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Fast inverse modeling technique to reconstruct a laminar flow: Application to a part of Versailles water network

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Abstract. *We propose a fast inverse modeling technique, based on the optimal control theory, to reconstruct a two-dimensional (2D) laminar flow in water networks. To determine the boundary control velocities, we minimize a data misfit functional. Using a priori information on the boundary velocity shape, it enables us to employ a moderate number of velocity sensors. Real-time flow reconstruction is achieved solving CFD problems during an offline stage. Once we get the sensor outputs, only a small linear system has to be solved online.*

Keywords: inverse problem; optimal control; fluid mechanics; water networks.

1 INTRODUCTION

The reconstruction of a flow in a bounded domain using several sensor outputs is a real challenge, especially in water networks. The knowledge of the velocity flow can be useful to detect leaks and to properly simulate the transport of reactants, such as chlorine, in water networks. In water pipe applications, one-dimensional (1D) flow equations are widely considered [1, 2, 3] because they predict the flow at a low computational cost. Nevertheless, the uniform velocity assumption in the pipe cross-section may fail when considering laminar flow. Indeed, in water networks, we can distinguish “transport mains”, defined as pipes that typically do not supply customers directly and “distribution mains” which support many domestic demands. A distribution network is usually designed on fire flow demand, that typically is much higher than domestic demand. Consequently, as it was reported in [4], the typical velocities can be very low (less than 1 cm/s). In this paper, we deal with the flow encountered in “distribution mains”.

In the present article, we focus on a two-dimensional (2D) representation of a water network including junctions where the flow is no more 1D. Two- or three- dimensional flows play a critical role to predict the evolution of the chlorine concentration (reactant) in water networks. In the proposed method, we aim at identifying the boundary control velocities which minimize L^2 error between sensor outputs and the numerical solution as commonly in Optimal control theory [5]. First, we use *a priori* information on the spatial velocity profile in water pipes. More precisely we consider the spatial shape of control velocities to be known. This drastically reduces the number of unknowns to identify. This strategy may also enable one to use a moderate number of sensors. Then, to fastly reconstruct the flow, CFD computations are performed during an offline stage. Online, one only has to get the sensor outputs and to solve a small linear system. The whole numerical procedure easily fits in a general purpose finite element software, thus featuring user’s friendliness for a wide engineering audience.

2 METHODOLOGY TO FASTLY RECONSTRUCT A LAMINAR FLOW

2.1 Direct problem

We consider a two-dimensional (2D) modelisation of a part of Versailles water network, represented in Figure 1. In this area, water companies observe that the flow can be laminar during off-peak periods. As a consequence, we

focus on laminar water flow which satisfies the incompressible Stokes equations :

$$\nabla p - \mu \Delta \underline{v} = \underline{0} \quad \text{in } \Omega \quad (1)$$

$$\text{div}(\underline{v}) = 0 \quad \text{in } \Omega \quad (2)$$

$$\underline{v} = \underline{v}_{c_i} \quad \text{on } \partial_{c_i} \Omega, \quad i \in \{1, 2, 3\} \quad (3)$$

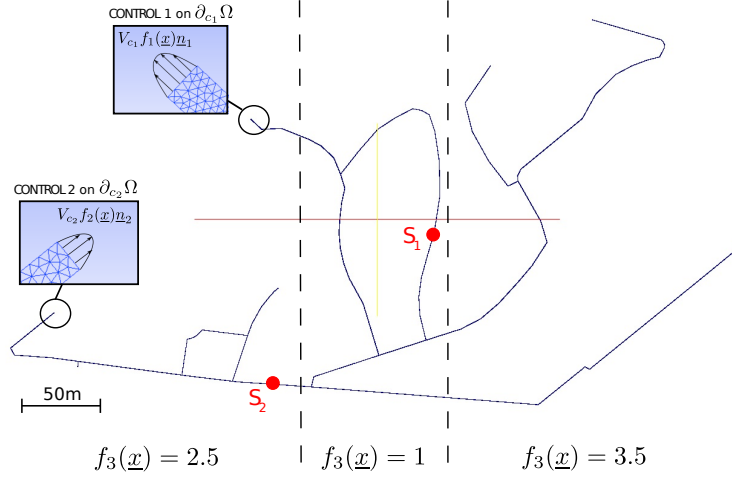


Figure 1: A part of the Versailles water network and the spatial shape function f_3 associated to the inhabitant water consumption, from left to right: intermediate area, residential area, dense area.

The boundary control velocities \underline{v}_{c_1} and \underline{v}_{c_2} are associated to the fictitious boundaries $\partial_{c_1} \Omega$ and $\partial_{c_2} \Omega$ (see Figure 1). To take into account the inhabitant water consumption, we consider an additional boundary control velocity \underline{v}_{c_3} which is defined at the wall boundary $\partial_{c_3} \Omega$. All the boundary control velocities are assumed normal to the boundaries and their spatial shape $f_i(\underline{x})$ is considered known.

$$\underline{v}_{c_i}(\underline{x}, t) = V_{c_i}(t) f_i(\underline{x}) \underline{n}_i, \quad i \in \{1, 2, 3\} \quad (4)$$

where \underline{n}_i is the exterior normal vector to the boundary $\partial_{c_i} \Omega$.

At the fictitious boundaries $\partial_{c_1} \Omega$ and $\partial_{c_2} \Omega$ the spatial shapes $f_1(\underline{x})$ and $f_2(\underline{x})$ are taken parabolic whereas the spatial shape $f_3(\underline{x})$ associated to inhabitant water consumption is chosen piecewise constant. According to the population density in the part of Versailles water network, we distinguish a residential area where $f_3(\underline{x}) = 1$, a dense area where $f_3(\underline{x}) = 3.5$ and an intermediate area where $f_3(\underline{x}) = 2.5$ (see Figure 1).

In what follows we set $F_i = \int_{\partial_{c_i} \Omega} f_i(\underline{x}) d\Gamma$ and the functions $V_{c_i}(t)$ will be called *control velocities*.

The flow being incompressible, we underline that only two control velocities are independent:

$$F_1 V_{c_1} + F_2 V_{c_2} + F_3 V_{c_3} = 0 \quad (5)$$

In the next section, we present the fast inverse technique to identify the control velocities V_{c_1}, V_{c_2} , then we deduce V_{c_3} from (5). Knowing the control velocities, we obtain the reconstructed flow.

2.2 Inverse problem

To determine the control vector $\underline{V}_c = \{V_{c_1}, V_{c_2}\}$, we minimize the data misfit functional:

$$\min_{\underline{V}_c} J(\underline{V}_c) = \min_{\underline{V}_c} \left\{ \frac{1}{2} \sum_{j=1}^2 \left(\underline{v}(\underline{V}_c) \cdot \underline{d}|_{\underline{x}=\underline{x}_j} - v_j^{mes} \right)^2 \right\} \quad (6)$$

In the studied part of Versailles water network, we use 2 velocity sensors. Their spatial location is given in Figure 1. We assume that each sensor is aligned with the pipe axis. Hence \underline{d} will be the unit normal vector to the cross section. As we deal with the reconstruction of a laminar flow using Stokes equations (linear and time-invariant problem), the superposition principle has been used to set up a fast inverse modeling technique.

Reference flow and sensor outputs In practice, we would use real sensor outputs. Nevertheless, herein, the sensor outputs are obtained numerically. To get a representative reference flow, we solve stationary Navier-Stokes equations with the finite element code FreeFem++ [6] assuming $(V_{c_1})_{ex} = -0.02$ m/s and $(V_{c_2})_{ex} = 0.015$ m/s. This computation gives the reference solution v_{ex} . The simulated sensor outputs are derived from v_{ex} by:

$$v_j^{mes} = \int_{\Omega} \psi_j^r v_{ex} \cdot \underline{d} \, d\Omega, \quad j = 1, 2 \quad (7)$$

Fast inverse technique using superposition principle Let (p_I, \underline{V}_I) and $(p_{II}, \underline{V}_{II})$ be the solutions of the elementary Stokes problem. More precisely (p_I, \underline{V}_I) satisfies (1-3) with the imposed boundary velocity $f_1(\underline{x})\underline{n}_1$ on $\partial_{c_1}\Omega$, zero on $\partial_{c_2}\Omega$ and $-(F_1/F_3)f_3(\underline{x})\underline{n}_3$ on $\partial_{c_3}\Omega$. Concerning $(p_{II}, \underline{V}_{II})$, it satisfies (1-3) with the imposed boundary velocity $\underline{0}$ on $\partial_{c_1}\Omega$, $f_2(\underline{x})\underline{n}_2$ on $\partial_{c_2}\Omega$ and $-(F_2/F_3)f_3(\underline{x})\underline{n}_3$ on $\partial_{c_3}\Omega$.

As Stokes problem (1-3) is linear and time-invariant, the solution \underline{v} can be obtained for any set of boundary conditions by superposition:

$$\underline{v} = V_{c_1}\underline{V}_I + V_{c_2}\underline{V}_{II} \quad (8)$$

Substituting the expression (8) in the data misfit functional (6), the minimization problem is:

Find $\underline{V}_c = \{V_{c_1}, V_{c_2}\}$ such that $\underline{A}\underline{V}_c = \underline{b}$ where

$$\underline{A} = \begin{bmatrix} \sum_{j=1}^2 \left(\int_{\Omega} \psi_j^r \underline{V}_I \cdot \underline{d} \, d\Omega \right)^2 & \sum_{j=1}^2 \left(\int_{\Omega} \psi_j^r \underline{V}_I \cdot \underline{d} \, d\Omega \right) \left(\int_{\Omega} \psi_j^r \underline{V}_{II} \cdot \underline{d} \, d\Omega \right) \\ \sum_{j=1}^2 \left(\int_{\Omega} \psi_j^r \underline{V}_I \cdot \underline{d} \, d\Omega \right) \left(\int_{\Omega} \psi_j^r \underline{V}_{II} \cdot \underline{d} \, d\Omega \right) & \sum_{j=1}^2 \left(\int_{\Omega} \psi_j^r \underline{V}_{II} \cdot \underline{d} \, d\Omega \right)^2 \end{bmatrix} \quad (9)$$

$$\underline{b} = \begin{bmatrix} \sum_{j=1}^2 \left(\int_{\Omega} \psi_j^r \underline{V}_I \cdot \underline{d} \, d\Omega \right) v_j^{mes} \\ \sum_{j=1}^2 \left(\int_{\Omega} \psi_j^r \underline{V}_{II} \cdot \underline{d} \, d\Omega \right) v_j^{mes} \end{bmatrix}$$

Note that the matrix \underline{A} only involves the elementary solutions \underline{V}_I and \underline{V}_{II} . As a consequence, this matrix can be precomputed offline from the elementary solutions. Contrary to the matrix \underline{A} , the second member \underline{b} has to be computed online using the sensor outputs and the elementary solutions.

To summarize, with the superposition principle method, the velocity and pressure fields of a laminar flow can be reconstructed by

1. solving numerically $n_c - 1$ elementary Stokes problems
2. computing offline the matrix \underline{A} once and for all
3. computing online the second member \underline{b}
4. solving online a linear system of $n_c - 1$ unknowns
5. deducing the last control velocity by means of the incompressibility condition (5)
6. post processing the velocity and pressure fields, according to (8).

2.3 Numerical results

After solving offline the 2 elementary Stokes problems with the finite element code FreeFem++, we construct the matrix \underline{A} . Then, we compute the second member \underline{b} using the elementary solutions $\underline{V}_I, \underline{V}_{II}$ and the sensor outputs simulated numerically from the Navier-Stokes reference flow.

The solution of the linear system (9) gives $V_{c_1} = -0.02$ m/s and $V_{c_2} = 0.015$ m/s. We deduce the reconstructed flow \underline{v} using (8). To evaluate the error between the reference Navier-Stokes flow v_{ex} and the reconstructed flow \underline{v} , we introduce the error field ϵ_v as the Euclidean norm of $v_{ex} - \underline{v}$

$$\epsilon_v(\underline{x}) = \sqrt{((v_{ex})_x - v_x)^2 + ((v_{ex})_y - v_y)^2} \quad (10)$$

The error field ϵ_v is represented in Figure 2. We observe that the error is less than $5 \cdot 10^{-4}$ m/s in most of the domain. As expected, the highest error is located in the bend pipes and in the junctions. In the bend pipes and in the junctions, 2D local Navier-Stokes computations should be used to get a more efficient reconstructed flow [7].

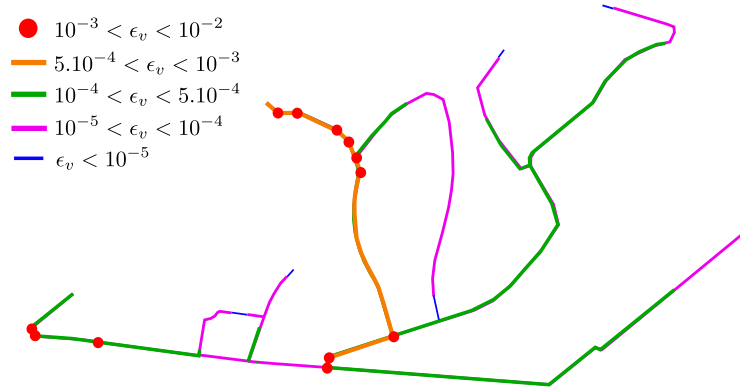


Figure 2: Error field ϵ_v (m/s) between the reference Navier-Stokes flow \underline{v}_{ex} and the reconstructed flow \underline{v} .

3 CONCLUSIONS

We presented a fast inverse technique to reconstruct a 2D laminar flow. The main idea is to identify boundary control velocities which minimize a data misfit functional. Velocity sensor outputs are considered. The method was illustrated on a real water network.

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REFERENCES

- [1] Cembrano, G., Wells, G., Quevedo, J., Perez, R., Argelaguet, R. Optimal control of a water distribution network in a supervisory control system. *Control Engineering Practice* **8**:1177-1188, 2000.
- [2] Constans, S., Bremond, B., Morel, P. Simulation and control of chlorine levels in water distribution networks. *Journal of Water Resources and Planning Management* **129**:135-145, 2003.
- [3] Fabrie, P., Gancel, G., Mortazavi, I., Piller, O. Quality modeling of water distribution systems using sensitivity equations. *Journal of Hydraulic Engineering* **136**:34-44, 2010.
- [4] Blokker, E., Vreeburg, J., Buchberger, S., Van Dijk, J. Importance of demand modelling in network water quality models: a review. *Drinking Water Engineering and Science* **1**:1-20, 2008.
- [5] J.L. Lions, J.L. *Contrôle Optimal des Systèmes Gouvernés par des Equations aux Dérivées Partielles*. Dunod (Paris), 1961 (in french).
- [6] Pironneau, O., Hecht, H., Morice, J., FreeFEM++, <http://www.freefem.org>.
- [7] J. Waeytens, P. Chatellier, F. Bourquin. Influence of discretization and model errors on inverse computational fluid dynamics- Part II: application to a real network including junctions. (*submitted*).