Material parameters identification for optimized loading path

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Abstract. *Presently, the characterization of metal materials is becoming increasingly important due to the development of new materials, which is often performed numerically. Therefore, the accuracy of software simulation totally depends on the quality of the constitutive models and of their parameters. Hence, in this work, it is performed a loading path which allows to attain the most unique and distinguishable information for the parameter identification. This methodology is presented for two kinds of material.*

Keywords: loading path; uniqueness; distinguishability; material parameters.

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1 INTRODUCTION

The characterization of materials is having an increasing attention due to the conception of new materials and computational simulation software. It is crucial that the differences between the experimental and numerical models results, in terms of material parameters identification, have to be the smallest as possible.
In this sense, a large number of technological mechanical tests are generally used to characterize the mechanical properties of materials, being very material and time-consuming. Moreover, similar materials may also have similar properties. Hence, many researchers are often confronted with the dilemma of what should be the best of numerical solution for all different results. Therefore, only one mechanical test which could characterize all the mechanical properties would be desired. Accordingly, using optimization strategies, it could be find the most unique and distinguishable solution for parameter identification. Thus, the aim of this work is to propose a procedure that allows to find the most unique and distinguishable loading path in order to identify the best set of mechanical parameters. This loading path has to be the most informative in order to displays normal and shear strain as distinctly as possible. To achieve this, the presented methodology uses Finite Element Analysis (FEA) and Singular Value Decomposition (SVD) coupled together with optimization strategies.
The FEA is a powerful tool that allows to divide a complex structure in several smaller elements. The initial problem (that might not have an analytical) is then subdivided into several subproblems. After solving all the subproblems, the solutions are assembled and they yield to an analytical solution for the initial problem.
SVD is a factorization of rectangular matrix used as a useful technique in many area such as Statistical applied [1], Data Compression, etc., which allows to discriminate variables, reordered sets of values [2], computed pseudoinverse, etc.
The presented methodology is applied to materials with anisotropic behaviour.

2 METHODOLOGY

2.1 First stage, evaluation of the loading path

Consider a specimen with boundary conditions and material parameters known *a priori*.
Using a commercial software for Finite Element Analysis, incremental strains and displacements are calculated. Then, applying the Principle of Minimum Potential Energy for FEA [3], from the increment of the potential energy,

it results that:

$$\delta\Delta U = \delta\Delta W.$$

Accordingly to the Matrices Theory, any data matrix $\mathbf{X}_{n\times p}$ with rank k ($k \leq \min(n,p)$) and r ($r \leq k$) non-zero singular value, $s_1, \ldots, s_r$, can be factorized as

$$\mathbf{X} = \mathbf{USV}^{\mathrm{T}}, \tag{1}$$

where Eq. 1 represents the Singular Value Decomposition of $\mathbf{X}$, $\mathbf{U} \in \mathbb{R}^{n\times n}$ and $\mathbf{V} \in \mathbb{R}^{p\times p}$ are orthogonal. Therefore, considering $\hat{\mathbf{G}}_{k\times p}$ the matrix which denotes the changes of strain energy for k-th step, $\hat{\mathbf{G}}$ can be factorized as

$$\hat{\mathbf{G}} = \hat{\mathbf{U}}\hat{\mathbf{S}}\hat{\mathbf{V}}^{\mathrm{T}},$$

where $\hat{\mathbf{U}} \in \mathbb{R}^{k\times p}$ and $\hat{\mathbf{V}} \in \mathbb{R}^{p\times p}$ are orthogonal and $\hat{\mathbf{S}} \in \mathbb{R}^{k\times p}$ is a diagonal matrix with non-negative singular values $s_i, \forall i \in \{1,\ldots,p\}$ [1]. To evaluate the quality of the loading path, distinguishability and uniqueness functions are calculated. The distinguishability function is computed as:

$$F^d \equiv \prod_{i=1}^{n} \hat{s}_i,$$

and the uniqueness function is computed as

$$F^u \equiv \frac{\hat{s}_{\min}}{\hat{s}_{\max}} \leq 1,$$

where $\hat{s}_{\max}$ is the maximum singular value and $\hat{s}_{\min}$ is the minimum singular value. Then, by calculating the pseudo-inverse matrix of $\hat{\mathbf{G}}$, the material parameters can be updated, as it shown in Figure 1. After this, all material parameters are updating in the Abaqus input file and successively.

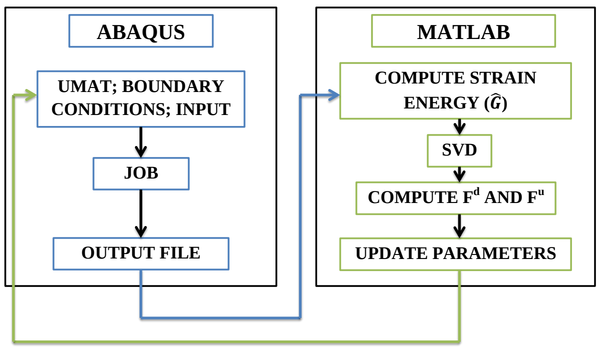

Figure 1: Procedure to obtain the most informative loading path for materials.

2.2 Second stage, optimization of the loading path

To find the most informative loading path, the distinguishability and uniqueness functions are both maximized. Hence, the objective function is biobjective and Multiobjective Programming (MOP) must be used. Therefore, the best solution is a compromise solution that has to satisfy the Pareto-optimal front [5]. Thus, in the second stage, after the distinguishability and uniqueness been computed, the direction of the loading path is optimized by:

$$\Lambda_k = \max\left((1-\mu)\,\frac{F_k^d - F_{k-1}^d}{\bar{F}^d} + \mu\frac{F_k^u - F_{k-1}^u}{\bar{F}^u}\right) \tag{2}$$

where $\mu \in [0;1]$ is a weighted factor and the boundary conditions are $0 \leqslant \mu \leqslant 1$. Notice that $\Lambda_{1:K}$ is the variable that controls the loading path from step 1 to step K [4] by the Weighted Sum Method. The formulation given in Equation (2) allows that $J^d = \dfrac{F_k^d - F_{k-1}^d}{\bar{F}^d}$ and $J^u = \dfrac{F_k^u - F_{k-1}^u}{\bar{F}^u}$ have smaller different scale magnitude, and also allows that the Pareto optimal front be convex or nearly-convex [5].
After this, the new direction is also updated in the FEA file.

3 CONCLUSIONS

This methodology was improved for an anisotropic elastic material. At the present the same strategy for plastic materials is being developed.

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