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A flexible and efficient DG discretization for Numerical Weather Prediction.

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Abstract. *As a first step towards construction of a DG based dynamical core for high resolution atmospheric modelling, a semi-implicit and semi-Lagrangian discontinuous Galerkin method for the SWE on the sphere and for nonhydrostatic vertical slice equations is proposed and analysed. The method is equipped with a simple p -adaptivity criterion, that allows to adjust dynamically the number of degrees of freedom employed. Numerical results in the framework of standard idealized test cases prove the accuracy and effectiveness of the method even at high Courant numbers.*

Keywords: Geophysical flows; discontinuous Galerkin methods; semi-implicit discretizations; semi-Lagrangian discretizations.

1 INTRODUCTION

The Discontinuous Galerkin (DG) method, after proving itself a very valuable tool for applications to high Mach number aerodynamics (see e.g. [2], [1], [4], [5], [6]), has also become increasingly popular for applications to geophysical flows. Indeed, its appealing combination of high order accuracy, local mass conservation and ease of massively parallel implementation have turned it into one of the possible choices for next generation climate and NWP dynamical cores, as well as for other environmental fluid dynamics applications, see e.g. [9], [11], [12]. One of the main drawbacks of DG discretizations, however, is that, when coupled to standard explicit time discretizations, they imply rather severe stability restrictions. Denoting by C the Courant number and by p the order of the polynomial basis employed, the equivalent of the standard Courant-Friedrichs-Lewy stability condition turns out to be $C \leq 1/(2p + 1)$, as proven e.g. in [4]. If high order spatial discretizations are envisaged, this is quite restrictive and implies a serious computational burden, that may lead to question the overall effectiveness of DG methods for real time, operational NWP models or for long range climate simulations. Traditionally, remarkable efficiency gains have been achieved in NWP models by application of semi - implicit (SI) and semi - Lagrangian (SL) techniques. The combination of these two approaches, starting with the seminal work by André Robert [16], has produced a number of efficient SISL models for a number of environmental applications, see e.g. [3], [7], [15], [17], [18], [19], [20], [8]. In previous work [13], [14], the first attempts have been presented at increasing the efficiency of DG methods by separate application of either SL or SI techniques. A complete SISL-DG discretization approach for low Mach number, compressible fluid dynamics problems has been already introduced in [21]. To further increase the efficiency of the proposed method, the flexibility of DG discretization with respect to the local number of degrees of freedom is fully exploited, by introducing a simple p -adaptivity strategy that allows to change dynamically the number of degrees of freedom used in each element, without degrading the overall accuracy of the method.

2 NUMERICAL FORMULATION

The proposed numerical formulation is described in details for the shallow water equations on Cartesian geometry in the previous work [21]. A similar strategy has been employed to solve the shallow water equations on the sphere

(and to produce results as those shown in the following). These equations are written in advective, vector form:

$$\frac{Dh}{Dt} + h\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\frac{D\mathbf{u}}{Dt} + g\nabla h + f\mathbf{k} \times \mathbf{u} = -g\nabla b, \quad (2)$$

where h represents the fluid depth, b the bathymetry elevation, f the Coriolis parameter, \mathbf{k} the unit vector locally normal to the earth surface and g the gravity force per unit mass on the earth surface, \mathbf{u} the velocity vector, $\frac{D}{Dt}$ the Lagrangian derivative. These equations are a standard test bed for numerical methods to be applied to the full equations of motion of atmospheric or oceanic circulation models, indeed among their possible solutions, they admit Rossby and external gravity wave propagation as well as the response of such waves to orographic forcing. It should be observed that the continuity equation (1) is written in advective form as the momentum one (2), therefore departing from the formulation used in [21]. This technique is now been applied by the authors to solve the nonhydrostatic vertical slice equations, the final goal being to arrive to a p-SISLDG solver for the fully 3D Euler equations.

3 PRELIMINARY RESULTS

The proposed numerical method has been implemented and tested on a number of relevant test cases using different initial conditions and bathymetry profiles, in order to assess its accuracy and stability properties and to analyze the impact of the p -adaptivity strategy on its efficiency. In order to assess the efficiency gain given by the the adaptivity strategy, for each model variable and for each time level t^n , the computational effort reduction has been measured as $\Delta_{dof}^n = \frac{\sum_{I=1}^N (p_I^n + 1)^2}{N(p_{max} + 1)^2}$, where N is the total number of elements, p_I is the local polynomial degree in the I -th element, while p_{max} is the maximum polynomial degree allowed. As a sample among the many test considered, some results for the well known test 5 and test 6 of [22] are quickly shown in the following. The test case 5 of [22] is given by a zonal flow impinging on an isolated mountain of conical shape. The initial geostrophic balance here is broken by the orographic forcing terms, with resulting development of a planetary wave propagating all around the globe. Figure 1 shows the numerical solution at 15 days, obtained with adaptivity on a 30×15 elements mesh, with maximum polynomial degree $p_{max}^h = 4$, $p_{max}^u = 5$, and $\Delta t = 900s$, corresponding to a Courant number in elements close to poles around 11. This numerical solution is smooth, does not exhibit spurious oscillations and appears very similar to spectral solutions available in literature. Figure 1 on the right top displays the time evolution of Δ_{dof}^n , showing that a saving of more than 25% of degrees of freedom was achieved throughout the adaptive integration. In test case 6 of [22] the initial datum consists of a Rossby-Haurwitz wave of wave number 4. Plots of the fluid depth h as well as of the velocity components u and v at 15 days with p-adaptive SISLDG are shown in figure 2. The resolution used corresponds to a mesh of 40×20 elements with $p^h = 5$, $p^u = 6$, and $\Delta t = 900s$, giving a meridional Courant number $C_{cel,y} \approx 1.75$ and a zonal Courant number close to value $C_{cel,x} \approx 20$ in elements close to poles. It can be observed that all the main features of the flow are correctly reproduced. The method appears to be effective in reducing the computational cost without affecting the overall accuracy of the solution, as it can be observed from the plots shown in the right top of figure 2 for the time evolution of Δ_{dof}^n , showing that a saving of more than 50% of degrees of freedom was achieved throughout the adaptive integration.

4 CONCLUSIONS

The first full semi - implicit, semi - Lagrangian discretization approach based on p -adaptive discontinuous finite elements is introduced. The proposed technique is aimed at applications to low Mach/Froude number compressible regimes, typical of environmental flows. The SISLDG time discretization method allows to improve substantially the efficiency of DG schemes for environmental applications and to extend previous results in this direction discussed in [13, 14]. Indeed, an approach to SISL discretization which is by now standard for finite difference and continuous finite element methods was proven here to be feasible also for discontinuous finite element methods. To further increase the efficiency of the proposed DG method, the flexibility of DG discretization with respect to the local number of degrees of freedom is fully exploited, by introducing a simple p -adaptivity approach that allows to change dynamically the number of degrees of freedom used in each element, without degrading the overall accuracy of the method. Numerical results in a number of relevant test cases have shown that the proposed method indeed allows to use much longer time steps than standard explicit schemes with no loss in accuracy. Furthermore, the p -adaptation algorithm reduces the computational cost by a considerable amount in all the tests performed. These results are quite encouraging and motivate the further extension of the present model to full three dimensional models of atmospheric flows. Indeed, it is planned to develop along this lines a non-hydrostatic, DG based dynamical core

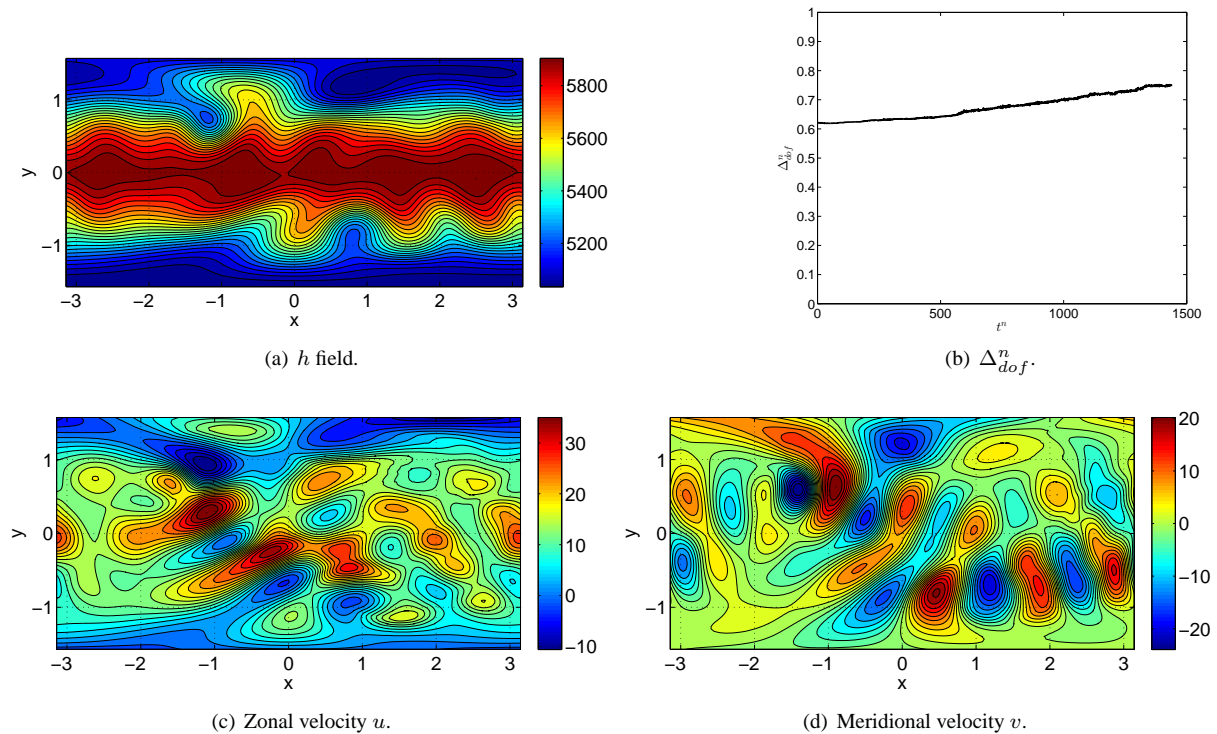


Figure 1: SISLDG solution at 15 days of the isolated mountain test ($C_{cel} \approx 11$ in elements close to poles).

for regional climate modelling, to be employed in the framework of the regional modelling system RegCM [10].

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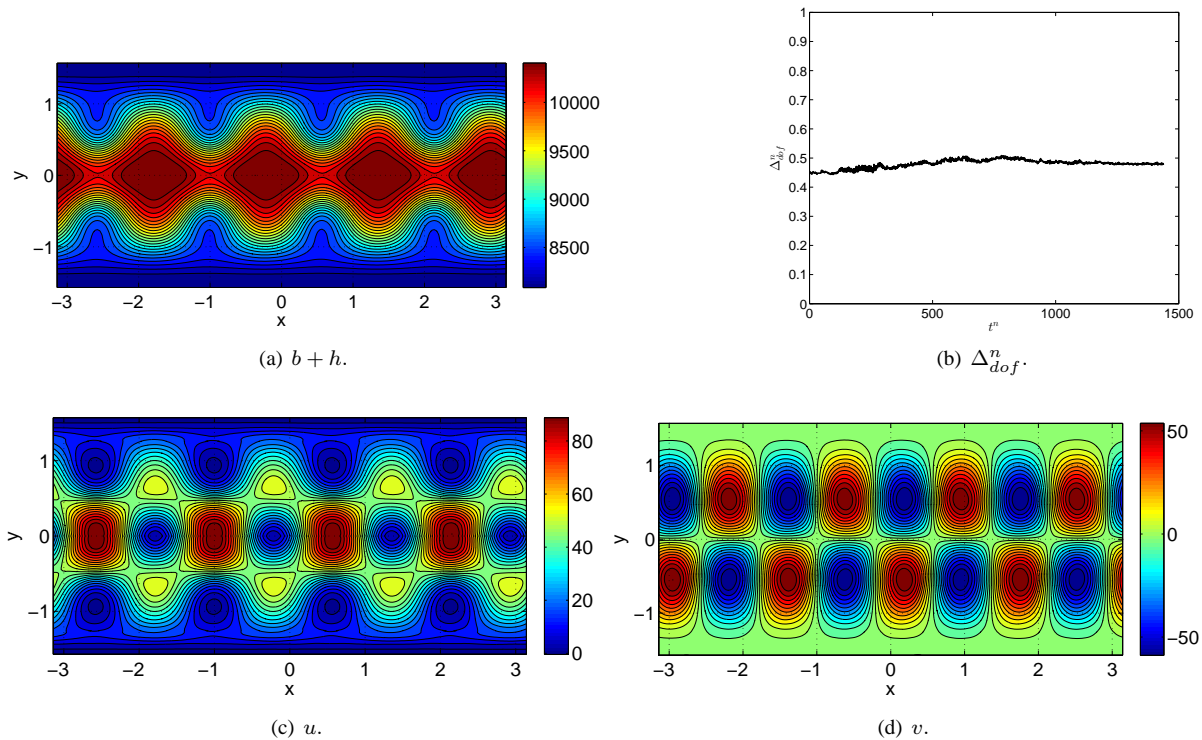


Figure 2: SISLDG solution at 15 days of the Rossby-Haurwitz test ($C_{cel} \approx 20$ in elements close to poles).

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