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► **To cite this version:**

Orestis Friderikos, Emmanuel Baranger. Dimensionality reduction of a complex constitutive law for Ceramic Matrix Composites. 2nd ECCOMAS Young Investigators Conference (YIC 2013), Sep 2013, Bordeaux, France. hal-00855895

HAL Id: hal-00855895

<https://hal.science/hal-00855895>

Submitted on 30 Aug 2013

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Dimensionality reduction of a complex constitutive law for Ceramic Matrix Composites

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Abstract. *Dimensionality reduction of a constitutive law describing the complex anisotropic material behavior of SiC/SiC Ceramic Matrix Composites (CMCs) is performed using a technique of multivariate analysis i.e., Principal Component Analysis (PCA). An approximation of the thermodynamic state potential of the original model with a simplified one that has a reduced set of internal damage variables is introduced. To this end, PCA is deployed to determine the optimal damage kinematics for a certain range of loading conditions via minimization of a prescribed error norm. The evolution laws linking the new damage variables with the associated conjugate thermodynamic forces are hereafter identified.*

Keywords: Anisotropic material behavior; CMCs; Reduced modeling; PCA.

1 INTRODUCTION

SiC/SiC Ceramic Matrix Composites exhibit complex mechanical behavior with degradation mechanisms due to formation of various networks of matrix cracks. An advanced mechanical macromodel for determining the different states of damage that can be observed under tension and compression, due to opening and closing of microcracks, is developed [1]. This damage model is taking also into account the microcracking mechanisms directed parallel or orthogonal to the fibers, as well as, normal to the loading direction. Therefore, fourth-order tensors are introduced as internal state variables to describe the evolution of the damage state, however, and as a by-product the complexity of the model and the computational cost are inevitably increased. The main objective of this work focuses on reducing the above described constitutive law under a certain range of loading conditions. The original thermodynamic potential is approximated with one constructed having a reduced number of state variables. A technique of multivariate analysis called Principal Component Analysis is used for the dimensionality reduction. PCA is in essence a vector space transform that is used to reduce multidimensional data sets with a large number of interrelated variables to lower dimensions, while retaining as much as possible of the variation present in the data set. The proposed approach uses the framework given by the thermodynamics of irreversible processes with internal state variables [2]. The simplified model is well suited for the application and implementation in standard finite element codes.

2 DIMENSIONALITY REDUCTION: PRINCIPAL COMPONENT ANALYSIS (PCA)

Considering the thermodynamic potential of the advanced damage model, the elastic strain energy density function is given by:

$$e_d(\boldsymbol{\sigma}, \mathbb{C}, \mathbb{Z}) = \frac{1}{2}Tr[\mathbb{C}\boldsymbol{\sigma}^+\boldsymbol{\sigma}^+] + \frac{1}{2}Tr[\mathbb{C}_0\boldsymbol{\sigma}^-\boldsymbol{\sigma}^-] + \frac{1}{2}Tr[\mathbb{Z}\boldsymbol{\sigma}\boldsymbol{\sigma}] \quad (1)$$

where \mathbb{C}_0 , \mathbb{C} and \mathbb{Z} are fourth order tensors representing the initial compliance, the damage compliance and a

compliance operator associated with shear damage, respectively. Additionally, a special spectral decomposition of the stress tensor into a positive and a negative part is introduced to ensure the continuity of the state law:

$$\begin{cases} \boldsymbol{\sigma}^+ = \mathbb{C}^{-1/2} : \langle \mathbb{C}^{1/2} : \boldsymbol{\sigma} \rangle_+ \\ \boldsymbol{\sigma}^- = \mathbb{C}_0^{-1/2} : \langle \mathbb{C}_0^{1/2} : \boldsymbol{\sigma} \rangle_- \end{cases} \quad (2)$$

where $\langle \rangle_{+-}$ denote the classical positive/negative part of a symmetric operator.

An approximation of the state potential (1) is introduced in the form:

$$\tilde{e}_d(\boldsymbol{\sigma}, d_i) = \frac{1}{2} Tr[(\mathbb{C}_0 + \sum_i d_i \mathbb{C}_i) \boldsymbol{\sigma} \boldsymbol{\sigma}] \quad (3)$$

where d_i is the new set of reduced variables. Note that the use of $\boldsymbol{\sigma}$ as a state variable still holds.

The Principal Component Analysis is used to find the optimal damage kinematic \mathbb{C}_i in a certain range of loading conditions, minimizing the error norm:

$$\min_{span(\mathbb{C}_i)} e^2 = \sum_{loads} Tr[\mathbb{C}_{err} \mathbb{C}_{err}] \quad (4)$$

where \mathbb{C}_{err} is given from the following expression:

$$\mathbb{C}_{err} = d_i \mathbb{C}_i - \left(\frac{\partial^2 e_d}{\partial \boldsymbol{\sigma}^2} - \mathbb{C}_0 \right) \quad (5)$$

PCA is performed on the matrix $\mathbb{X} = \sum_{loads} \left(\frac{\partial^2 e_d}{\partial \boldsymbol{\sigma}^2} - \mathbb{C}_0 \right)$. The principal components are the six orthonormal eigenvectors \mathbb{C}_i (in Voigt notation) which are ordered so that the first few retain most of the variation present in all of the original variables. Therefore, as the error decreases if more principal components are included, it becomes apparent that choosing the number of principal components is a trade-off between error norm and dimensionality reduction. Considering the simple proportional loading paths presented in Figure 1., the percentage of variance explained by the first two principal components is approximately 97.4 %, whereas the relative error in the potential is at the level of 4.5 %.

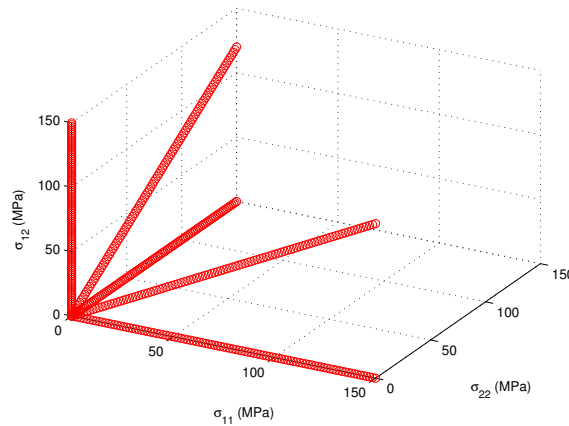


Figure 1: Proportional loading paths.

3 DAMAGE EVOLUTION LAWS OF THE REDUCED MODEL

The conjugate thermodynamic forces (energy release rates) associated with the variations of the new damage variables d_i are obtained from the strain energy function:

$$y_i = \frac{\partial \tilde{e}_d}{\partial d_i} = \frac{1}{2} \text{Tr}[\mathbb{C}_i \boldsymbol{\sigma} \boldsymbol{\sigma}] \quad (6)$$

Using the thermodynamic forces associated to \mathbb{C} and \mathbb{Z} from the original complex model, y_i is given by the relation:

$$y_i = \text{Tr}[\mathbb{C}_i^T \mathbb{Y}'] \quad (7)$$

where the following expression for \mathbb{Y}' is set:

$$\mathbb{Y}' = \frac{\partial e_d}{\partial \mathbb{Z}} = \frac{1}{2} \boldsymbol{\sigma} \otimes \boldsymbol{\sigma} \quad (8)$$

Figure 2. presents the evolution of the damage variable d_2 versus the associated thermodynamic forces y_1 and y_2 for the considered loading paths. On that basis, an approximated evolution law has been identified and plotted on the same figure.

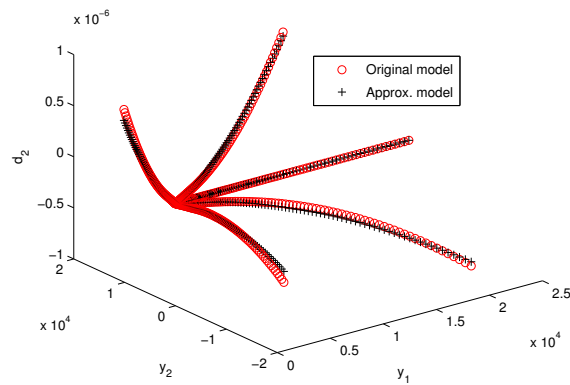


Figure 2: Evolution law for damage variable d_2 .

REFERENCES

- [1] Ladevèze, P.. An anisotropic damage theory with unilateral effects: applications to laminate and three- and four-dimensional composites. In: Allix O, Hild F, editors. Continuum damage mechanics of materials and structures. Elsevier, 205-33, 2002.
- [2] Germain P., Nguyen Q., Suquet P.. Continuum thermodynamics. *ASME Journal of Applied Mechanics* **50**:1010-1021, 1983.