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# Numerical investigation of interaction between shock waves and dusty flows

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## Abstract.

We numerically investigate the interaction between gas-particles flows and shock waves that form during the explosive decompression of underexpanded jets. Simulations are carried out using a validated second-order accurate numerical model for multiphase flows. We quantify theoretically the expected non-equilibrium effects between the gas and the solid phase in terms of the ratio between the particle relaxation time and the characteristic time scale of the jet. We show how multiphase effects affect the average jet dynamics. We carry on the analysis for monodisperse mixtures and we extend it to bidisperse and polydisperse mixtures.

**Keywords:** Dusty flows; supersonic jets; shock waves; numerical modeling; finite volume methods.

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## 1 INTRODUCTION

We investigate the problem of underexpanded jet decompression when the injected fluid is a mixture of a gaseous phase and different classes of solid particles. Whereas the case of homogeneous jets has been studied deeply in the literature, both experimentally, theoretically and numerically, the case of dusty jets still presents some open issues. It has been proven theoretically and experimentally that nozzles with supersonic or sonic velocity and gas pressure greater than the atmospheric one result in a rapid expansion and acceleration of the fluid to high Mach number. A series of expansion waves form and are reflected as compression waves at the flow boundary. The compression waves coalesce to form a standing normal shock wave (Mach disk), across which the fluid is rapidly compressed and decelerated to subsonic speeds (Figure 1). When solid particles are added to the gas flow, new phenomena associated to kinetic and thermal non-equilibrium between gas and particulate phases arise.

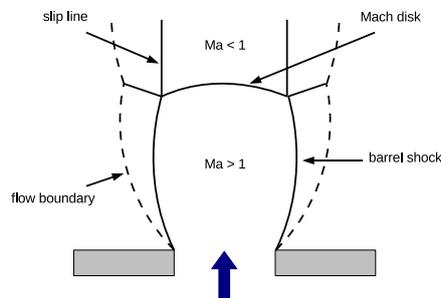


Figure 1: Decompression structure in underexpanded supersonic jets.

In the present work we try to identify and to quantify these non-equilibrium effects, that are related to the interaction between the decompression structure inside the jet and solid particles. We introduce a ratio between the particle relaxation time and the characteristic jet time scale to identify different regimes. The change in the value of this

ratio can be related to a displacement of the wave pattern inside the underexpanded jet and a disequilibrium between the gas and the particle dynamics. The analysis is carried out for the case of monodisperse mixture, then it is extended to the case of bidisperse and polydisperse mixtures.

Numerical tests are carried out using the PDAC (Pyroclastic Dispersal Analysis Code) model presented in [1]. On the laboratory scale we obtain a good qualitative and quantitative agreement with experimental results available in the literature. However we concentrate mainly on larger spatial scales, such as the characteristic scales of explosive volcanic eruptions, during which the erupted multiphase mixture can manifest the features of supersonic flows [2, 3], since we are interested ultimately in the effects that the interaction between shock waves and solid particles has on the whole eruption dynamics.

## 2 MULTIPHASE FLOW MODEL

In this work we adopt the PDAC model equations proposed in [1] to describe the injection and dispersal of a high-velocity gas-particle mixture in a standard atmosphere. The mixture is composed of different gaseous components and  $N$  classes of solid particles, that are described as interpenetrating continua, following an Eulerian-Eulerian approach. The gas phase is compressible and obeys the ideal gas law, whereas each class of solid particles is assumed to consist of spherical particles of equal radius and density, with fixed specific heat and thermal conductivity.

### 2.1 Basic equations

We denote with the subscript  $s = 1 \dots N$  the classes of solid particles and with  $g$  the gas phase. The PDAC model consists of  $5(N + 1)$  coupled partial differential equations for the independent variables gas pressure  $p_g$ , gas density  $\rho_g$ , volume fractions  $\epsilon_s$ , velocities  $\mathbf{v}_s$ , enthalpies  $h_s$  (or temperatures  $T_s$ ), with  $s = g, 1 \dots N$ . The system of equations that represents the mass conservation, the momentum balance and the energy balance in terms of the enthalpies for the gas phase  $g$  and the  $s$ -th solid phase has the following form:

$$\begin{aligned}
\frac{\partial(\epsilon_g \rho_g)}{\partial t} + \nabla \cdot (\epsilon_g \rho_g \mathbf{v}_g) &= 0 \\
\frac{\partial(\epsilon_s \rho_s)}{\partial t} + \nabla \cdot (\epsilon_s \rho_s \mathbf{v}_s) &= 0 \\
\frac{\partial(\epsilon_g \rho_g \mathbf{v}_g)}{\partial t} + \nabla \cdot (\epsilon_g \rho_g \mathbf{v}_g \mathbf{v}_g) &= -\epsilon_g \nabla p_g + \epsilon_g \rho_g \mathbf{g} + \sum_{s=1}^N D_{s,g} (\mathbf{v}_s - \mathbf{v}_g) \\
\frac{\partial(\epsilon_s \rho_s \mathbf{v}_s)}{\partial t} + \nabla \cdot (\epsilon_s \rho_s \mathbf{v}_s \mathbf{v}_s) &= -\epsilon_s \nabla p_g + \epsilon_s \rho_s \mathbf{g} + \sum_{p=1}^N D_{p,s} (\mathbf{v}_p - \mathbf{v}_s) + D_{g,s} (\mathbf{v}_g - \mathbf{v}_s) \\
\frac{\partial(\epsilon_g \rho_g h_g)}{\partial t} + \nabla \cdot (\epsilon_g \rho_g h_g \mathbf{v}_g) &= \epsilon_g \left( \frac{\partial p_g}{\partial t} + \mathbf{v}_g \cdot \nabla p_g \right) + \sum_{s=1}^N Q_s (T_s - T_g) \\
\frac{\partial(\epsilon_s \rho_s h_s)}{\partial t} + \nabla \cdot (\epsilon_s \rho_s h_s \mathbf{v}_s) &= Q_s (T_g - T_s).
\end{aligned} \tag{1}$$

Here,  $D_{p,s}$  represents the drag coefficient describing the interaction between the phase  $p$  and the phase  $s$ ,  $\mathbf{g}$  denotes the gravitational acceleration vector and  $Q_s$  is the volumetric heat transfer rate between the gas and the  $s$ -th solid phase. As reported in [4], viscous dissipation effects can be neglected due to their second-order effect in comparison with advection, conduction and gas-particle heat and momentum exchange in the regimes of interest.

By definition of the volumetric and mass fractions, one also has the relations  $\epsilon_g + \sum_{s=1}^N \epsilon_s = 1$ , with  $0 \leq \epsilon_g \leq 1, 0 \leq \epsilon_s \leq 1$ . The gas phase is compressible and we suppose that thermodynamic quantities are related by the ideal gas law  $p_g = R \rho_g T_g$ , where  $R$  is the gas constant of the gaseous phase. Particulate solid phases are considered incompressible. Consequently, their microscopic density is assumed to be constant and denoted by  $\rho_s$ ,  $s = 1 \dots N$ . The temperature of each phase is derived from its enthalpy as  $T_s = \frac{h_s}{c_{p,s}}$ ,  $s = g, 1 \dots N$ , where the constant pressure specific heats of the particles  $c_{p,s}$  are assumed to be constant and to correspond to average values, due to their minor sensitivity on temperature. Specific heats, interphase drag coefficients and heat transfer rates are derived from semi-empirical correlations for dilute and dense regimes [5, 6, 7].

### 2.2 The numerical method

The model equations are discretized in time by a second order scheme described in [8] and solved using a semi-implicit version of the implicit multifield (IMF) algorithm proposed in [9]. The model equations are discretized

in space by a finite volume approach on an orthogonal, non uniform mesh, adopting a second-order fully multidimensional scheme for advective terms, as described in [8]. The PDAC model and the numerical procedure adopted in this work have been validated against theoretical and experimental results and the model is able to reproduce correctly well-known benchmarks, as shown in [8].

### 3 ASSESSMENT OF MULTIPHASE EFFECTS

#### 3.1 Characteristic time scales in dusty supersonic jets

When solid particles are added to the gas flow, phenomena associated to kinetic and thermal non-equilibrium between gas and particulate phases arise. Such effects are controlled by drag and heat exchange terms in the momentum and energy equations. To quantify these effects, dilute gas-particle flow can be characterized by a time scale, the particle relaxation time  $\tau_s$ , determined by the balance between particle inertia and gas-particle viscous drag. The relaxation time  $\tau_s$  gives an order of magnitude of the time delay with which a particle equilibrates to a time-varying gas flow. We can compare the particle relaxation time with the formation time  $\tau_{Ma}$  of the Mach disk [3].

When particle relaxation time is much lower than the jet time scale, particles are tightly coupled to the gas phase. Under such assumption, kinematic and thermal equilibrium between the phases can be assumed and the gas-particle mixture can be described as an homogeneous pseudogas, characterized by average thermodynamic and rheologic properties. Numerical simulations carried out in this regime with the multiphase model are in quantitative agreement with results obtained with a pseudogas model proposed [10].

When the particle relaxation time and the formation time of the Mach disk are comparable, multiphase effects become important and affect the average dynamics of the underexpanded jet. Numerical results are in agreement with the experimental results proposed in [11].

#### 3.2 Monodisperse mixture

We start considering a mixture of gas and only one class of solid particles with a given mixture density  $\rho_{mix} = \epsilon_g \rho_g + \epsilon_s \rho_s$  and studying the ratio  $r = \tau_s / \tau_{Ma}$  between the particle relaxation time and the jet time scale we identify a threshold particle diameter such that  $r \simeq 1$ . Then we investigate numerically how the jet dynamics change as the ratio  $r$  vary as a function of the particle diameter, covering the spectrum from few micrometers to one millimeter. Numerical tests are carried out both on the laboratory scale, obtaining a good quantitative and qualitative agreement with experimental results presented in the literature, and on larger scales, such as the typical scale of an explosive volcanic eruption. We observe that when  $r \ll 1$ , solid particle dynamics is strongly coupled with the gas dynamics, i.e. they have the same velocities and temperature, whereas as  $r$  becomes of the order of 1, the solid phase and the gas phase show different dynamics, i.e. different velocity and temperature profiles along the jet axis. Moreover, the increase in the particle diameter can be related to the change in the decompression structure inside the underexpanded jet in terms of normal shock location and intensity.

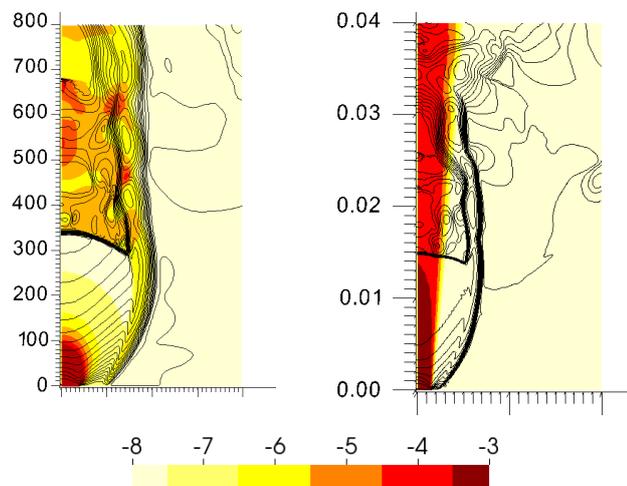


Figure 2: Examples of different regimes inside multiphase underexpanded jets. Plot of logarithm to the base 10 of particle volume fraction and contour of gas vertical velocity. Left.  $r \ll 1$  Right.  $r \gg 1$ .

### 3.3 Bidisperse and polydisperse mixtures

The analysis is extended to the case of bidisperse mixture, in which we consider the gas phase and two classes of solid particles,  $s_1$  and  $s_2$ . In particular, we consider  $s_1$  such that  $r_{s_1} \ll 1$  and  $s_2$  such that  $r_{s_2} \simeq 1$ . From the previous results, we can describe the mixture composed by the gas phase and the solid particles  $s_1$  as a pseudogas. Maintaining a constant mixture density  $\rho_{mix}$ , we change the composition of the mixture by changing the volumetric fractions of the two classes of solid particles ( $\epsilon_{s_1}, \epsilon_{s_2}$ ). We introduce a particle relaxation time  $\tau_{s_2}$  for the particle  $s_2$  moving in a pseudogas composed by the gas and the solid particles  $s_1$ . We study how the internal structure of the multiphase underexpanded jet changes as the ratio  $\tau_{s_2}/\tau_{Ma}$  changes as a function of the mixture composition ( $\epsilon_{s_1}, \epsilon_{s_2}$ ). Finally, we propose a possible extension of the previous analysis to the general case of  $N$  classes of solid particles.

## 4 CONCLUSIONS

In the presented work we have investigated the decompression structure inside a gas-particle underexpanded jet and we have analysed the non-equilibrium effects between the gas phase and the solid particulate phase. The objective has been to identify different regimes and to quantify non-equilibrium effects as a function of different characteristic time scales of the supersonic jet and the solid particles. A ratio  $r$  between the particle relaxation time and the characteristic jet time scale has been introduced. This ratio allows to identify a pseudogas regime, in which non-equilibrium effects between the gas and the solid phase are negligible. Moreover, the value of the parameter  $r$  is related to the change in the decompression structure of the underexpanded jet, in particular to the location and the intensity of the standing normal shock. The analysis has been developed for the case of monodisperse mixtures and then extended to the case of bidisperse and polydisperse mixtures of gas and solid particles.

We are now investigating possible extensions of the presented analysis in order to consider the continuous spectrum of solid particles that are ejected during explosive volcanic eruptions. The final aim will be to correlate the ejected mixture composition to the global dynamics of explosive volcanic eruptions.

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