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Combining proper orthogonal decomposition and residual estimates for fast time integration of PDEs

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Abstract. *A method to accelerate time-dependent solvers is presented. The method is based on the combination of a numerical code and a Galerkin system in a sequence of alternate time intervals. The governing equations are Galerkin projected onto a set of modes, obtained by performing POD on sets of snapshots calculated by the numerical code. The reduced model is integrated until an a priori error estimate indicates that the approximation is no longer accurate. High order modes truncation instabilities are prevented using an estimate of the residual.*

Keywords: Reduced order models; Proper orthogonal decomposition; Galerkin projection; Low dimensional dynamics.

1 THE METHOD

Traditional computational fluid dynamics (CFD) approaches in aerodynamics design, such as direct numerical simulation or turbulence models require huge computational resources and CPU time. Reducing the computational effort of CFD solvers is therefore a critical point to facilitate both their industrial use [1, 3, 8], and the analysis of the underlying physical problems [2, 6].

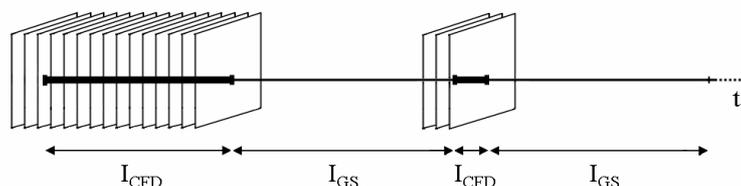


Figure 1: The local POD plus Galerkin projection method.

Local proper orthogonal decomposition (POD) and Galerkin projection have been recently combined to accelerate time-dependent solvers of PDEs. The method was presented in [5] for one-dimensional parabolic equations and extended in [7] for two-dimensional fluid dynamics problems. The method is based on the combined use of a numerical code and a Galerkin system in a sequence of alternate time intervals (see Figure 1). Modes are obtained performing POD on sets of snapshots calculated by the numerical code in the CFD intervals. The governing equations are Galerkin projected onto the most energetic POD modes, providing a reduced system of equations for the evolution of the modal amplitudes. The reduced model was integrated in [5, 7] until an a priori error estimate indicated that the approximation was no longer accurate. The integration of a second Galerkin system with a larger number of modes detected high-order modes truncation instabilities. The drawback of this strategy is that

integrating two Galerkin systems at least doubles the CPU cost.

In this work we discuss an alternative strategy to detect truncation instabilities based on the monitoring of an estimate of the residual of the reduced order model that replaces the integration of the second Galerkin system.

2 IMPROVING THE METHOD

The major computational effort is due to the calculation of the snapshots in the first CFD interval, where the POD manifold is completely calculated. This manifold is only updated in the subsequent CFD intervals, which can be rather small.

A strategy to drastically reduce the size of the first CFD interval was presented in [4]. The method is based on the fact that the POD manifold depends only weakly on the particular values of the parameters of the problem. Thus, the key point is to use libraries constructed from POD modes calculated in previous runs, for other parameter values. These libraries might not contain the true dynamics for the current set of parameters, but the POD manifold can be updated by adding a few snapshots for the true parameter values. In this way, the numerical code has to be used only in a small time interval to compute such snapshots.

In this work we will combine this strategy with our local POD plus Galerkin and Residual method to develop a robust and highly efficient method from the computational point of view. Some numerical experiments will be shown.

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