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# Verifying floating-point programs with constraint programming and abstract interpretation techniques

Olivier Ponsini · Claude Michel ·  
Michel Rueher

**Abstract** Static value analysis is a classical approach for verifying programs with floating-point computations. Value analysis mainly relies on abstract interpretation and over-approximates the possible values of program variables. State-of-the-art tools may however compute over-approximations that can be rather coarse for some very usual program expressions. In this paper, we show that constraint solvers can significantly refine approximations computed with abstract interpretation tools. More precisely, we introduce a hybrid approach that combines abstract interpretation and constraint programming techniques in a single static and automatic analysis. We compared the efficiency of the system we developed—named RAICP—with state-of-the-art static analyzers: RAICP produces substantially more precise approximations and is able to check program properties on both academic and industrial benchmarks.

**Keywords** Program verification · Floating-point computation · Constraint solving over floating-point numbers · Constraint solving over real number intervals · Abstract interpretation-based approximation

## 1 Introduction

Programs with floating-point computations control complex and critical systems in numerous domains, including cars and other transportation systems, nuclear energy plants, or medical devices. Floating-point computations are derived from mathematical models on real numbers (Goldberg, 1991), but computations on floating-point numbers are different from computations on real numbers. For instance, with binary floating-point numbers, some real numbers are not representable (e.g. 0.1 does not

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Olivier Ponsini · Claude Michel · Michel Rueher  
Université de Nice Sophia Antipolis, CNRS, I3S, 06900 Sophia Antipolis, France  
E-mail: [firstname.lastname@unice.fr](mailto:firstname.lastname@unice.fr)

have any exact representation). Floating point arithmetic operators are neither associative nor distributive, and may be subject to phenomena such as absorption and cancellation. Furthermore, the behavior of programs containing floating-point computations varies with the programming language, the compiler, the operating system, or the hardware architecture.

For all these reasons, floating-point computations are an additional source of errors in embedded programs. But there is much more, including the fact that most programs are written with the semantics of real numbers in mind. That's why it is very important to estimate the accuracy of floating-point computations with respect to the same sequence of operations in an idealized semantics of real numbers. The goal of this estimation is to identify suspicious values, i.e. values for which the behavior of the program over the floating-point numbers is different from the behavior one could expect over the real numbers. Identifying such suspicious values is a critical issue in embedded program verification.

### 1.1 Value analysis

Static value analysis is a classical approach for verifying programs with floating-point computations. Value analysis can deal with properties ranging from the absence of run-time errors to simple user assertions (Cousot et al, 2007). Value analysis consists in approximating variable *domains*, i.e. the set of possible values that each variable can take at a program point. Approximations are mainly worked out with abstract interpretation techniques. They are used to estimate the accuracy of floating-point computations with respect to the same sequence of operations in an idealized semantics of real numbers.

Value analysis is also used to check program properties: if none of the values in variable domains can violate a property, then the property holds. However, value analysis over-approximates domains and thus, some values in a domain may not actually be reachable for the corresponding variable. Therefore, value analysis usually cannot ensure that a property is violated: when some values may violate a property, the analysis just reports a potential error in the program. If such a potential error is reported for a property that turns out to be true, it is called a false alarm. This issue is intensified by the fact that state-of-the-art systems for value analysis may compute rather coarse approximations for very usual programming constructs and expressions (Ghorbal et al (2010); see also example in Sect. 1.3).

### 1.2 Contribution

The main goal of the approach introduced in this paper is to compute tight approximations for value analysis, and thus to reduce the number of false alarms it may generate. To achieve this goal, we introduce a hybrid approach for value analysis of floating-point programs that combines abstract interpretation (AI) and constraint programming (CP) techniques. More precisely, we propose to exploit the refutation

capabilities of constraint solvers to refine domains computed by abstract interpretation. We show that constraint solvers over floating-point numbers and over real numbers can significantly improve the precision of the value analysis. Experiments on academic programs demonstrate that our system—named RAICP—is substantially more precise than FLUCTUAT (Delmas et al, 2009), a state-of-the-art AI analyzer dedicated to floating-point computations; especially on programs that are difficult to handle with abstract interpretation techniques.

We also evaluated RAICP on a set of 55 benchmarks proposed by D’Silva et al (2012) to demonstrate the capabilities of CDFL, a program analysis tool that embeds an abstract domain in the conflict driven clause learning algorithm of a SAT solver. RAICP was on average 4 times faster than CDFL, and it did not produce any false alarm whereas FLUCTUAT did generate 11 false alarms.

We also applied RAICP to check a property of a real time software application embedded in a car provided by Geensys/Dassault Systems<sup>1</sup>. RAICP proved the property for a realistic system service time. RAICP also compared well on this example with CBMC, a state-of-the-art bounded model checker based on a SAT solver.

To sum up, RAICP is a promising framework for computing accurate domain approximations in floating-point programs and thus for proving properties of hybrid systems with floating-point and integer computations.

Before going into the details, we illustrate in the next subsection how our approach works on a small example.

### 1.3 Motivating example

The program in Fig. 1 contains only linear expressions and a sequence of two conditional statements. This quite simple program is difficult to handle for AI-based analyses. On floating-point numbers—as well as on real numbers—this function returns a value in the interval  $[0, 50]$ . Indeed, the pre-condition and the assignment of line 5 state the following constraints on  $g$  and  $y$ :  $\{x = f + 2 * g \text{ and } f, g \in [-10, 10]\}$ . Thus, from the conditional statement of line 7 we can derive the following information:

- `then` branch, line 8:  $g \in [-10, 5]$ , and thus  $y \in [-10, 5]$  at the end of this branch;
- `else` branch, line 11:  $g \in ]-5, 10]$ , and thus  $y \in [-10, 5[$  at the end of this branch.

Then, from the conditional statement of line 14, we obtain  $z \in [0, 50]$ .

However, FLUCTUAT fails to compute a good approximation for  $z$ . With *zonotope*-based abstract domains, FLUCTUAT over-approximates  $z$  to  $[0, 100]$ , both over the real numbers and the floating-point numbers. The difficulty for AI techniques is to intersect the abstract domains computed for  $x$  at lines 5 and 7. Actually, AI techniques are unable to derive from these statements any constraint on  $g$ . As a consequence, FLUCTUAT estimates that  $g$  ranges over  $[-10, 10]$  in the `then` and `else` branches of the first conditional statement. FLUCTUAT’s analysis of the second conditional statement is more precise, but the upper bound of  $z$  is overestimated since it relies on the coarse over-approximation of  $g$  and  $y$  computed previously.

<sup>1</sup> <http://www.3ds.com/>

---

```

1 /* Pre-condition: f,g ∈ [-10,10] */
2 float foo(float f, float g) {
3   float x, y, z;
4
5   x = f + 2 * g;
6
7   if (x <= 0) {
8     y = g;
9   }
10  else {
11    y = -g;
12  }
13
14  if (y >= 0) {
15    z = 10*y;
16  }
17  else {
18    z = -y;
19  }
20
21  return z;
22 }

```

---

**Fig. 1** Function foo

On this example, RAICP managed to shrink the domain of  $z$  to  $[0, 50]$ . To do so, it successively used AI techniques and CP techniques between consecutive merge points of the control flow graph of the program. The key idea is to build one constraint system for each path between successive merge points, and to apply CP filtering techniques on each of these systems to refine the approximations computed by AI on the corresponding path. Merge points of program `foo` are at lines 13 and 21; for the sake of uniformity, we consider also the program's entry point as a merge point.

There are two paths between the program's entry point and the first merge point. Consider the path through the `then` branch of the first conditional statement. AI techniques compute on this path the following approximations:  $f, g, y \in [-10, 10]$ ,  $x \in [-10, 0]$ . So, the constraint system built for this path is:

$$C_1 = \{x = f + 2 * g \wedge x \leq 0 \wedge y = g \wedge -10 \leq f \wedge f \leq 10 \wedge -10 \leq g \wedge g \leq 10 \\ \wedge -10 \leq y \wedge y \leq 10 \wedge -10 \leq x \wedge x \leq 0\}$$

CP filtering techniques reduce the domain of  $g$  to  $[-10, 5]$  and shrink the domain of  $y$  to  $[-10, 5]$  with constraint system  $C_1$ . In a similar way, for the path going through the `else` branch of the first conditional statement, we obtain the constraint system:

$$C_2 = \{x = f + 2 * g \wedge x > 0 \wedge y = -g \wedge -10 \leq f \wedge f \leq 10 \wedge -10 \leq g \wedge g \leq 10 \\ \wedge -10 \leq y \wedge y \leq 10 \wedge 0 < x \wedge x \leq 10\}$$

Here, CP techniques shrink the domain of  $y$  to  $[-10, 5[$  with constraint system  $C_2$ .

We merge the domains computed for every variable on the different paths reaching a merge point. So, at line 13, the domain of  $y$  becomes  $[-10, 5]$ , that is the smallest

closed interval including all the values in  $[-10, 5] \cup [-10, 5[$ . Note that this domain is sharper than the one computed by FLUCTUAT, i.e.  $y \in [-10, 10]$ . These new domains are then used for analyzing the rest of the program.

On program `foo`, the analysis goes on from line 13 to the next merge point at line 21. Again, we generate a constraint system for each of the two paths. For the path through the `then` branch of the second conditional statement, AI techniques shrink the domain of  $y$  to  $[0, 5]$  and of  $z$  to  $[0, 50]$ . Hence, the constraint system for this path is  $\{y \geq 0 \wedge z = 10 * y \wedge -10 \leq y \wedge y \leq 5 \wedge 0 \leq z \wedge z \leq 50\}$ . CP filtering techniques cannot reduce anymore the domain of  $z$  with this constraint system. Likewise, for the path going through the second conditional statement, RAICP builds the constraint system  $\{y < 0 \wedge z = -y \wedge -10 \leq y \wedge y \leq 5 \wedge 0 < z \wedge z \leq 10\}$ . Here again, CP filtering techniques cannot achieve any reduction of the domain of  $z$ . Finally, at the last merge point, RAICP computes the union of domains and we obtain  $z \in [0, 50] \cup ]0, 10] = [0, 50]$ .

It is worth noting that RAICP does not generate one constraint system for each execution path in the control flow graph (CFG) of a program. We split programs according to the merge points in the CFG and we generate one constraint system per path going from one merge point to the next merge point. Thus, for a program with a succession of  $n$  conditional statements, we would only generate  $2n$  constraint systems whereas the program includes  $2^n$  execution paths. At each merge point, we use CP filtering techniques to shrink the domains computed by abstract interpretation. Then the analysis goes on with the reduced domains. Note also that the CFG exploration is performed on-the-fly: exploration stops as soon as we detect that the constraint system of the current path is inconsistent, i.e. when we detect that the current path is infeasible.

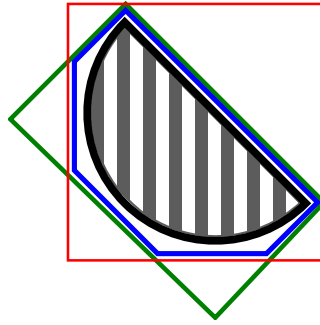
Now, assume we want to verify a post-condition  $p_1$  that states that the value returned by function `foo` is always less than 75. Since AI-based analysis approximates the domain of  $z$  by  $\in [0, 100]$ , it would infer that the post-condition may not hold, and hence generating a false alarm. In contrast, RAICP can ensure that post-condition  $p_1$  holds. Here, the proof is trivial since the upper bound of  $z$  is strictly smaller than 75. However, in practice this proof may be more difficult and we apply the following process: to check a property defined over the program variables, we add the negation of this property to each of the constraint systems generated between the last merge point and the end of the program. If all these systems are inconsistent, we can conclude that the post-condition holds; otherwise, the post-condition may be violated.

In program `foo`, we have two paths from the merge point at line 13 to the end of the program. So, to prove post-condition  $p_1$ , we generate the following constraint systems:

$$\begin{aligned} &\{y \geq 0 \wedge z = 10 * y \wedge z \geq 75 \wedge z \leq 50 \wedge \dots\} \\ &\{y < 0 \wedge z = -y \wedge z \geq 75 \wedge z \leq 50 \wedge \dots\} \end{aligned}$$

Both systems are trivially inconsistent and thus, we can ensure that post-condition  $p_1$  holds.

For program properties specified as assertions inside the program, we apply the same reasoning as for a post-condition: we consider the constraint systems that cor-



**Fig. 2** Half-disk approximations by intervals (in red), zonotope (in green), and polyhedron (in blue)

respond to paths reaching the assertion from previous merge points together with the negation of the assertion.

#### 1.4 Outline of the paper

In Sect. 2, we recall basics on abstract interpretation and constraint programming. Sect. 3 concerns related works. RAICP is described in details in Sect. 4. Section 5 gives some insights into the implementation and analyses the experiments and their results.

## 2 Background

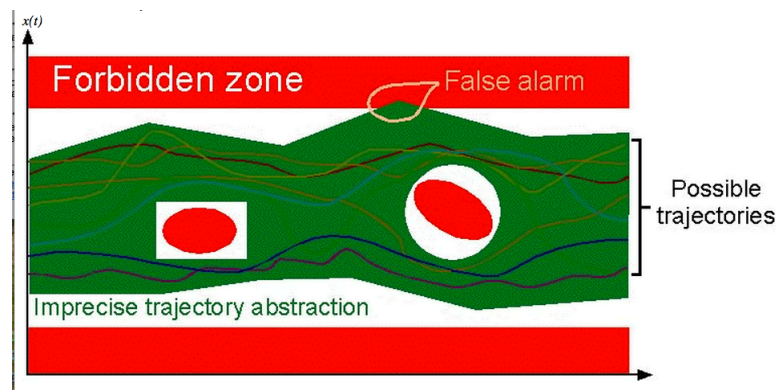
Before going into the details, we recall basics on abstract interpretation and constraint programming techniques that are useful to understand the rest of this paper. Readers familiar with these techniques may skip the corresponding sections.

### 2.1 Abstract interpretation

Abstract interpretation methods define an abstract semantics that approximates the concrete semantics of programs. An abstract semantics is built upon an abstract domain that determines a trade-off between precision and speed of the analysis.

An abstract domain approximates the concrete state of a program by considering only some specific properties of the state. Then, all concrete operations are mapped to corresponding operations in the abstract domain. Special operations allow to approximate program loops in very short time. These operations are designed to preserve all the concrete behaviors of the program.

The choice of an abstract domain is a critical issue. As we can observe on Fig. 2, the approximation of the half-disk in black by a polyhedron is much more precise than the approximation by a box of intervals. The issue is that operations like intersection between polyhedra require computationally expensive algorithms whereas these operations are trivial on intervals. Zonotopes (Goubault and Putot, 2006) offer a good



**Fig. 3** A false alarm occurs when the abstract semantics intersects the forbidden zone while the concrete semantics does not intersect this zone. Forbidden zones are in red, the abstract semantics is in green, and the concrete semantics is the set of curves.

trade-off between performance and precision. Zonotopes are sets of affine forms that preserve linear correlations between variables and keep track of the statements involved in the loss of accuracy of floating-point computations. Zonotopes have nevertheless some drawbacks: approximations of some common program constructs, such as conditionals and nonlinear expressions, are not accurate.

An abstract semantics is a super-set of the concrete program semantics, and thus AI-based analyses are sound but incomplete. In other words, since the domains of the variables are over-approximated by value analysis, properties proved true with the abstract semantics are actually true on the concrete one, but properties violated with the abstract semantics may hold with the concrete one. The latter case is called a *false alarm* when properties represent desired behaviors of the program (see Fig. 3 extracted from Cousot's informal introduction to abstract interpretation<sup>2</sup>).

To sum up, AI techniques provide a good trade-off between precision and performance. They scale well, but they lack of precision for programs with non-linear expressions and with numerous conditionals.

## 2.2 Constraint programming

Constraint Programming (CP) is a *way of modeling and solving combinatorial optimization problems*. CP combines techniques from artificial intelligence, logic programming, and operations research. Several industrial solvers and academic solvers are available, e.g. ILOG/IBM<sup>3</sup>, Gecode<sup>4</sup>. There are many successful industrial applications, e.g. timetabling (Dutch railway), hardware verification (Hentenryck et al, 2009), scheduling, planning (Rossi et al, 2006, Part II).

<sup>2</sup> <http://www.di.ens.fr/~cousot/AI/IntroAbsInt.html>

<sup>3</sup> <http://www-01.ibm.com/software/integration/optimization/cplex-optimizer>

<sup>4</sup> <http://www.gecode.org>



The key features of CP are *domain filtering* and *search strategies*. Domain filtering algorithms consider each constraint separately and remove values that are trivially inconsistent. Search strategies try to exploit the structure of the problem to guide the variable instantiation process. In this paper we mainly use constraint techniques over continuous domains<sup>5</sup>.

A numeric constraint satisfaction problem  $(\mathcal{X}, \mathcal{D}, \mathcal{C})$  is defined by:

- $\mathcal{X} = \{x_1, \dots, x_n\}$ , a set of variables;
- $\mathcal{D} = \{D_{x_1}, \dots, D_{x_n}\}$ , a set of domains.  $D_{x_i}$  contains all acceptable values for variable  $x_i$ ;
- $\mathcal{C} = \{c_1, \dots, c_m\}$ , a set of constraints

The constraint programming framework over continuous domains is based on a *branch & prune* schema which is best viewed as an iteration of two steps:

1. Pruning the search space
2. Making a choice to generate two (or more) sub-problems

The *pruning step* reduces an interval when it can prove that the upper bound or the lower bound does not satisfy some constraint. The *branching step* splits the interval associated to some variable in two or more intervals (often with the same width).

Pruning techniques on continuous domains are based on partial consistencies, that is to say a consistency of a relaxation of the system. More precisely, it is a property that holds on a subset of variables or constraints and that is associated with a filtering algorithm. Informally speaking, a constraint system  $C$  satisfies a partial consistency property if *a relaxation of  $C$  is consistent*. For instance, consider  $X = [x, \bar{x}]$  and  $C(x, x_1, \dots, x_n) \in \mathcal{C}$ : if  $C(x, x_1, \dots, x_n)$  does not hold for any values  $v \in [x, x']$ , then  $X$  may be shrunken to  $X = [x', \bar{x}]$ .

*2B-consistency* (Lhomme, 1993) states a local property on the bounds of the domains of a variable at a single constraint level. In other words, the domain of variable  $x$  is *2B-consistent* if, for any constraint  $c$ , there exists at least one value in the domains of all other variables such that  $c$  can be satisfied when  $x$  is set to the upper or lower bound of its domain.

*Example:* Let  $S_1 = \{x + y = 2, y \leq x - 1, x \in [0, 100], y \in [0, 100]\}$  and  $S_2 = \{x + y = 2, y \leq x - 1, x \in [1, 2], y \in [0, 1]\}$  be two constraint systems.  $S_1$  is not *2B-consistent*. Indeed, the domain of  $x$  is not *2B-consistent* since  $100 + y = 2$  is not satisfiable when  $D_y = [0, 100]$ .  $S_2$  is *2B-consistent*. Indeed,  $D_x$  is *2B-consistent* since  $1 + y = 2$ ,  $2 + y = 2$ ,  $y \leq 1 - 1$  and  $y \leq 2 - 1$  are all satisfiable when  $D_y = [0, 1]$ . A similar reasoning can show that  $D_y$  is *2B-consistent* in  $S_2$ .

*2B-consistency pruning* algorithms successively narrow the domains of the variables. The approximation of the projection of a constraint  $c$  on its variables is the basic tool for narrowing domains. The projection  $\Pi_x(c)$  of the constraint  $c(x, x_1, \dots, x_n)$  on  $x$  is the set defined as follows:

$$\Pi_x(c) = \{v \in D_x \mid \exists (v_1, \dots, v_n) \in D_{x_1} \times \dots \times D_{x_n} \text{ s.t. } c(v, v_1, \dots, v_n) \text{ holds}\}.$$

<sup>5</sup> For an informal introduction, see [http://www.it.uu.se/research/group/astra/CPmeetsCAV/slides/rueher\\_Continuous\\_Domains.pdf](http://www.it.uu.se/research/group/astra/CPmeetsCAV/slides/rueher_Continuous_Domains.pdf)

The approximation of  $\Pi_x(c)$  is the interval  $[\min(\Pi_x(c)), \max(\Pi_x(c))]$ . In practice, this approximation is often computed on constraints where the multiple occurrences of variables have been replaced by fresh variables with the same domain.

When the domain of a variable has been narrowed, all the constraints in which this variable occurs will be processed again. The filtering ends when none of the domains cannot be narrowed anymore. If at least one domain becomes empty, the system is not  $2B$ -consistent.

Stronger consistencies have also been defined. For instance,  $3B$ -consistency (Lhomme, 1993) checks whether  $2B$ -Consistency can be enforced when the domain of a variable is reduced to the value of one of its bounds in the whole system. Roughly speaking,  $3B$ -pruning algorithms are based on a shaving process that tries to shrink the interval of a given variable. For instance, let  $x$  be a variable the domain of which is  $X = [\underline{x}, \bar{x}]$ . A standard  $3B$ -pruning algorithm will first try to show that the constraint system is not  $2B$ -consistent when  $X$  is set to  $[\underline{x}, \frac{\underline{x}+\bar{x}}{2}]$ ; if it succeeds, it will remove  $[\underline{x}, \frac{\underline{x}+\bar{x}}{2}]$  from  $X$  and the process goes on; otherwise, the pruning process restarts with a smaller domain for  $X$ , e.g.,  $[\underline{x}, \frac{\underline{x}+\bar{x}}{4}]$ . The process stops when the domain of  $X$  becomes smaller than a given  $\epsilon$ .

To sum up, the strong points of CP are its refutation capabilities and its great flexibility. However, the pruning algorithm may be time consuming on large domains.

### 3 Related works

Various methods address static validation of programs with floating-point computations: abstract interpretation based analyses, proofs of programs with proof assistants or with decision procedures in automatic solvers.

Analyses based on abstract interpretation capture rounding errors due to floating-point computation in their abstract domains. They are usually fast, automatic, and scalable. However, they may lack of precision. *ASTRÉE* (Cousot et al, 2007) is one of the most famous tool in this family of methods: it estimates the value of the program variables at every program point and can show the absence of run-time errors, e.g. division by zero, arithmetic overflow. *FLUCTUAT* (Delmas et al, 2009) estimates in addition the accuracy of the floating-point computations: it bounds the difference between the values taken by variables when the program is given a real semantics and when it is given a floating-point semantics.

Proof assistants like *Coq* (Boldo and Filliâtre, 2007) or *HOL* (Harrison, 1999) allow their users to formalize floating-point arithmetic. Proofs of program properties are done manually in the proof assistants that only guarantee the correctness of the proof. Even though some parts of the proofs may be automatized, the user must still make a significant effort to conduct the proof. Moreover, when a proof strategy fails to prove a property, the user often does not know whether the property is false or whether he could prove it with another strategy. The *Gappa* tool (de Dinechin et al, 2011) combines interval arithmetic and term rewriting from a base of theorems. The theorems rewrite arithmetic expressions so as to compensate for the shortcomings of interval arithmetic, e.g. loss of dependency between variables. Whenever the computed intervals are not precise enough, theorems can be manually introduced or the

input domains can be subdivided. Again, the cost for the user of this semi-automatic method is considerable. Ayad and Marché (2010) propose axiomatizing floating-point arithmetic within first-order logic to automate the proofs conducted in proof assistants such as Coq by calling external SMT (Satisfiability Modulo Theories) solvers and Gappa. Their experiments show that human interaction with the proof assistant is still required.

The classical bit-vector approach of SAT solvers is ineffective on programs with floating-point computations because of the size of the domains of floating-point variables and the cost of bit-vector operations. An abstraction technique was devised for CBMC by Brillout et al (2009). It is based on under and over-approximation of floating-point numbers with respect to a given precision expressed as a number of bits of the mantissa. However, this technique remains slow. D’Silva et al (2012) developed recently CDFL, a program analysis tool that embeds an abstract domain in the conflict driven clause learning algorithm of a SAT solver. CDFL is based on a sound and complete analysis for determining the range of floating-point variables in loop-free control software. The authors state that CDFL is more than 200 times faster than CBMC (D’Silva et al, 2012). In Section 5.3, we compare the performances of CDFL and RAICP on a set of benchmarks proposed by D’Silva et al.

Links between abstract interpretation and constraint logic programming have been studied at a theoretical level for a long time (Codognet and Filé, 1992). More recently, Denmat et al (2007) introduced a new global constraint to model iterative arithmetic relations between integer variables. The associated filtering algorithm is based on abstract interpretation over polyhedra. Pelleau et al (2013) designed a generic constraint solver parametrized by abstract domains. They focus on mixed discrete-continuous problems over the integer and real numbers. In this paper, we show how abstract interpretation and constraint programming techniques can complement each other for the static analysis of floating-point programs.

#### 4 RAICP, a hybrid approach

As said before, RAICP, the approach we introduce in this paper, is based on a piecewise exploration of a program CFG that alternates path analysis and merging steps. Nodes of the CFG where two branches join are selected as merge points. We build one constraint system per path between two successive merge points. We use CP filtering techniques on these systems to reduce variable domains first computed with AI techniques. At merge points, the reduced domains for the different paths are merged and exploration goes on with the next part of the CFG.

In Sect. 4.1, we detail the notions of merge point and path exploration of a CFG. Then, in Sect. 4.2, we give the algorithms implemented in RAICP to perform piecewise exploration of the CFG and compute domain approximations on each piece of the CFG.

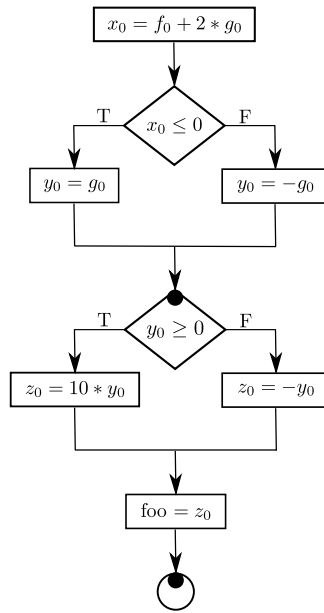


Fig. 4 CFG of program `foo` from Fig 1: nodes with black circles are merge points

#### 4.1 Control flow graph exploration

A control flow graph is a standard graph representation of computations and control flow in a program. Nodes in the graph are basic blocks of the program, that is a sequence of consecutive statements without any branching in it. Directed edges represent possible flow of control from the end of one block to the beginning of another. A control flow graph contains one entry node, a node without incoming edge, and one exit node, a node without outgoing edge.

In our CFGs, we will consider the following types of nodes:

- *assignment* nodes containing a program assignment;
- *assertion* nodes containing a logical expression to be checked;
- *while* nodes containing a loop condition and a loop body;
- *if* nodes containing a branching condition;

We define specific locations in the program that correspond to nodes in the CFG where two branches join. We call these locations *merge points*. In addition, the exit node is always a merge point. Our CFGs are acyclic graphs since we unfold loops a bounded number of times before enclosing them in a *while* node.

For instance, the graph in Fig. 4 is the CFG of function `foo` described in Fig. 1. Edges labeled T (resp. F) represent the control flow when the associated condition is true (resp. false). The merge points are the nodes with a black circle. The second merge point is not the assignment node that follows the branch junction but the exit node: a merge point is always the last node of a straight sequence of nodes after a

**Algorithm 1: RAICP**


---

**Data:**  
 $Q$ , a queue of merge points.  
 $D$ , an array of sets s.t.  $D[m]$  is the set of variable domains at merge point  $m$ .  
 $\mathcal{D}^I$ , the initial variable domains.  
 $n^I$ , the CFG's entry node.  
 $n^E$ , the CFG's exit node.

**Result:**  
 $D[n^E]$  is the set of variable domains at the end of the program.  
error is a set of domains when an assertion may be violated; otherwise it is the empty set.

```

1 error ← explorePaths( $n^I$ ,  $\mathcal{D}^I$ ,  $\emptyset$ )
2 while error =  $\emptyset$  and  $Q \neq \emptyset$  do
3    $n \leftarrow \text{pop}(Q)$ 
4   if  $n \neq n^E$  then
5     error ← explorePaths( $n$ ,  $D[n]$ ,  $\emptyset$ )
6   end
7 end

```

---

junction. Note that program expressions were put into DSA-like form<sup>6</sup> to facilitate constraint generation.

As said before, we only explore paths between two successive merge points. Of course, this process may be less precise than an exploration of the full length paths, but it is sound: variable domains are over-approximated for value analysis and properties found to be true hold over the full length paths too.

The CFG is explored using a forward analysis starting at the beginning of the program. We generate one constraint system for each path between two consecutive merge points. At any point of a path, the possible states of the program are represented by a constraint system over the program variables. To this end, the semantics of each program statement is expressed by constraints. Variable domains are intervals over the integers, the floating-point numbers, or the real numbers depending on the type of the variable. This technique for representing programs by constraint systems was introduced for bounded program verification in CPBPV by Collavizza et al (2010).

## 4.2 RAICP algorithm

In this section, we detail how RAICP explores the CFG between consecutive merge points. We also describe the process for computing domain approximations.

### 4.2.1 Exploring paths

Algorithm 1 launches the exploration of the paths from each merge point. It uses a queue of merge points ordered by increasing depth in the CFG, i.e. the number of nodes from the entry node to the merge point. RAICP stores in  $D$  the result of the

<sup>6</sup> DSA (Dynamic Single Assignment) is a semantic preserving program transformation in which each variable is assigned at most once on each program path (Barnett and Leino, 2005).

value analysis of the program. Initially, all elements of  $D$  are empty sets and at the end  $D[m]$  contains the domain approximations computed at merge point  $m$ .

When the program contains a user assertion, RAICP stores in set `error` the result of the assertion checking process: `error` is empty if the assertion holds; otherwise, `error` contains values that may violate the assertion.

Algorithm 1 calls function `explorePaths` to explore all the paths between a given node and the next merge points. Exploration of the CFG stops when a property may be violated or all merge points were considered. Function `explorePaths` updates the domains stored in  $D$  during path exploration. To this end, the function generates on-the-fly one constraint system per path while visiting successively the nodes of the path. At an *if* node, `explorePaths` explores successively the paths in each branch of the control flow. Note that the function checks the consistency of the constraint system of a branch before exploring it.

At each merge point  $m$ , `explorePaths` calls function `approximate` for computing an approximation of the domains for the current path. Function `approximate` combines AI and CP techniques (see Sect. 4.2.2). Function `explorePaths` updates  $D[m]$  with the smallest closed interval including all the values in the union of the domains computed for the different paths.

For *while* nodes, `explorePaths` uses AI techniques to approximate the domains at the end of the loop. The function then goes on exploring the path with these domains and the negation of the loop condition in the constraint system. Approximating loops with AI techniques ensures that the length of paths are bounded, and as a result the constraint system generation always terminates.

When `explorePaths` reaches an *assertion* node, it will check whether the assertion holds on the current path. To this end, `explorePaths` calls function `approximate` with a constraint system made up of the negation of the assertion to check and of the constraints collected on the path starting at the previous merge point. When function `approximate` can detect an inconsistency, the assertion holds and exploration goes on with the next node on the path. Otherwise, the property checking process is inconclusive: path exploration stops and the domains computed by `approximate` are returned.

#### 4.2.2 Computing approximations

Function `approximate` computes an approximation of the variable domains for a given path between two successive merge points. It takes the domains defined at the beginning of the path ( $\mathcal{D}$ ) and the constraints collected on the path ( $\mathcal{C}$ ). The function returns domains reduced according to the constraints, or an empty set if an inconsistency of the constraint system has been detected.

Function `approximate` starts by checking whether the set of constraints  $\mathcal{C}$  is not trivially inconsistent: `consistentsyn` just checks whether a constraint and its syntactic negation are in  $\mathcal{C}$ . This removes some slow convergence issues that may occur when trying to solve pathological systems such as  $\{a \geq b \wedge a < b\}$ . Note that  $a$  and  $b$  must be identical expressions in both constraints: we do not perform any formal expression simplification.

---

**Function approximate**


---

**Input:**

$\mathcal{D}$ , current variable domains.  
 $\mathcal{C}$ , current set of constraints.

**Output:** A set of domains. If  $\mathcal{C}$  is found inconsistent, the returned set is empty.

```

1 Function approximate( $\mathcal{D}$ ,  $\mathcal{C}$ ) is
2   if  $\neg\text{consistent}_{\text{sym}}(\mathcal{C})$  then
3     return  $\emptyset$ 
4   else
5      $D_{AI} \leftarrow \text{filter}_{AI}(\mathcal{D}, \mathcal{C})$ 
6     if  $D_{AI} = \emptyset$  then
7       return  $\emptyset$ 
8     else
9       return  $\text{filter}_{CP}(D_{AI}, \mathcal{C})$ 
10    end
11  end
12 end

```

---

Function  $\text{filter}_{AI}$  calls an AI library to analyze the part of the program corresponding to the path between the two considered merge points. It returns an empty set when it detects that the path is infeasible. Function  $\text{filter}_{CP}$  applies strong partial consistencies to the constraint system of the path updated with the domains computed by  $\text{filter}_{AI}$ .

## 5 Experiments

In this section, we first describe the prototype of RAICP we have implemented. Then, we report the experiments we have performed to evaluate RAICP. We compare RAICP with FLUCTUAT on academic programs, and we evaluate the property checking capabilities of RAICP both on a set of academic benchmarks provided by the authors of CDFL and on an industrial benchmark.

Academic programs are available at [http://users.polytech.unice.fr/~rueher/Benchs/ASE\\_RAICP](http://users.polytech.unice.fr/~rueher/Benchs/ASE_RAICP). All results were obtained on an Intel Core 2 Duo at 2.8 GHz with 4 GB of memory running Linux using FLUCTUAT version 3.1247, REALPAVER version 0.4, CPLEX version 12.3, CBMC version 4.5 and the downloadable version of CDFL.

### 5.1 Implementation

We implemented a prototype of RAICP that uses:

- FLUCTUAT for AI-based computations,
- REALPAVER for constraint solving over real numbers, and
- FPCS for constraint solving over floating-point numbers.

More precisely, RAICP takes as input a C program and builds the corresponding CFG. Each explored path of the CFG between two merge points is transformed into both

a set of constraints and a C program. RAICP calls the FLUCTUAT library on these generated C programs. Then, RAICP passes the domains returned by FLUCTUAT and the set of constraints to the constraint solver FPCS (resp. REALPAVER) to reduce the domains over the floating-point numbers (resp. the real numbers). The domains returned by the constraint solver will be used by RAICP for the next steps of the analysis.

Neither REALPAVER nor FPCS can deal with constraints over integers. As a workaround, the prototype handles constraints over integers with the MILP solver IBM ILOG CPLEX in separate constraint systems. The current prototype does not yet handle variables that appear both in constraints over integers and floating-points.

Our prototype uses *2B*-like partial consistencies<sup>7</sup> to cut infeasible paths during CFG exploration and *3B*-like partial consistencies<sup>8</sup> to reduce domains at merge points. This choice is motivated by performance: *2B*-like consistency algorithms are much faster than *3B*-like consistency algorithms, but the latter may achieve a much stronger pruning.

RAICP analyzes C programs that conform to IEEE 754 standard with the following restrictions: size of arrays are bounded; pointers, bitwise operators and statements that interrupt the control flow (`goto`, `continue`, and `break`) are not handled. However, all aspects of computations over floating-point numbers are not specified in the IEEE 754 standard and so are implementation-dependent. We assume here that the C programs will be compiled with GCC without any optimization option and run on an x86 architecture managed by a 32-bit Linux operating system<sup>9</sup>. In the current implementation, we handle basic arithmetic operations, comparisons and some classical functions like square root.

### 5.1.1 AI-based static analyzer

FLUCTUAT is a static analyzer for C programs that proceeds by abstract interpretation. It is specialized in estimating the precision of floating-point computations (Delmas et al, 2009). FLUCTUAT is developed by CEA-LIST<sup>10</sup> and was successfully used for industrial applications of several tens of thousands of lines of code in transportation, nuclear energy, or avionics areas. FLUCTUAT compares the behavior of the analyzed program over real numbers and over floating-point numbers. In other words, it allows to specify ranges of values for the program input variables and computes for each program variable  $v$ :

- bounds for the domain of variable  $v$  considered as a real number;
- bounds for the domain of variable  $v$  considered as a floating-point number;
- bounds for the maximum error between real and floating-point values;
- the contribution of each statement to the error associated with variable  $v$  ;

<sup>7</sup> The prototype uses REALPAVER's *HC4*-consistency or FPCS's *2B(w)*-consistency.

<sup>8</sup> The prototype uses REALPAVER's *BC5*-consistency in paving mode or FPCS's *3B(w)*-consistency.

<sup>9</sup> All computations are done using 80 bits floating point numbers (long double). Inputs are first converted from their base type to long double while the computation result is converted from double to the awaited result base type.

<sup>10</sup> [http://www-list.cea.fr/validation\\_en.html](http://www-list.cea.fr/validation_en.html)



- the contribution of the input variables to the error associated with variable  $v$ .

FLUCTUAT uses the weakly relational abstract domain of zonotopes (Goubault and Putot, 2006). Zonotopes are sets of affine forms that preserve linear correlations between variables. They offer a good trade-off between performance and precision for floating-point and real number computations. Indeed, the analysis is fast and scales well, processes accurately linear expressions, and keeps track of the statements involved in the loss of accuracy of floating-point computations. To increase the analysis precision, FLUCTUAT allows to use arbitrary precision numbers or to subdivide up to two input variable intervals. However, over-approximations computed by FLUCTUAT may be very large because the abstract domains do not handle well conditional statements and non-linear expressions.

### 5.1.2 Constraint solver over the real numbers

REALPAVER is an interval solver for numerical constraint systems over the real numbers<sup>11</sup> (Granvilliers and Benhamou, 2006). It handles non-linear constraints defined with the usual arithmetic operations as well as transcendental elementary functions.

REALPAVER computes reliable approximations of continuous solution sets using correctly rounded interval methods and constraint satisfaction techniques. More precisely, the computed domains are closed intervals bounded by floating-point numbers. REALPAVER implements several partial consistencies. An approximation of a solution is described by a box, i.e., the Cartesian product of the domains of the variables. REALPAVER either proves the unsatisfiability of the constraint system or computes small boxes that contains all the solutions of the system.

The REALPAVER modeling language does not provide strict inequality and not-equal operators, which can be found in conditional expressions in programs. As a consequence, in the constraint systems generated for REALPAVER, strict inequalities are replaced by non strict ones and constraints with a not-equal operator are ignored. This may lead to over-approximations but it is safe since no solutions are lost.

We experimented with various consistencies implemented in REALPAVER: *BC5*, a combination of *2B* and box consistencies with interval Newton method, provided the best trade-off between time cost and domain reduction.

### 5.1.3 Constraint solver over the floating-point numbers

Constraint solvers over the real numbers based on interval arithmetic cannot handle constraints over the floating-point numbers because of the specific properties of the floating-point numbers. The tricky point is that constraints that do not have any solutions over the real numbers may hold over the floating-point numbers. Moreover, relations that hold over the real numbers may not hold over the floating-point numbers. Finite domain solvers are ineffective for handling constraints over the floating-point numbers due to the huge size of the domains.

<sup>11</sup> REALPAVER web site: <http://pagesperso.lina.univ-nantes.fr/info/perso/permanents/granvil/realpaver/>

That’s why we use here FPCS, a constraint solver designed to solve a set of constraints over floating-point numbers without losing any solution (Michel, 2002; Marre and Michel, 2010). FPCS implements  $2B$ -consistency with projection functions adapted to floating-point arithmetic (Michel et al, 2001; Botella et al, 2006).

Inverse projection functions that keep all the solutions are the most difficult to implement. Indeed, direct projections only requires a slight adaptation of classical results on interval arithmetic, but inverse projections do not follow the same rules because of the properties of floating-point arithmetic. More precisely, each constraint is decomposed into an equivalent binary or ternary constraint by introducing new variables if necessary. A ternary constraint  $x = y \odot_f z$ , where  $\odot_f$  is an arithmetic operator over the floating-point numbers, is decomposed into three projection functions:

- the direct projection,  $\Pi_x(x = y \odot_f z)$ ;
- the first inverse projection,  $\Pi_y(x = y \odot_f z)$ ;
- the second inverse projection,  $\Pi_z(x = y \odot_f z)$ .

A binary constraint of the form  $x \odot_f y$ , where  $\odot_f$  is a relational operator among  $=$ ,  $!$ ,  $<$ ,  $<=$ ,  $>$ , and  $>=$ , is decomposed into two projection functions:  $\Pi_x(x \odot_f y)$  and  $\Pi_y(x \odot_f y)$ . The computation of the approximation of these projection functions is mainly derived from interval arithmetic and benefits from floating-point numbers being a totally ordered finite set.

FPCS also implements stronger consistencies—e.g.,  $kB$ -consistencies (Lhomme, 1993)—to deal with the classical issues of multiple occurrences and to reduce more substantially the bounds of the domains of the variables.

The floating-point domains handled by FPCS also include infinities. Moreover, FPCS handles all the basic arithmetic operations, as well as most of the usual mathematical functions. Type conversions are also correctly processed.

On our experiments,  $3B$ -consistency pruning worked well with FPCS whereas  $2B$ -consistency was not strong enough to reduce the domains computed by FLUCTUAT.

## 5.2 Comparison with FLUCTUAT for value analysis

We report here experiments on a set of academic programs with conditionals, nonlinearities, and loops. We show that RAICP is more efficient than FLUCTUAT alone on these benchmarks.

### 5.2.1 Conditionals

The first benchmark concerns conditional statements, for which abstract domains need to be intersected with the condition of the conditional statement. The function `gsl_poly_solve_quadratic` comes from the GNU scientific library and contains several conditional statements. It computes the two real roots of a quadratic equation  $ax^2 + bx + c$  and puts the results in variables `x0` and `x1`.

Table 1 shows analysis times and approximations of the domains of variables `x0` and `x1` for a given configuration of the input variables. The first two rows present

**Table 1** Domains of the roots of the `gs1_poly_solve_quadratic` function

		$a \in [-1, 1] \quad b \in [0.01, 1] \quad c \in [0.01, 1]$		
		x0	x1	Time
$\mathbb{R}$	FLUCTUAT	$[-\infty, \infty]$	$[-\infty, \infty]$	0.02 s
	RAiCP	$[-\infty, 0]$	$[-200.1, \infty]$	1.66 s
$\mathbb{F}$	FLUCTUAT	$[-\infty, \infty]$	$[-\infty, \infty]$	0.02 s
	RAiCP	$[-\infty, 0]$	$[-312.51, \infty]$	0.95 s

**Table 2** Domains of the return value of `sinus` and `rump` functions

		sinus $x \in [-1, 1]$		rump $x \in [7 \times 10^4, 8 \times 10^4]$ $y \in [3 \times 10^4, 4 \times 10^4]$	
		Domain	Time	Domain	Time
$\mathbb{R}$	FLUCTUAT	$[-1.009, 1.009]$	0.02 s	$[-1.168 \times 10^{37}, 1.992 \times 10^{37}]$	0.02 s
	RAiCP	$[-0.842, 0.842]$	0.93 s	$[-1.144 \times 10^{36}, 1.606 \times 10^{37}]$	1.82 s
$\mathbb{F}$	FLUCTUAT	$[-1.009, 1.009]$	0.02 s	$[-1.168 \times 10^{37}, 1.992 \times 10^{37}]$	0.02 s
	RAiCP	$[-0.855, 0.85]$	0.86 s	$[-1.168 \times 10^{37}, 1.992 \times 10^{37}]$	0.86 s

the results of FLUCTUAT and RAiCP (with REALPAVER) over the real numbers. The next two rows present the results of FLUCTUAT and RAiCP (with FPCS) over the floating-point numbers. FLUCTUAT's over-approximation is so large that it does not give any information on the domain of the roots, whereas RAiCP drastically reduce these domains both over  $\mathbb{R}$  and  $\mathbb{F}$ .

### 5.2.2 Non-linearity

The abstract domain used by FLUCTUAT is based on affine forms that do not allow an exact representation of non-linear operations: the image of a zonotope by a non-linear function is not a zonotope in general. Non-linear operations are thus over-approximated very roughly. FPCS handles the non-linear expressions better. This is illustrated on function `sinus` (see Table 2, column `sinus`). This function computes the 7<sup>th</sup>-order Taylor series of function `sinus`:  $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040}$ .

FPCS and REALPAVER also use approximations to handle non-linear terms and thus cannot always achieve a significant pruning. This is outlined in Table 2 by program `rump`. This program computes a very particular polynomial designed by Rump (2010) to illustrate a catastrophic cancellation phenomenon:

$$333.75y^6 + x^2(11x^2y^2 - y^6 - 121y^4 - 2) + 5.5y^8 + \frac{x}{2y}$$

### 5.2.3 Loops

FLUCTUAT unfolds loops a bounded number of times<sup>12</sup> before applying a widening operator to find a fixed point for the domains at the end of the loop. In RAiCP, by default, we let FLUCTUAT compute the domains for a loop. However, RAiCP can also

<sup>12</sup> Default value is ten times.

**Table 3** Domain of the return value of the `sqrt` and `bigLoop` functions

		sqrt #1: $x \in [4.5, 5.5]$		sqrt #2: $x \in [5, 10]$		bigLoop	
		Domain	Time	Domain	Time	Domain	Time
$\mathbb{R}$	FLUCTUAT	[2.116, 2.354]	0.02 s	[2.098, 3.435]	0.06 s	$[-\infty, \infty]$	0.03 s
	RAICP	[2.121, 2.346]	0.97 s	[2.232, 3.165]	1.06 s	[0, 10]	1.09 s
$\mathbb{F}$	FLUCTUAT	[2.116, 2.354]	0.02 s	$[-\infty, \infty]$	0.06 s	$[-\infty, \infty]$	0.03 s
	RAICP	[2.120, 2.351]	1.5 s	[2.232, 3.193]	4.97 s	[0, 10]	0.94 s

<pre> /* Pre-condition: x ∈ [4.5, 5.5] */ double sqrt(double x) {   double xn, xn1;    xn = x/2.0;   xn1 = 0.5*(xn + x/xn);    while (xn-xn1 &gt; 1e-2) {     xn = xn1;     xn1 = 0.5*(xn + x/xn);   }   return xn1; } </pre>	<pre> /* Pre-condition: x ∈ [0, 10] N ∈ [1, 1000000] */ double bigLoop(double x, int N) {   double a = 0.1;   int i = 1;   double y = x*x-x;    if (y &lt; 0) {     if (x &gt; 1.2) {       a = -2;     }   }    while (N &gt; i) {     x = a * x;     i = i + 1;   }    return x; } </pre>
(a)	(b)

**Fig. 5** Programs (a) `sqrt` with input domain #1 and (b) `bigLoop`

unfold loops until either the exit condition of the loop becomes true or a given bound is reached. In the latter case, we rely again on FLUCTUAT to compute the domains for the loop after the unfolding process.

Program `sqrt` (see Fig. 5a) is based on the so-called Babylonian method that computes an approximate value, with an error of  $1 \times 10^{-2}$ , of the square root of a number greater than 4. Ten unfoldings are sufficient to exit the loop with the two different input domains used in this benchmark (see Table 3). FLUCTUAT obtains accurate results except in the second configuration over  $\mathbb{F}$  where it could not achieve any reduction: the different interpretation of a conditional statement over  $\mathbb{R}$  and over  $\mathbb{F}$  leads to different paths in the program. With an unfolding bound of ten—like in FLUCTUAT—RAICP shrinks the domain over  $\mathbb{F}$  to [2.232, 3.193] in the second configuration.

Program `bigLoop` (see Fig. 5b) contains very simple non-linear expressions followed by a loop that iterates one million times. FLUCTUAT alone fails to analyze accurately the loop in this program because of the over-approximation of the non-linear expressions before the loop. CP techniques alone run out of time and memory

**Table 4** Execution times and number of false alarms of CDFL, FLUCTUAT and RAICP

	CDFL	FLUCTUAT	RAICP
<b>Total execution time</b>	208.99 s	16.06 s	56.25 s
<b>False alarms</b>	0	11	0

since it is far too expensive to unfold completely such loops. However, CP techniques computed a good approximation of the non-linear expressions at the beginning of the program. That's why RAICP refined significantly the domains of the variables. This example illustrates well that a tight cooperation between CP and AI techniques can be very efficient.

### 5.3 Property checking on academic benchmarks

We used RAICP to check simple assertions that state numeric bounds on floating-point program variables. These assertions come from benchmarks proposed by D'Silva et al (2012) to evaluate CDFL<sup>13</sup>. CDFL is a program analysis tool that embeds the interval abstract domain in the conflict driven clause learning algorithm of a SAT solver. The benchmarks are made from 12 programs by varying the input variable domains, the loop bounds, and the constants in the properties to check. All the programs are based on academic numerical algorithms, except *Sac* which is generated from a Simulink controller model. We discarded 2 out of 57 benchmarks: one that is related to integers only, and another one that merge integers and floats in the same expressions.

On these benchmarks, CDFL was much more efficient than CBMC and much more precise than ASTRÉE for approximating floating-point variable domains (D'Silva et al, 2012). We compare in Table 4 the efficiency of RAICP, FLUCTUAT and CDFL on these benchmarks. RAICP is on average 3.5 times slower than FLUCTUAT used alone, but it is much more precise than FLUCTUAT: FLUCTUAT produced 11 false alarms whereas RAICP successfully eliminated all these false alarms and reported correctly all the 33 true properties.

In other words, RAICP is as effective as CDFL on these benchmarks for checking assertions that state numeric bounds on floating-point program variables. On top of it, RAICP is on average 3.5 times faster than CDFL.

It is however important to note that all of these systems may produce false alarms in the general case.

### 5.4 Property checking on an industrial benchmark

Finally, we applied RAICP to an industrial system provided by Geensys/Dassault Systems. The anti-lock braking system (ABS) is a real time software application running on an electronic unit embedded in a car. The system was designed with Simulink and the embedded code was automatically generated from the Simulink model. The

<sup>13</sup> These benchmarks are available at <http://www.cprover.org/cdfpl>

**Table 5** Validity results and execution times of CBMC, FLUCTUAT and RAICP on property  $P_I$  when varying unfoldings

Number of unfoldings	CBMC		FLUCTUAT		RAICP	
	Validity	Time	Validity	Time	Validity	Time
1	valid	0.4 s	valid	0.04 s	valid	0.9 s
2	-	> 3600 s	unknown	0.03 s	valid	1 s
100	-	-	unknown	1.47 s	valid	19.2 s
1000	-	-	unknown	77.3 s	valid	338.7 s
2000	-	-	unknown	413.9 s	valid	1217.8 s

code contains computations over integer and floating-point variables and consists of an infinite loop that repeatedly reads inputs and computes the output every 0.01 s. Since we bound the number of unfoldings of the real-time loop, we can only check assertions for a limited service time of the system. ABS will be active for at most 20 s when braking on a wet road with a maximum vehicle speed of 180 kilometers per hour and a cautious deceleration value of 2.5 meters per squared second. This means that at most 2 000 unfoldings of the real-time loop are required.

ABS prevents wheel lock when braking. It monitors wheel speed through sensors and acts on an hydraulic valve. ABS looks for the tendency to lock of a wheel. It computes the skidding rate of the slowest wheel as  $r_s = 1 - \frac{v_{\text{slow}}}{v_{\text{car}}}$ . ABS tries to maintain the optimal rate  $r_o = 20\%$ <sup>14</sup>. When  $r_s$  is greater than  $r_o$ , ABS starts controlling braking.

Our industrial partner had specified property  $P_I$  as follows: ABS enters controlled braking as soon as skidding rate is greater than 20%. The state of the ABS is an internal variable, `abs_state`, that can take two predefined values: CONTROLLED or UNCONTROLLED. The assertion to be checked for  $P_I$  is then:

$$(v_{\text{slow}} < 0.8 * v_{\text{car}}) \implies (\text{abs\_state} = \text{CONTROLLED})$$

We compared CBMC, FLUCTUAT, and RAICP on the checking of property  $P_I$ . We did not manage to run CDFL on these benchmarks. For checking this property, the user was only interested by the behavior of the program with a semantics over the floating-point numbers. We fixed a time-out of one hour. Table 5 shows that RAICP could prove quite efficiently that property  $P_I$  holds up to the fixed 2 000 unfoldings limit. Property  $P_I$  trivially holds at the first unfolding which corresponds to the initialization phase of the ABS. CBMC reached the time limit on the second unfolding. This is probably due to the fact that CBMC falls into a slow convergence process. FLUCTUAT is very fast, but it computes such coarse over-approximations that one cannot determine whether the property holds or not.

## 6 Conclusion

In this paper, we introduced a new approach for computing tight intervals of floating-point variables of C programs. The prototype of RAICP we developed relies on the

<sup>14</sup> Actually, optimal rate depends on the road surface and varies between 30% and 10% .

static analyzer FLUCTUAT, on the floating-point solver FPCS, and the real number solver REALPAVER. Thanks to these solvers, RAICP can exploit the refutation capabilities of constraint techniques to refine the domains computed by FLUCTUAT.

This integration of AI and CP works well because the approximation of variable bounds computed by AI is often small enough to allow efficient pruning with partial consistencies. Even though the same domain reductions could sometimes be achieved without starting from the approximation computed by FLUCTUAT, our experiments show that the approximation computed by FLUCTUAT is required in programs with loops. In FLUCTUAT, sets of affine forms abstract non-linear expressions and constraints. These sets constitute better approximations of linear constraint systems than the boxes used in interval-based constraint solvers. Nevertheless, they are less adapted for non-linear constraint systems where filtering techniques used in numeric CSP solving offer a more flexible and extensible framework.

We showed that RAICP is fast and efficient on programs that are representative of the difficulties of FLUCTUAT (conditional constructs and non-linearities). The computed approximations both over the real numbers and the floating-point numbers are much sharper than the ones computed by AI techniques. The user has therefore more facilities to identify suspicious values for which the behavior of the program over the floating-point numbers is different from the behavior the user could expect over the real numbers. Experiments on a significant set of benchmarks showed also that RAICP is as precise and faster than CDFL, a state-of-the-art tool for bound analysis and assertion checking on programs with floating-point computations.

Further work concerns a tighter integration of abstract interpretation and constraint solvers and the generation of counter-examples. For instance, the integration of AI and CP could be done at the abstract domain level instead of the interval domain level. Likewise, the constraint systems generated by RAICP could be used for generating counter-examples when we cannot prove that a property holds.

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