



HAL
open science

Behaviour of upwind schemes in low Mach number flow

Simon Delmas, Vincent Perrier, Pascal Bruel

► **To cite this version:**

Simon Delmas, Vincent Perrier, Pascal Bruel. Behaviour of upwind schemes in low Mach number flow. 2nd ECCOMAS Young Investigators Conference (YIC 2013), Sep 2013, Bordeaux, France. hal-00855904

HAL Id: hal-00855904

<https://hal.science/hal-00855904v1>

Submitted on 30 Aug 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Behaviour of upwind schemes in low Mach number flow

S. Delmas^{a,*}, V. Perrier^b, P. Bruel^c

^a Université de Pau et des Pays de l'Adour, LMAP UMR CNRS 5142 and Inria Cagire Team
Université de Pau et des Pays de l'Adour, Avenue de l'Université, 64013 Pau, France

^b Inria Bordeaux Sud-Ouest, CAGIRE team and LMAP UMR CNRS 5142
Université de Pau et des Pays de l'Adour, Avenue de l'Université 64 013 Pau, France

^c CNRS, LMAP UMR CNRS 5142 and Inria CAGIRE Team
Université de Pau et des Pays de l'Adour, Avenue de l'Université 64 013 Pau, France

*simon.delmas@inria.fr

Abstract. *In the present work, we are interested in the direct numerical simulation of the compressible Euler and Navier Stokes equations at low Mach number regime. First, we propose a review of existing work on the subject in order to identify the issues raised by the simulation of in this kind of flow, and the existing relevant solutions. Then, we will test different selected compressible low Mach solvers using the discontinuous Galerkin space discretisation and discuss about their behaviour.*

Keywords: Compressible flow; Discontinuous Galerkin method; High order; Low Mach number

1 INTRODUCTION

Many flow configurations of practical interest such as jet engine combustion chambers are characterised by the simultaneous presence of a low Mach and acoustic waves. Thus, the capability of accurately simulating this type of systems is of crucial importance when designing the new generation of engines. Tackling this issue numerically is quite a challenging problem. Indeed, it is not only necessary to preserve the acoustic character of the Euler or Navier Stokes equations, which means keeping the compressible property of the flow but also to handle properly the asymptotic behaviour of the equations as the Mach number tends to zero.

Analysing the continuous system by performing double scale asymptotic expansion in power of the Mach number reveals that the limit of the compressible model when the Mach number tends to zero is the incompressible model [8]. However, numerical simulation using classical compressible solvers give solutions which do not converge to the solution of the incompressible system when the Mach number tends to zero [1, 2, 3, 4, 5, 7, 8, 9, 10]. These studies reveal different problems such as the large disparity between the material velocity and the acoustic wave speed which induces stiffness [10], or problems in the scaling of the numerical dissipation introduced by upwind finite volumes low Mach number schemes [4].

In the present work, we propose to perform computations using the discontinuous Galerkin space discretisation and to observe its behaviour for both steady and unsteady low Mach flows.

2 REVIEW OF KNOWN PROBLEMS

2.1 Stiffness problem

The large disparity of wave speeds at low Mach number between flow and acoustic phenomena makes explicit time discretisation inefficient due to very small CFL condition as shown by Turkel [10]. Dealing with this stiffness, this author proposed to precondition the system in order to accelerate the convergence of the solution. Semi-implicit time discretisation as SIMPLE type algorithms are other solutions to cure this problem by implicitly solving the stiff

part of the system [3, 7, 10].

2.2 Accuracy problem

As far as stability is concerned, it is well known that upwind schemes introduce viscosity. This numerical dissipation which depends on the Mach number was studied by Guillard and Viozat [4] for the Roe solver. They noticed by performing an asymptotic analysis of both the continuous and discrete systems that the scaling of the numerical dissipation in the Roe solver and the scaling of dissipation in the continuous case were different. This explains the accuracy problem for this solver when the Mach number becomes small. A preconditioning of the flux was proposed for overcoming this problem. Dellacherie [2] extended this analysis to all Godunov type schemes and proposed a cure to all of them.

The study of the unsteady case are rarer, we can cite the recent study by Moguen et al. [7]. In the unsteady case, the pressure-velocity coupling coefficient is of different scale order than that of the steady case. By performing numerical test comparisons these authors show that this coefficient must explicitly depend on the time step.

2.3 Geometry of cells and high order

Feistauer and Kucera [3] performed computations using the discontinuous Galerkin method for space discretisation. Their tests seem to prove the accuracy of their method. However, Rieper and Bader [9] and Guillard [5] have proved that the convergence problem at low Mach number disappear on triangular grids. As the tests carried out in [3] were realised on triangular meshes, accuracy of this method in general geometry can therefore be questioned.

Bassi et al. [1] experimentally confirm the property of triangular meshes with discontinuous Galerkin method. Tests on quadrangular grids are also performed using preconditioned time derivative term and flux function. This method significantly improves the convergence of the solution with polynomial approximations of degrees 1 to 3.

PROPOSED APPROACH

The work proposed here, is to test the behaviour of discontinuous Galerkin methods on compressible low Mach number flow, using the AEROSOL library developed at INRIA [6]. We will particularly focus on the issues presented above by testing the effectiveness the different solutions proposed, like preconditioning methods. Then, we also propose to test the behaviour of higher order methods which are known to be less diffusive.

REFERENCES

- [1] F. Bassi, C. De Bartolo, R. Hartmann, A. Nigro, A discontinuous Galerkin method for inviscid low Mach number flows *Journal of Computational Physics* **228** 3996-4011 (2009)
- [2] S. Dellacherie, Analysis of Godunov type schemes applied to the compressible Euler system at low Mach number *Journal of Computational Physics* **229** 978-1016 (2010)
- [3] M. Feistauer, V. Kucera, On a robust discontinuous Galerkin technique for the solution of compressible flow *Journal of Computational Physics* **224** 208-221 (2007)
- [4] H. Guillard, C. Viozat On the behaviour of upwind schemes in the low Mach number limit *Computers & Fluids* **28** 63-86 (1999)
- [5] H. Guillard, On the behavior of upwind schemes in the low Mach number limit. IV: P0 approximation on triangular and tetrahedral cells *Computers & Fluids* **38** 1969-1972 (2009)
- [6] D. Amenga Mbengou, D. Genet, C. Lachat, E. Martin, M. Mogé, V. Perrier, F. Renac, F. Rué, M. Ricchiuto Comparison of algorithm in Aerosol and Aghora for compressible flows *INRIA research report 8200* (2013)
- [7] Y. Moguen, T. Kousksou, P. Bruel, J. Vierendeels, E. Dick, Pressure-velocity coupling allowing acoustic calculation in low Mach number flow *Journal of Computational Physics* **231** 5522-5541 (2012)
- [8] B. Müller. *In Lecture Series 1999-03*. von Karman Institute for Fluid Dynamics, 1999.
- [9] F. Rieper, G. Bader, The influence of cell geometry on the accuracy of upwind schemes in the low Mach number regime *Journal of Computational Physics* **228** 2918-2933 (2009)
- [10] E. Turkel, Review of preconditioning methods for fluid dynamics *Applied Numerical Mathematics* **12** 257-284 (1993)