_r) C_f (t_u + t_p) \cos \phi_i \cos \alpha_{pr}
\]

(11)

\(\theta_r\) is the angle of sheared surface of the front tires. \(C_f\) is the lateral stiffness of the front tires. \(t_u\) and \(t_p\) are respectively mechanical and pneumatic trail of the tire. \(\phi_i\) and \(\alpha_{pr}\) are respectively the trail angle and pivot angle of the front wheels.

The equation describing the steering dynamics of the front wheels becomes:

\[
J_u \ddot{\theta}_u + B_u \dot{\theta}_u + \theta_u + \frac{d_{arm} K_{bh} \theta_a}{R_p \cos \phi_i \cos \alpha_{pr}} = \frac{d^2 u}{R_p \cos \phi_i \cos \alpha_{pr}} + C_f d + \theta_r [C_f d] + \frac{d_{arm} K_{bh} \theta_a}{R_p \cos \phi_i \cos \alpha_{pr}}
\]

(12)

With:

\[
\begin{align*}
K &= K_{ss} R_p + K_u S
\end{align*}
\]

Concerning the efforts applied on the sheared surface around the pivot axis of the front wheel, the dynamic equation is given by:

\[
J_u \ddot{\theta}_u = -B_u \dot{\theta}_u + \tau_{fr} - F_{fr} (t_u + t_p) \cos \phi_i \cos \alpha_{pr}
\]

(13)

Where \(F_{fr}\) is the friction force created on the pneumatic contact of the tire, shown previously. \(B_{fr}\) is the viscous
friction caused by the pneumatic contact on the road. The distance \((t_w + t_p) \cos \phi \cos \alpha_{pv}\) represents the arm of the friction force applied in the sheared surface. The dynamic equation of the movement of sheared surfaces around pivot axis becomes:

\[
J_w \ddot{\theta}_w + B_w \dot{\theta}_w + \theta \left[ C_f (t_w + t_p) \cos \phi \cos \alpha_{pv} \right] = (C_f \theta_w - F_h) \left[ (t_w + t_p) \cos \phi \cos \alpha_{pv} \right]
\]  

(14)

To obtain the instruction torque of the steering motor, an analogy with the conventional steering system with a steering column is used. In this case, the steering engine fills the absence of the steering column, this condition should be satisfied:

\[
\theta_n = \frac{\theta_{nc}}{R_{nc}} \Rightarrow \theta_n = \theta_n R_{nc}
\]  

(15)

By replacing this condition in the dynamic equation of the steering motor given previously, we obtain the formula of the torque command of steering engine according to the handwheel angle and the reference angle of front wheels \(\theta_u^{ref}\):

\[
\tau_{se} = J_w R_{nc} \ddot{\theta}_w + B_w R_{nc} \dot{\theta}_w + \frac{K_{hy} \theta_n}{R_{nc}} + \frac{d_{sym} K_{sy} \theta_n^{ref}}{R_{nc} R_p \cos \phi \cos \alpha_{pv}} \theta_n^{ref}
\]  

(16)

The reference angle of front wheels \(\theta_u^{ref}\) is obtained by replacing \(\theta_n = \theta_n R_{nc}\) in the Equation (12).

Using a similar procedure, we obtain the reference torque of the restitution engine on the handwheel. This torque represents an image of the external efforts due to the friction forces (in case of stopped vehicle) and lateral vehicle dynamics (in case of moving vehicle), which goes back to the handwheel via the steering column on a conventional steering system [15]. The reference torque is given by:

\[
\tau_{rc} = -\frac{\delta_{solv} K_{sy} \theta_n + \tau_{add}}{R_{rc}} - \frac{1}{R_{rc}} \left[ \frac{K_{hy} \theta_n}{R_{nc}} - \frac{K_{hy} d_{sym} \theta_n^{ref}}{R_p \cos \phi \cos \alpha_{pv}} + \tau_{add} \right]
\]  

(17)

By replacing: \(\theta_u = \theta_n R_{nc}\), one obtains:

\[
\tau_{rc} = -\frac{K_{hy} \theta_n}{R_{nc}} \left[ \frac{d_{sym} \theta_n^{ref}}{R_p \cos \phi \cos \alpha_{pv}} + \tau_{add} \right]
\]  

(18)

\(\tau_{add}\) is the additional torque which allows the adjustment of the handwheel feeling and its inertia, also to control the steering assistance on the handwheel according to the vehicle velocity by using a control law. This additional torque is given by this expression:

\[
\tau_{add} = J_{add} \ddot{\theta}_h + B_{add} \dot{\theta}_h
\]  

(19)

\(J_{add}\) and \(B_{add}\) are parameters which allow the adaptation of handwheel felt.

The suggested model for the Steer-by-wire dynamics in the case of a stopped vehicle is summarized in the form of a general diagram provided in the Fig. 4.

Fig. 4. General diagram of the proposed steer-by-wire model

Thereafter, some simulation results of the dynamic model suggested for a steering-by-wire system are presented.

4. SIMULATION RESULTS OF THE PROPOSED STEER-BY-WIRE DYNAMIC MODEL

In order to understand the steering dynamic model proposed when the vehicle is stopped, two simulations are carried out for three different profiles of the driver torque.

4.1 Driver torque profile in teeth of saw form

In the first simulation, a driver torque profile in teeth of saw form, showed on the Fig. 5, in order to visualize the difference between the steering angle of the front wheels and the angle of sheared surface of the tire.
force due to the deformation of the tire in the pneumatic contact, while sheared surface remains in the same position.

4.2 Driver torque in right-left steering profile

Driver torque in right-left steering profile is used in this second simulation, showed on Fig.7. The objective is to show the process of the steering hysteresis form.

The driver torque according to the handwheel angle is shown in Fig. 8. We achieve a hysteresis diagram which characterizes perfectly real effect of feedback felt on the handwheel on real steering system when vehicle is stopped.

The centre shift of the obtained hysteresis curve Fig.8, is due to the reaction force created by the deformation of the tire in the contact zone, after slackening the handwheel at the end of each steering (right- left).

The front wheels steering angle and the angle of sheared surface, are shown in the Fig. 9.
The complete replacement of the steering linkages by the X-
the developed friction and reaction forces due to the
torque on the handwheel has been tested using the developed
tested experimentally by using the developed prototype. With
the adjustment of the various parameters we could obtain a
dynamic Steer-by-wire model suggested in the case of a
surface at the wheel contact zone was explained, by showing
stopped vehicle. The process of the creation of a sheared
explored. Simulation results have been presented. They have
shown the coherence of the dynamic behaviour of the
proposed model with hydraulic assistance in the case of
stopped vehicle. Detailed dynamics study of this steering type with hydraulic assistance has been
carried out, by giving all the relating dynamic equations.

A general Steer-by-wire architecture diagram has been
explored. Simulation results have been presented. They have
shown the coherence of the dynamic behaviour of the
proposed model with hydraulic assistance in the case of
stopped vehicle. In addition, the control law for the feedback
torque on the handwheel has been tested using the developed
prototype. The obtained results are very promising and the
haptic feedback perceived on the handwheel is very realistic.

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Fig. 9. Wheels and sheared surface angles

We can notice that the obtained results in simulation for the
dynamic Steer-by-wire model suggested in the case of a
stopped vehicle are very conclusive and very coherent with
the behaviour of a real steering system of vehicle with
hydraulic assistance.

It's very important to notice that this proposed model was
tested experimentally by using the developed prototype. With
the adjustment of the various parameters we could obtain a
handwheel feedback very near to that felt on the handwheel of
a real steering system in the case of stopped vehicle. The
effect of the handwheel back move to the position of the
sheared surface is relevant.

5. CONCLUSION

The complete replacement of the steering linkages by the X-
by-wire system poses problems related to the cost,
standardization, power and reliability. The challenge of this
technology lies in its capacity to detect and manage the
breakdowns, by ensuring more safety of users. The
constructors and researchers continue their developments in
this field with an aim of increasing reliability and safety.

In this paper, we have first presented the context of the
conducted research. Then, we have described the behaviour
of the tire during the steering operations in the case of
stopped vehicle. The process of the creation of a sheared
surface at the wheel contact zone was explained, by showing
the developed friction and reaction forces due to the
def ormation of the tire. The relevant part of this paper has
been dedicated to the description of a proposed Steer-by-wire
model in the case of stopped vehicle. Detailed dynamics
study of this steering type with hydraulic assistance has been
carried out, by giving all the relating dynamic equations.