Application of the integral approach and wavelet transform in simulating and analysis of guided waves excitation by axisymmetric partially debonded piezoactuators

Mikhail Vladimirovich Golub, Alisa Shpak

To cite this version:
Mikhail Vladimirovich Golub, Alisa Shpak. Application of the integral approach and wavelet transform in simulating and analysis of guided waves excitation by axisymmetric partially debonded piezoactuators. 2nd ECCOMAS Young Investigators Conference (YIC 2013), Sep 2013, Bordeaux, France. hal-00855903

HAL Id: hal-00855903
https://hal.science/hal-00855903
Submitted on 30 Aug 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Application of the integral approach and wavelet transform in simulating and analysis of guided waves excitation by axisymmetric partially debonded piezoactuators

M.V. Golub\textsuperscript{a,*}, A.N. Shpak \textsuperscript{a}

\textsuperscript{a} Institute for Mathematics, Mechanics and Informatics, Kuban State University, 350040, Krasnodar, Russia, Stavropol’skaya str. 149

*mgolub@inbox.ru

Abstract. \textit{The aim of the present work is to simulate and understand dynamics of partially debonded axisymmetric piezoactuators. The mathematical and computer models are based on a semi-analytical integral approach. Simulation results are analyzed and compared with experimental data. The wavefields have integral representations of Fourier transforms of Green’s matrix of an elastic layer and surface load generated by the actuator. Wavelet transform is used in order to analyze experimental and numerical results.}

Keywords: guided waves; Lamb wave; debonding; structural health monitoring; piezoactuator; wavelet transform; integral approach.

1 INTRODUCTION

Due to increasing complexity of everyday used civil objects such as airplanes, nuclear power plants, rotor blades of wind turbines, damage detecting technologies are of a great importance. In view of possible catastrophic failures, cracks and debondings are among the main goals of inspection. Since the presence of defects becomes evident from scattered waves, the ascertainment of their typical patterns is of great importance for the detectability of delaminations. Lamb-wave testing has already shown good efficiency for Structural Health Monitoring \cite{1}. Active health monitoring techniques that are based on Lamb waves often use a certain number of piezoelectric waver active sensors (PWAS). A detailed understanding of the dynamics of PWAS is of great importance in order to optimize the diagnostic system. In recent years, a considerable amount of literature has been published on PWAS modeling and experimental analysis trying to explain the transducer dynamics, e.g. \cite{2, 3, 4}. Failure of actuators might lead to significant problems and evidently the monitoring of actuators themselves is necessary. The aim of the present work is development of mathematical and computer model of elastic layered waveguides with PWAS generating Lamb waves in order to collect knowledge about the effects and dynamics of partially debonded PWAS.

2 EXPERIMENTAL SETUP

Circular piezo wafer active sensor with disk-wrapped electrode are widely used in practical applications therefore they are chosen as a subject of present research \cite{4}. In order to reveal the difference in the wavefield, generated by the bonded and debonded actuators a Laser Doppler Vibrometer setup is used. Sixteen PWASs were glued with various degree of debonding to an aluminum plate of the thickness of 2 mm. When a transducer is excited by a Hann-windowed toneburst voltage signal

\[
p(t) = \frac{1}{2} \cos(2\pi\omega_0 t) \left( 1 - \cos \left( \frac{2\pi\omega_0 t}{N} \right) \right), \quad 0 < t < \frac{N}{\omega_0}
\]  

(1)

of central frequency $\omega_0$ Lamb waves are produced and the laser vibrometer measures the out of plane velocities on the surface of the plate. The measurements are made with various central frequencies from 30 kHz to 180 kHz.
An experimental setup with partially debonded actuators is used to investigate wave excitation in an aluminum plate. An example of debonded PWAS is given in Figure 1. Phenomena accompanying wave excitation by debonded actuators are also under investigation. Collected data are analyzed below in order to identify existence, location and shape of a debonded part of the actuator.

3 MATHEMATICAL MODEL

Two classes of the mathematical models can be marked out in literature. The class of coupled models takes into account the coupling between the actuator and the substrate; this leads to the corresponding boundary-value problems, e.g. [2, 3, 4]. The second class of so-called uncoupled models uses a predetermined form of the stresses generated by actuators. This approach arising from the classical pin-force model [5] uses concentrated forces applied at the points of patch edges as the approximation of stresses generated by PWASs. Due to this assumption the approach is simpler in exploitation and it has proved adequacy in practical applications.

The Cartesian coordinates \( x = (x, y, z) \) are introduced in such a way, that the origin of the coordinates coincides with the center of the PWAS. The \( z \)-axis is an outer normal to the surface of the layer of thickness \( H \). Dimensionless coordinates are used in order to simplify the calculation. Dimensionless frequency is introduced as \( \omega = 2\pi f H/v_s \), where \( f \) is dimensional frequency, \( H \) is thickness of the plate and \( v_s \) is velocity of the motion.

3.1 Integral approach

The linear theory of elasticity assumes a displacement vector \( \mathbf{u}(x, t) \) governed by the Lame equations written in Cartesian coordinates \( x \). The boundary conditions on displacements and velocities

\[
\mathbf{u}(x, t < 0) = 0; \quad \frac{\partial \mathbf{u}(x, t < 0)}{\partial t} = 0
\]

\[
\mathbf{u}(x, t < 0) = \frac{\partial \mathbf{u}(x, t < 0)}{\partial t} = 0
\]

force the plate to be at rest until the moment \( t = 0 \) when the surface load \( q(x, y) \) induced by the PWAS on the top surface \( z = 0 \) of the layer \( \{(-\infty < x, y < \infty, -H < z < 0)\} \) is applied

\[
\tau(x, y, z = 0, t) = q(x, y)p(t),
\]

whereas the bottom of the layer is stress-free

\[
\tau(x, y, z = -H, t) = 0.
\]

The vector \( \tau \) is composed of normal stress \( \sigma_{zz} \) and tangential stresses \( \sigma_{xz}, \sigma_{yz} \) on the surface with normal \( (0, 0, 1) \). The stresses \( \sigma_{ij} \) are related to displacements via Hooke’s law; \( p(t) \) is given by (1).

The solution in time domain \( \mathbf{u}(x, t) \) can be represented as the integral transform of the harmonic solution \( \mathbf{u}(x, \omega) \).

Thus, the velocities can be computed as the following inverse Laplace transform

\[
\frac{\partial \mathbf{u}(x, t)}{\partial t} = -\frac{1}{\pi} \text{Re} \int_{-\infty}^{\infty} i\omega \mathbf{u}(x, \omega) P(\omega)e^{i\omega t} d\omega
\]
Here $P(\omega)$ is the Fourier transform of input impulse function arising from the condition (2). The Fourier transform of the time-harmonic solution $u(x, \omega)$ has an integral representation

$$u(x, y, z, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K(\alpha_1, \alpha_2, z, \omega)Q(\alpha_1, \alpha_2)e^{i(\alpha_1 x + \alpha_2 y)} d\alpha_1 d\alpha_2$$

written via the Fourier transform $Q(\alpha_1, \alpha_2)$ of the surface load $q(x, y)$ and the two-dimensional Fourier transform $K(\alpha_1, \alpha_2, z, \omega)$ of the Green’s matrix for the 3D case has been given in [6]. The integral representation is very convenient in simulation: another structures can be modeled by substitution of $K$ matrix only, asymptotics can be derived in the far-field zone.

### 3.2 Wavelet transform

Continuous wavelet transform is applied to the recorded signal in order to identify the debonded part of the investigated PWAS. And more specifically to compute the carrier frequency of the generated wavefield which could significantly differ from the central frequency of the input signal especially in the case of debonded PWAS. The Gabor wavelet is selected as a kernel function [7] due to its waveform is similar to the generated signal used in the experiment.

$$\psi_G = \frac{1}{\sqrt{\pi}} \frac{\omega_0}{\gamma} \exp \left[ -\frac{1}{2} G \left( \frac{\omega_0}{\gamma} \right)^2 t^2 + i\omega_0 t \right]$$

Here $\psi_G$ is Gabor wavelet function with parameter $G$ optimized for the problem under consideration and $\gamma = \pi \sqrt{2}/\ln 2$. Continuous wavelet transform is performed with the following formula

$$W(\omega, t) = \sqrt{\frac{\omega}{\omega_0}} \int_{t_1}^{t_2} u(\xi)\psi_G \left( \frac{(\xi - t)\omega}{\omega_0} \right) d\xi,$$

where $u(\xi)$ is recorded signal, $\xi \in [t_1, t_2]$ and $\omega_0$ is the central frequency of the input voltage signal. Maximum of the function $W(\omega, t)$ from the formula above is the value of the carrier frequency $\omega_c$ of the recorded signal.

### 4 Numerical Analysis

![Figure 2: Geometry of the problem.](image)

For a certain sufficiently debonded PWAS some interesting abnormalities are detected for higher frequencies. A sizable increase in the amplitude of the velocity of the motion was observed (up to 300 percent in comparison to perfectly bonded PWAS). It was revealed that the velocities of the motion and carrier frequencies $f_c$ depend on shape of the debonded part of the PWAS. An example demonstrating this phenomenon is given in Figure 2. It can be observed a certain deviation of the carrier frequency $f_c$ from central frequency of the input signal. Thus, it is known that elastodynamic scattering by a crack may be resonant at certain frequencies depending on crack’s size and location.
5 CONCLUSIONS

The knowledge on the dynamics of debonded PWAS, e.g. resonance frequencies or carrier frequency shift according to central frequency, is important for estimation whether PWAS is damaged or not. Wave pattern is also useful for optimizing the transducer positions within SHM system especially at higher frequencies. Wavelet transform can be applied to determination of wave characteristics of elastic layer–piezoactuator system. Accordingly, ongoing and future work is concentrated on full mathematical model, which takes into account interaction of PWAS with elastic layer and axisymmetric electric field applied to the PWAS.

ACKNOWLEDGEMENT

The authors are grateful to Professors E.V. Glushkov, N.V. Glushkova and C.-P. Fritzen for valuable discussions and support. The experiments were performed in Institute of Mechanics and Control Engineering-Mechatronics (University of Siegen, Germany) many thanks to Dr. J. Moll and MSc. I. Buthe. The work is supported by the Russian Foundation for Basic Research (Projects 12-01-33011, 12-01-00320, 13-01-96516).

REFERENCES