Interface finite elements for parametric study of structures with thin constrained layer damping

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Interface finite elements for parametric studies of structures with thin constrained layer damping

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Abstract. *The design and the optimization of structures with constrained layer damping (CLD) treatment requires an efficient numerical tool. The frame of this work is the modelling of thin constrained viscoelastic layers embedded in a massive structure meshed in 3D. A zero-thickness interface finite element is developed in order to model this viscoelastic layer, which enables to modify both its placement and thickness without remeshing the whole structure.*

Keywords: Interface finite element; CLD treatment; Fractional derivative model.

1 INTRODUCTION

Viscoelastic layer treatment has traditionally been used in industrial applications to reduce structural noise and vibration by dissipating part of the strain energy. Constrained layer damping (CLD) treatment usually dissipate more energy than free layer treatment, but its design must be integrated in the design process of the structure to be damped. Parametric studies are then necessary to determine the optimal viscoelastic material forming the CLD layer, and the optimal placement and thickness of the layer. The prediction of the vibration level of a structure with CLD treatment modelled by finite elements and excited by an harmonic external force $\mathbf{F}$ requires the resolution of a system of the form:

$$\left[\mathbf{K}_{\mathrm{e}} + G^*(\omega)\mathbf{K}_{\mathrm{v}} - \omega^2 \left(\mathbf{M}_{\mathrm{e}} + \mathbf{M}_{\mathrm{v}}\right)\right] \mathbf{X} = \mathbf{F} \tag{1}$$

where $\mathbf{K}_{\mathrm{e}}$ and $\mathbf{M}_{\mathrm{e}}$ (resp. $\mathbf{K}_{\mathrm{v}}$ and $\mathbf{M}_{\mathrm{v}}$) are the stiffness and mass matrices of the elastic (resp. the viscoelastic) part of the structure, and $G^*(\omega)$ is the complex modulus representing the frequency-dependent behaviour of the viscoelastic material. The fractional derivative model

$$G^*(\omega) = \frac{G_0 + G_\infty(\mathrm{i}\omega\tau)^\alpha}{1 + (\mathrm{i}\omega\tau)^\alpha}, \tag{2}$$

generally offers a good representation of the viscoelastic behaviour in the frequency domain [1]. The parameters G_0, G_∞, τ and α can then be identified for several damping materials so that a change of material in a parametric study only consists in changing these four parameters in the scalar complex modulus in Equation (1). For structures with complex geometry, 3D finite elements are required and a change in the thickness or the placement of the viscoelastic layer imply a complete re-meshing of the structure. The calculation of new stiffness and mass matrices for each set of structural parameter represents an additional significant computational cost in a parametric study. The goal of this work is to present the interface elements developed to model thin constrained damping layer [2, 3]. These elements are compared with 3D elements for validation. Such a representation enables the testing of various configurations without having to re-mesh.

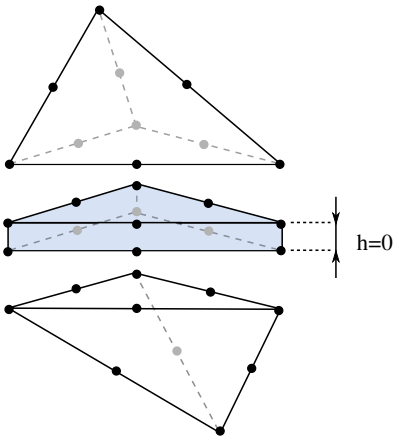

Figure 1: Interface finite element connected to quadratic tetrahedra in the initial configuration.

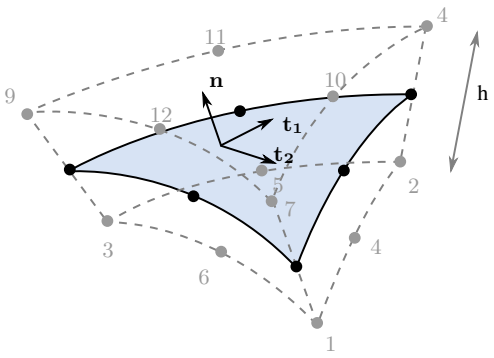

Figure 2: Interface finite element showing its characterizing mean surface and fictive thickness h.

2 INTERFACE FINITE ELEMENT IN THREE-DIMENSIONS

2.1 Element formulation

The interface element is a 12-nodes wedge-shape element connecting two quadratic tetrahedra, as shown on Figure 1. The two triangular surfaces of the interface element lie together in the initial configuration so that the initial thickness is zero. The interface element is then characterized by a surface and a fictive thickness h which is assumed to remain constant (see Figure 2). Each node i has three degrees of freedom: U_x^i, U_y^i and U_z^i, the nodal displacement in the global coordinate system (x, y, z). The three-dimensional field (u_x, u_y, u_z) is approximated by:

$$\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} \mathbf{N} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{N} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{N} \end{bmatrix} \begin{bmatrix} \mathbf{U}_x \\ \mathbf{U}_y \\ \mathbf{U}_z \end{bmatrix} = \mathbb{N}\mathbf{U} \tag{3}$$

where $\mathbf{N}$ is the vector of shape functions for a 12-nodes wedge finite element. The Jacobian matrix of the transformation from the reference coordinate system (r, s, t) to the global coordinate system is calculated as:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial s} & \frac{\partial z}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial \mathbf{N}}{\partial r}\mathbf{X}^{\mathrm{f}} & \frac{\partial \mathbf{N}}{\partial s}\mathbf{X}^{\mathrm{f}} & \frac{\partial \mathbf{N}}{\partial t}\mathbf{X}^{\mathrm{f}} \\ \frac{\partial \mathbf{N}}{\partial r}\mathbf{Y}^{\mathrm{f}} & \frac{\partial \mathbf{N}}{\partial s}\mathbf{Y}^{\mathrm{f}} & \frac{\partial \mathbf{N}}{\partial t}\mathbf{Y}^{\mathrm{f}} \\ \frac{\partial \mathbf{N}}{\partial r}\mathbf{Z}^{\mathrm{f}} & \frac{\partial \mathbf{N}}{\partial s}\mathbf{Z}^{\mathrm{f}} & \frac{\partial \mathbf{N}}{\partial t}\mathbf{Z}^{\mathrm{f}} \end{bmatrix}, \tag{4}$$

where $[\mathbf{X}^{\mathrm{f}}, \mathbf{Y}^{\mathrm{f}}, \mathbf{Z}^{\mathrm{f}}]^T = [\mathbf{X}, \mathbf{Y}, \mathbf{Z}]^T \pm \frac{h}{2}\mathbf{n}$ are the coordinates of the fictive nodes obtained from the coordinates of the physical nodes $[\mathbf{X}, \mathbf{Y}, \mathbf{Z}]^T$ after calculation of the normal $\mathbf{n}$ to the mean surface at each node. The elementary stiffness and mass matrices are defined as:

$$\begin{aligned} \mathbb{K}_v &= \int_{V_e} \mathbb{B}^T \mathbb{C}^*(\omega) \mathbb{B} \det(\mathbf{J}) \mathrm{d}V \\ \mathbb{M}_v &= \int_{V_e} \mathbb{N}^T \rho \mathbb{N} \det(\mathbf{J}) \mathrm{d}V \end{aligned} \tag{5}$$

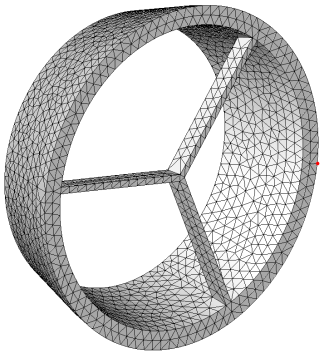

Figure 3: Tridimensionnal structure to be damped by a constrained damping layer at the core of the cylindrical part.

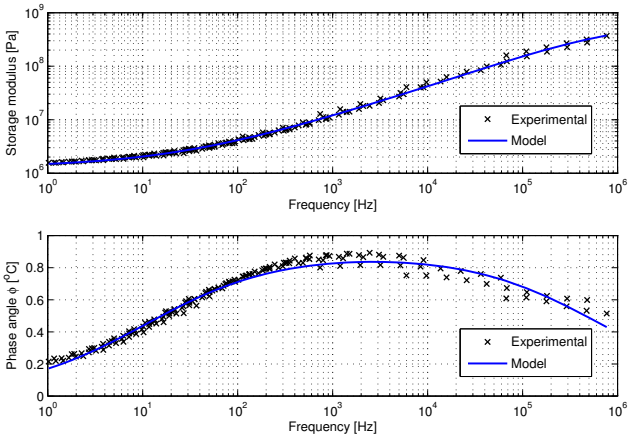

Figure 4: Experimental master curves of Deltane 350 at reference temperature $T = 12^o$ (crosses) and fractional derivative model identified (solid line).

with $\mathbb{B}$ is the discretized gradient matrix, V_e is the volume of the element in the reference coordinate system, ρ is the density of the viscoelastic material and $\mathbb{C}$ is the elasticity matrix which can be written as:

$$\mathbb{C}^*(\omega) = G^*(\omega) \begin{bmatrix} \frac{2(1-\nu)}{1-2\nu} & \frac{2\nu}{1-2\nu} & \frac{2\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{2\nu}{1-2\nu} & \frac{2(1-\nu)}{1-2\nu} & \frac{2\nu}{1-2\nu} & 0 & 0 & 0 \\ \frac{2\nu}{1-2\nu} & \frac{2\nu}{1-2\nu} & \frac{2(1-\nu)}{1-2\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

where ν is the Poisson ratio, which is assumed to remain constant. The integration of the elementary matrices $\mathbb{K}_v$ and $\mathbb{M}_v$ over the volume V_e makes use of 12 quadrature points, considering two quadrature points in the through-thickness direction.

2.2 Element validation

The interface element is validated by calculating the frequency response function of the structure shown on Figure 3, where CLD treatment is applied at the core of the cylindrical part. The thickness of the damping layer represents about $1/50$ of the total thickness of the structure. The elastic structure is made of steel ($E = 2.1e11$ Pa, $\nu = 0.3$, $\rho = 7800$ kg/m^3) and the viscoelastic material used for damping is Deltane 350, a polymer from Paulstra®. The frequency dependent properties of Deltane 350 are represented by the fractional derivative model of Eq.(2), whose identified parameters are:

$$G_0 = 1.29 \text{ MPa} \qquad G_\infty = 0.72 \text{ GPa} \qquad \tau = 0.24\ \mu\text{s} \qquad \alpha = 0.5 \tag{7}$$

The identified fractional derivative model gives a good representation of the mechanical properties of the viscoelastic material, as shown on Figure 4.

The damped structure, which aims at representing a turbine nozzle, is fixed at the intersection of the radial stiffeners and a unit load is applied. Quadratic tetrahedra are used to model the 3D structure while interface finite elements model the damping layer.

The frequency response calculated is compared to the one obtained by modelling the viscoelastic layer with 3D finite elements. Figure 5 shows that both curves are practically superposed, thus validating the interface finite element previously formulated.

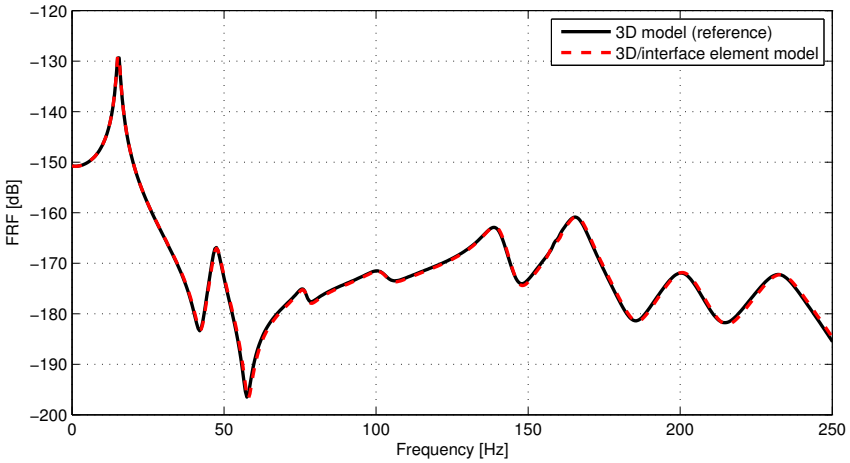

Figure 5: Frequency response function of the damped structure on the interval $[0, 250]$ Hz, with a frequency step of 0.5 Hz, for a constrained damping layer modelled by quadratic tetrahedra (solid line) and by interface finite elements (dashed line).

3 CONCLUSIONS

A 12-nodes interface finite element has been formulated and programmed to model thin constrained viscoelastic layers. This element can be connected to quadratic tetrahedra, which allows the modelling of structures with complex geometry, and gives equivalent results to a three-dimensional representation of the damping layer. Since the interface element is characterized by a mean surface and a fictive thickness, no re-meshing is necessary to test several positions or thickness of the viscoelastic layer, and so the stiffness and mass matrices of the elastic structure need not to be re-calculated.

Moreover, the computation of the frequency response of the damped structure can be reduced by a factor of about 10 using Padé approximations. Instead of solving Equation (1) at each frequency, the direct calculation is performed at selected frequencies and the solution is reconstructed around each selected frequencies thanks to Padé approximations.

The strategy consisting in combining the advantages of the developed interface finite element with the efficiency of the Padé-based reconstruction method is appropriate for parametric studies.

ACKNOWLEDGEMENT

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