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# Numerical methodologies for sheet metal forming inverse problems

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**Abstract.** *The use of optimization methods in engineering is increasing. Process and product optimization, inverse problems, shape optimization and topology optimization are frequent problems both in industry and science communities. In this paper, numerical methodologies for computational mechanics inverse problems, such as the parameter identification and the shape optimization problems in metal forming, are presented and discussed. The parameter identification problem refer to the definition of input parameters to be used in constitutive models for numerical simulations, based on experimental data. The initial blank and tool design problem aim to estimate the initial shape of a blank or a tool in order to achieve the desired geometry after the forming process. special emphasis is done to the right choice of the objective function in this problems. A new objective function is also presented.*

**Keywords:** Inverse problems; parameter identification; inverse shape optimization.

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## 1 INTRODUCTION

The use of optimization methods in engineering is increasing. Process and product optimization, inverse problems, shape optimization and topology optimization are frequent problems in both industry and scientific communities. In engineering inverse problems, specifically in computational mechanics, parameter identification and inverse shape optimization take a special role [1].

Parameter identification problems have emerged due to the increasing demanding of precision in the numerical results obtain by Finite Element Method (FEM) software. High result precision can only be obtained with confident input data and robust numerical techniques. Unfortunately, the large majority of the robust numerical techniques are inherently more complex. Constitutive material models developed to simulate with increasing accuracy the behaviour of different materials are an example of these techniques that became more complex. However, the accuracy of the model is much dependent on the model input data (constitutive model parameters) given by the user. Generally, the number of parameters to be determined increases with the model complexity and, consequently, the difficulty of the parameter identification problem. The determination of parameters should always be performed confronting numerical and experimental results leading to the minimum difference between them (minimization of the cost function that is defined as the difference between experimental and numerical results). This problem can be reduced to a curve-fitting problem if physical constraints are not taken into account. However, most material constitutive models have physical constraints such as material parameter boundary values and mathematical relations between them, guaranteeing the physical meaning of the material parameters [1].

The aim of shape optimization problems is to find the shape which is optimal (minimizes a certain cost functional while satisfying given constraints) for a determined objective. However, in inverse shape optimization problems in metal forming, the final shape of the manufactured product is already known. The goal is to find the initial shape of the specimen (blank) or the tool in order to reach the desired final geometry after the forming process without any defect.

Frequently, the cost functional (objective function) cannot be evaluated without performing a time consuming simulation analysis. Additionally, multiple solutions can be obtained.

It should be notice that the inverse shape optimization problem can be similar to the parameter identification problem

if the shape to be optimized is defined by a finite number of parameters. Therefore, both problems can be solved by the same approach.

The minimization of the cost function, defined as the difference between experimental and numerical results (or the difference between the obtain geometry and the desired one), can be a hard task. The function of a set of arguments (the material or geometry parameters) may have many isolated local minima, non-isolated minimum hypersurfaces, or even more complex topologies. No finite minimization method can guarantee to locate the unique, global, minimum of the parameter set without supplying additional information about the cost function by the user. There is no best algorithm for finding the minimum of a cost function. Several approaches and optimization methods can be used to solve the mentioned non-linear optimization problems with small computational costs and to efficiently determine the best suited material parameter set. For many optimization problems there are techniques which will locate a local minimum which may be close enough to the global minimum and there are techniques which will find the global minimum in a large fraction of the time (in a probabilistic sense). The most common optimization methods are the gradient-based methods followed by the genetic, evolutionary and nature-inspired algorithms. The immune algorithms and the methods based in neural networks and artificial intelligence are also often used. Without doubt, the most effective methods are gradient-based but their performance is known to be dependent on the starting set of parameters. Nature-inspired techniques provide a different way to determine an optimized set of parameters. Therefore, the difficulty of choosing a starting set of parameters is minor for this process. However, these proved to be computationally more expensive than the gradient-based methods.

This paper describes synthetically some aspects of the numerical methods used to solve both mentioned problems. Further issues will be presented at the conference in order to discuss the potential improvements in this specifically scientific field.

## 2 THE IMPORTANCE OF THE OBJECTIVE FUNCTION IN INVERSE PROBLEMS

The determination of parameters should always be performed confronting numerical and experimental results leading to the minimum difference between them. However, the success of this task is dependent of the specification of the cost/objective function, defined as the difference between the experimental and the numerical results [3]. Recently, various objective functions have been formulated to assess the errors between the experimental and computed data. The objective functions should be able to efficiently lead the optimisation process. An ideal objective function should have the following properties: (i) all the experimental data points on the curve and all experimental curves should have equal opportunity to be optimised; and (ii) different units and/or the number of curves in each sub-objective should not affect the overall performance of the fitting. These two criteria should be achieved without manually choosing the weighting factors. However, for some non-analytical specific problems, this is very difficult in practice. Null values of experimental or numerical models also turn the task difficult. In this work, a set of novel objective functions for constitutive model parameter identification are proposed. The proposed objective function or optimisation criterion is formulated using a correspondent computed value based on the total length of the curve in order to enclose all available data.

Consider a curve defined analytically as  $y = f(x)$ . The length  $l$  of this curve can be calculated as

$$l = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}. \quad (1)$$

Numerically, the curve can be defined by a set of points. Therefore, the information of the analytical description  $f(x)$  of the curve is not available. In these cases, the equation of the length of a curve (equation 1) can be approximated by the following expression

$$\begin{aligned} l &= \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \int_a^b \sqrt{dx^2 + dy^2} \\ &\approx \sum_{i=1}^N \sqrt{dx^2 + dy^2} \\ &= \sum_{i=1}^N \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}. \end{aligned} \quad (2)$$

If  $x$  and  $y$  have different units and magnitude, the length (or dimension) of the larger variable will *monopolize* the length distance. Therefore, it is convenient to normalize these distances. The concept of calculation of the corre-

spondent computed-experimental point is called *weighted length*, in opposition to the *weighted distance* proposed in [2]. This concept takes into account negative values and curve length equidistant values. Therefore, the weighted lengths of each experimental and computed data points are, respectively, [3]

$$l_{ij}^e = \sum_{k=1}^i \sqrt{\left( \frac{\varepsilon_{k+1,j}^e - \varepsilon_{k,j}^e}{\varepsilon_{|\max|}^e} \right)^2 + \left( \frac{\sigma_{k+1,j}^e - \sigma_{k,j}^e}{\sigma_{|\max|}^e} \right)^2} \quad (3)$$

and

$$l_{ij}^c = \sum_{k=1}^i \sqrt{\left( \frac{\varepsilon_{k+1,j}^c - \varepsilon_{k,j}^c}{\varepsilon_{|\max|}^c} \right)^2 + \left( \frac{\sigma_{k+1,j}^c - \sigma_{k,j}^c}{\sigma_{|\max|}^c} \right)^2} \quad (4)$$

where  $\varepsilon_{|\max|}$  and  $\sigma_{|\max|}$  are the maximum absolute values. The correspondent computed value is given by the equivalent length,

$$l_{ij}^{eq} = l_{ij}^e \left( \frac{l_{N_j}^c}{l_{N_j}^e} \right), \quad (5)$$

that is responsible by the absolute curve scaling.

## 2.1 Overall Multi-objective function

The use of the natural logarithm function in the OF for each data point was introduced in [2] as a "true" error definition. This function slowly grows to positive infinity as x-axis increases and rapidly goes to negative infinity as x approaches 0. The use of this function can be also related to the use of logarithm scale when the OF results are analysed, increasing the sensitivity of the result when approaching the minimum. Other advantage is related to the smoothing of very large values. Very large variations values, such as  $1 \times 10^{30}$ , can create serious difficulties to some optimisation algorithms. The logarithm function can smooth this very large values ( $1 \times 10^{30}$  becomes  $\ln(1 \times 10^{30}) = 69.077$ ).

Considering the least-square structure and the true error definition, the objective function suggested for each  $i^{th}$  data point in the  $j^{th}$  curve is given for a stress-strain mechanical model [3]

$$r_{ij}^2 = \left\{ \sqrt{\omega_{ij}^\sigma} \ln \left[ 1 + \left| \frac{\sigma^c(l_{ij}^{eq}) - \sigma_{ij}^e}{\bar{\sigma}_{ij}^e} \right| \right] \right\}^2 + \left\{ \sqrt{\omega_{ij}^\varepsilon} \ln \left[ 1 + \left| \frac{\varepsilon^c(l_{ij}^{eq}) - \varepsilon_{ij}^e}{\bar{\varepsilon}_{ij}^e} \right| \right] \right\}^2. \quad (6)$$

The denominators in the previous equation ( $\bar{\sigma}^e$  and  $\bar{\varepsilon}^e$ ) can take the following values [3]:

1. (New OF1)  $\bar{\sigma}_{ij}^e = \sigma_{ij}^e$  and  $\bar{\varepsilon}_{ij}^e = \varepsilon_{ij}^e$ . In this case, a relative error is used.
2. (New OF2)  $\bar{\sigma}_{ij}^e = \frac{1}{N} \sum_{i=1}^N \sigma_{ij}^e$  and  $\bar{\varepsilon}_{ij}^e = \frac{1}{N} \sum_{i=1}^N \varepsilon_{ij}^e$  for  $i = 1, \dots, N$  and  $j = 1, \dots, M$ . In this case, the normalization is made using the average value of the experimental data set. The absolute maximum value can also be an alternative.

The use of the module or absolute function in the objective function for each point prevents a non-proportional error when the experimental value and the computed value present different signs. Although the logarithm function is well prepared for large values when  $(\bar{\varepsilon}^e, \bar{\sigma}^e) \rightarrow 0$ , zero divisions should be avoided. Therefore, for the first case (New OF1), the denominators of equation 6 should respect the following rule

$$\text{if } |\sigma^e| < \frac{\sigma_{|\max|}}{N^2} \quad \text{then} \quad \sigma_{\text{denominator}}^e = \min \left( 1, \frac{\sigma_{|\max|}}{N^2} \right). \quad (7)$$

For the case of normalization using the average value of the experimental data set, if this value is zero, the absolute maximum can be used instead.

The global objective function follows the following structure:

$$f = \frac{1}{M} \sum_{j=1}^M \left( \frac{1}{N_j} \sum_{i=1}^N r_{ij}^2 \right) \quad (8)$$

that allows giving the same opportunity to all curves to be optimised. In order to different units and different magnitudes not interfere with the equality criterion for each data point, weighting coefficients should be used.

It is of utmost importance that the weighting coefficients are chosen automatically without any manual or empirical influence. In this work, for the OF1, the weighting coefficients suggested by Cao and Lin [2] will be extended to the possibility of using negative values curves. Hence, the weighting coefficient for the strain is defined as

$$\omega_{ij}^{\varepsilon} = \theta \cdot \frac{|\varepsilon_{ij}^e|}{\sum_{j=1}^M \sum_{i=1}^{N_j} |\varepsilon_{ij}^e|}. \quad (9)$$

A similar expression is used for the stress component. The use of absolute values allows to maintain the fact that, for  $\theta = \sum_{j=1}^M N_j$ ,  $\omega_{11} + \omega_{21} + \dots + \omega_{N_j M} = \theta$ . The weighting coefficients balance the experimental data points in order to reduce the weight of the denominator. Large experimental points, as denominators, diminish and scale the absolute difference between the computed and the experimental data point.

### 3 INVERSE SHAPE OPTIMIZATION

Other category of inverse problems are the shape and/or process optimization problems. These approaches extend the concepts beyond the constitutive model, and shows a great potential in helping engineers to efficiently solve design problems in actual mechanical processes. In most forming processes, such as in the deep drawing of a car fender, the aimed final geometry is known a-priori, and the initial shape of the blank and/or the tools are needed to be determined. Moreover, the forming process must be designed to ensure that, after deformation, the blank will reach the prescribed shape of the desired safe final part, within thig geometrical tolerances.

Shape optimization problem formulation is not so different from the previous problems formulation. The major difference is that, in parameter identification, the variables can have different magnitudes due to their different physical or numerical meaning, while in shape optimization all variables define geometrical shapes and contours, being all at the same magnitude (now  $\mathbf{A}$  are called the design variables). In a general form, the objective function of these problems,  $\mathcal{L}(\mathbf{A})$ , can be written as in the previous equations. However, it can be much more simpler as

$$\mathcal{L}_q(\mathbf{A}) = \frac{1}{M_q} \sum_{i=1}^{M_q} [F_i(\mathbf{A}, \sigma, \varepsilon, \dots)]^2, \quad (10)$$

where  $F_i$  is the specific function that characterizes the geometric shape as a function of structural properties such as, for instance, the stress  $\sigma$  and strain  $\varepsilon$  fields.

The problem inverse shape optimization problems is the multiple solutions that can be obtained. This aspect will be adressed in the conference.

### 4 CONCLUSIONS

In this paper, specific engineering inverse problems, such as the parameter identification and the shape optimization problems, are presented. Some numerical aspects are presented in order to be presented at the YIC2013 conference. Other issues related to this thematic will be also adressed in the conference.

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