

Adaptive Multigrid Methods for Fluid-Structure Interaction (FSI) Optimization in an Aircraft and design of integrated Structural Health Monitoring (SHM) Systems

Bhuiyan Shameem Mahmood Ebna Hai ^{a,*} and Markus Bause ^a

^a Department of Mechanical Engineering, Helmut Schmidt University,
University of the Federal Armed Forces Hamburg,
Hostenhofweg 85, 22043 Hamburg, Germany.

*shameem.ebna.hai@hsu-hh.de

Abstract. *Nowadays, advanced composite materials such as carbon fiber reinforced plastics (CFRP) are being applied to many aircraft structures in order to improve performance and reduce weight. Most composites have strong, stiff fibres in a matrix which is weaker and less stiff. However, aircraft wings can break due to Fluid-Structure Interaction (FSI) oscillations or material fatigue. Material inspection by piezoelectric induced ultrasonic waves is a relatively new and an intelligent technique to monitor the health of CFRP for damage detection in Non-Destructive Test (NDT). To design a Structural Health Monitoring (SHM) systems, it is important to understand phenomenologically and quantitatively wave propagation in CFRP and the influence of the geomaterial and mechanical properties of the structures. The principal aim of this research is to explore and understand the behaviour of engineering artefacts in a maritime environment, with a view to better integrating their design and operation from safety and economic viewpoints. To accelerate the design of SHM systems, the FSI effect on the wave propagation has to be considered. Due to the nonlinear properties of fluids and solids as well as the shape of the structures, only numerical approaches can be used to solve FSI and wave propagation problems. Part of this research will focus on the analysis of Navier-Stokes and equations of elastodynamics in the arbitrary Lagrangian-Eulerian (ALE) framework, simultaneously we will study the mathematical modeling and numerical approximation of the propagation of time-harmonic elastic waves in a CFRP composite material. Also this project aims to develop efficient numerical methods for fluid-structure interaction and wave propagation phenomena, which combine modern techniques from PDE-constrained optimization, adaptive and multigrid simulation methods.*

Keywords: CFRP, Navier-Stokes, Elastodynamic, Lift, Drag, FSI, ALE, Vibration, SHM, Elastic wave equation, Wave propagation, Asymptotic homogenisation, DIS, FEM, DOpElib, deal.II.

1 INTRODUCTION

This paper is intended to provide the motivation that will act as driving force for the execution of this research work. With the scope and objectives together, we will give the outline of this research to help the reader understand the context of the present proposal.

1.1 Motivation

Currently, composite materials have shown remarkable resilience for lightweight structures, construction in ballistics protection, engineering, and other similar applications. Composite materials are formed by combining two or more materials in such a way that the constituents are still distinguishable and not fully blended. The objective is usually to make a component which is strong and stiff, often with a low density. But there is a possibility of structural damage due to Fluid Structure Interaction (FSI). FSI is the interaction of some movable or deformable structure with an internal or surrounding fluid flow, which describes the coupled dynamics of fluid mechanics and

structure mechanics. These types of problems are known as Classical Multi-Physics problems. The problems can be stable or oscillatory. Simulation of the FSI, where the dynamics of these currents dominate, poses a formidable challenge for even the most advanced numerical techniques and it is currently at the forefront of an ongoing work in Computational Fluid Dynamics (CFD). In this research work, we will focus on the analysis of the incompressible Navier-Stokes and elastodynamic equations in the arbitrary Lagrangian-Eulerian (ALE) framework and present a numerical simulation of the FSI effect on an aircraft wing, in which these equations shall describe the underlying physics. The process of implementing a Damage Identification Strategy (DIS) for aerospace, civil and mechanical engineering infrastructures is referred to as Structural Health Monitoring (SHM). Here, damage is defined as changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the system's performance. A wide variety of highly effective local Non-Destructive Test (NDT) evaluation tools are available for such monitoring. Piezoelectric or ultrasonic inspection (pulse-echo) is one of the most promising NDT tool for locating structural damage. In this method, high frequency sound waves are introduced into a material and they are reflected back from surfaces or flaws. Reflected sound energy is displayed versus time, and inspectors can visualize a cross section of the specimen showing the depth of features that reflect sound. High resolution images can be produced by plotting signal strength or time-of-flight using a computer controlled scanning system to identify the location of the damage. For this purpose, it is important to understand Wave Propagation Phenomena in composite material. One of the main goal of this research is to provide a framework for reliable and accurate numerical simulation of Elastic Wave Propagation in composite material. In this case we need an appropriate treatment for the microstructure of the composite material which typically cannot be resolved by finite element meshes. We use homogenization techniques for upscaling the microscopic material properties and for obtaining macroscopic material parameters by solving cell problems on the microscale.

1.2 Specific aims and Anticipated Outcome

The principal aim of this research is to explore and understand the behaviour of engineering artefacts in extreme environments. To achieve the main ambition of this work, we split this research into two parts. The first part will consider the following: (a) Determine the effect of fluid flow over a sample aircraft, (b) Study the displacement of a control point under incompressible fluid flow, (c) Identify the list of critical design points on a sample aircraft. The second part will focus on: (e) Numerical approximation of wave propagation in composite material, (d) Design of an integrated SHM system for an aircraft. The anticipated outcome of this research will be: (a) An overview of FSI effect on the sample aircraft, (b) Modern techniques for FSI and wave propagation problem optimization, (c) Optimum locations for sensors of SHM systems, and (d) An outlook on the expected future of SHM systems.

2 MATHEMATICAL MODELS

2.1 The Fluid-Structure Interaction (FSI) problem

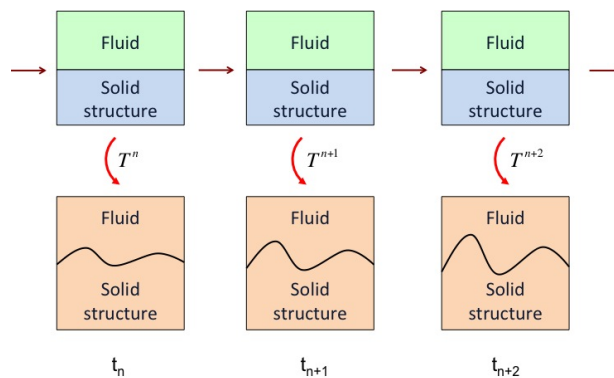


Figure 1: Typical FSI Problem in the ALE framework [7, 8, 9, 10]

Let us assume that $\widehat{\Omega} \subset \mathbb{R}^d$, $d = 2, 3$, be a bounded domain of the fluid-structure interaction problem in reference configuration at time $t = 0$ with the Lipschitzian boundary [1, 5]. The outer unit normal vector at the boundary is denoted by n . Assume that $\Omega := \Omega(t)$ is split into two time-dependent subdomains $\Omega_f(t)$ (for an incompressible fluid flow) and $\Omega_s(t)$ (for an elastic structure), which is slicked to at the boundary of the domain $\widehat{u}_s = 0$. Here, the boundaries of Ω , Ω_f and Ω_s are denoted by $\partial\Omega$, $\partial\Omega_f$ and $\partial\Omega_s$, respectively. The variational ALE formulation of the

fluid part is transformed from its Eulerian description into an arbitrary Lagrangian framework [7, 8, 9] and stated on the (arbitrary) reference domain $\widehat{\Omega}_f$, while the structure part is formulated in Lagrangian coordinates [7, 8, 9] on the domain $\widehat{\Omega}_s$, where $\widehat{\Omega} = \widehat{\Omega}_f \cup \widehat{\Gamma}_i \cup \widehat{\Omega}_s$. Moreover, here we solve the Laplace equation for the definition of the ALE mapping. Here, the continuity of velocity $\widehat{v}_f = \widehat{v}_s$ and $\widehat{u}_f = \widehat{u}_s$ across the common fluid-structure interface on $\widehat{\Gamma}_i = \widehat{\Omega}_f \cup \widehat{\Omega}_s$. We search for $\widehat{u} \in H^1(\widehat{\Omega}_f, \widehat{\Gamma}_D)^d$ and $\widehat{v} \in H^1(\widehat{\Omega}_f, \widehat{\Gamma}_D)^d$, where the local quantities are defined by restrictions: $\widehat{v}_f := \widehat{v}|_{\widehat{\Omega}_f}$, $\widehat{v}_s := \widehat{v}|_{\widehat{\Omega}_s}$ and $\widehat{u}_s := \widehat{u}|_{\widehat{\Omega}_s}$. It is noted that this extension of the pressure is an inconsistency. While the fluid's pressure is of low regularity $\widehat{p}_f \in L^2_0(\widehat{\Omega}_f)$, the Laplace equation yields $\widehat{p}_s \in H^1(\widehat{\Omega}_s)$. This additional regularity will be fed back into the fluid domain if the extension is not properly decoupled. Since the ALE mapping is defined in accordance to the Lagrange-Euler structure mapping via $\widehat{T} := \widehat{x} + \widehat{u}_f$, we can define the following on all Ω : $\widehat{T} := \widehat{x} + \widehat{u}$, $\widehat{F} := I + \widehat{\nabla}\widehat{u}$, and $\widehat{J} := \det(\widehat{F})$. In the structure domain, \widehat{T} takes the place of the Lagrangian-Eulerian coordinate transformation, while in the fluid domain, \widehat{T} has no physical meaning but serves as ALE mapping.

Let us consider for a given set X , the Lebesgue space $L_X := L^2(X)$ and $L^0_X := L^2(X)/\mathbb{R}$. The functions in L_X with first-order distributional derivatives in L_X make up the Sobolev space $H^1(X)$. Furthermore, we can use the function spaces $V_X := H^1(X)^d$, $V^0_X := H^0_1(X)^d$, and for time-dependent functions:

$$\begin{aligned} \mathcal{L}_X &:= L^2[0, T; L_X], & \mathcal{V}_X &:= L^2[0, T; V_X] \cap H^1[0, T; V^*_X], \\ \mathcal{L}^0_X &:= L^2[0, T; L^0_X], & \mathcal{V}^0_X &:= L^2[0, T; V^0_X] \cap H^1[0, T; V^*_X], \end{aligned}$$

The fluid-structure interaction problem in ALE framework

Find $\widehat{v} \in \widehat{v}^D + \widehat{V}^0_{\widehat{\Omega}}$, $\widehat{u} \in \widehat{u}^D + \widehat{V}^0_{\widehat{\Omega}}$ and $\widehat{p} \in \widehat{\mathcal{L}}_{\widehat{\Omega}}$, such that $\widehat{u}(0) = \widehat{u}^0$ and $\widehat{v}(0) = \widehat{v}^0$, for almost all time steps $t \in I$ holds:

$$\begin{aligned} & \left(\widehat{J}_f \widehat{\rho}_f \partial_t \widehat{v}_f, \widehat{\phi}^v \right)_{\widehat{\Omega}_f} + \left(\widehat{J}_f \widehat{\rho}_f (\widehat{F}_f^{-1}(\widehat{v}_f - \partial_t \widehat{T}_f) \cdot \widehat{\nabla}) \widehat{v}_f, \widehat{\phi}^v \right)_{\widehat{\Omega}_f} \\ & + \left(\widehat{J} \widehat{\sigma}_f \widehat{F}^{-T}, \widehat{\nabla} \widehat{\phi}^v \right)_{\widehat{\Omega}_f} - \langle \widehat{h}, \widehat{\phi}^v \rangle_{\widehat{\Gamma}_N} - \left(\widehat{J} \widehat{\rho}_f \widehat{f}_f, \widehat{\phi}^v \right)_{\widehat{\Omega}_f} \\ & + \left(\widehat{\rho}_s \partial_t \widehat{v}, \widehat{\phi}^v \right)_{\widehat{\Omega}_s} - \left(\widehat{\rho}_s \widehat{f}_s, \widehat{\phi}^v \right)_{\widehat{\Omega}_s} - \left(\widehat{J} \widehat{\sigma}_s \widehat{F}^{-T}, \widehat{\nabla} \widehat{\phi}^v \right)_{\widehat{\Omega}_s} = 0 \quad \forall \widehat{\phi}^v \in \widehat{V}^0_{\widehat{\Omega}} \quad (1) \\ & \left(\widehat{div}(\widehat{J} \widehat{F}^{-1} \widehat{v}), \widehat{\phi}^p \right)_{\widehat{\Omega}_f} + \left(p_s, \widehat{\phi}^p \right)_{\widehat{\Omega}_s} = 0 \quad \forall \widehat{\phi}^p \in \widehat{\mathcal{L}}_{\widehat{\Omega}} \\ & \left(\partial_t \widehat{u} - v, \widehat{\phi}^u \right)_{\widehat{\Omega}_s} + \left(\widehat{\sigma}_g, \widehat{\nabla} \widehat{\phi}^u \right)_{\widehat{\Omega}_f} - \langle \widehat{\sigma}_g, \widehat{n}^f, \widehat{\phi}^u \rangle_{\widehat{\Gamma}_i} = 0 \quad \forall \widehat{\phi}^u \in \widehat{V}^0_{\widehat{\Omega}} \end{aligned}$$

The stress tensors for the fluid and structure are implemented in $\widehat{\sigma}_f$, $\widehat{\sigma}_s$, and $\widehat{\sigma}_g$, where the stress tensors are given by $\widehat{\sigma}_f(\widehat{x}) = -p_f I + \widehat{\rho}_f \nu_f \left(\widehat{\nabla} \widehat{u}_f \widehat{F}_f^{-1} + \widehat{F}_f^{-T} \widehat{\nabla} \widehat{u}_f^T \right)$, and $\widehat{\sigma}_s = \widehat{J}^{-1} \widehat{F} \left(2\mu_s \widehat{E} + \lambda_s \text{tr}(\widehat{E}) I \right) \widehat{F}^T$. In this formulation, for momentum equations, integration by parts in both subdomains yields the boundary term on $\widehat{\Gamma}_i$ as: $\left(\widehat{n}_f \cdot (\widehat{J} \widehat{\sigma}_s \widehat{F}^{-T}), \widehat{\phi}^v \right)_{\widehat{\Gamma}_i} + \left(\widehat{n}_s \cdot (\widehat{J} \widehat{\sigma}_f \widehat{F}^{-T}), \widehat{\phi}^v \right)_{\widehat{\Gamma}_i} = 0$. We refer to [5, 7, 8, 9, 10] for more details about the functional spaces and FSI formulation in ALE formulation.

2.2 Wave Propagation problem

Let us consider a periodic bounded and convex domain Ω in \mathbb{R}^n ($n \in \{2, 3\}$), which is for simplicity assumed to be a polygon for $n = 2$ or a polyhedron for $n = 3$, for the following wave equation (second-order hyperbolic PDE system):

Find $\mathbf{u} : \mathbb{R}^3 \times \mathbb{R} \supset \Omega \times I \rightarrow \mathbb{R}^3$, such that $\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0^0(\mathbf{x})$ and $\partial_t \mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0^1(\mathbf{x}) \forall \mathbf{x} \in \Omega$, for almost all time steps $t \in I$ holds:

$$\begin{aligned} \rho \partial_t^2 \mathbf{u}(\mathbf{x}, t) - \partial_x \boldsymbol{\sigma} &= \mathbf{f}(\mathbf{x}, t) & \text{for } (\mathbf{x}, t) \in \Omega \times I, \\ \mathbf{u}(\mathbf{x}, t) &= \mathbf{g}_i(\mathbf{x}, t) & \text{for } (\mathbf{x}, t) \in \partial\Omega_D \times I, \\ \boldsymbol{\sigma}(\mathbf{x}, t) \cdot \mathbf{n} &= \mathbf{h}_i(\mathbf{x}, t) & \text{for } (\mathbf{x}, t) \in \partial\Omega_N \times I, \end{aligned} \quad (2)$$

with a positive density function ρ . Here, $I = (0, t]$, $t > 0$ is a finite time interval. If d be the number of components of the solution function \mathbf{u} , for the initial values, $\mathbf{u}_0^0 \in H^1_0(\partial\Omega_D; \Omega)^d$, $\mathbf{u}_0^1 \in L^2(\Omega)^d$, and the source function

$\mathbf{f} \in L^2(I; H^{-1}(\partial\Omega_D; \Omega)^d)$, where $H^{-1}(\partial\Omega_D; \Omega)^d$ is dual space of $H_0^1(\partial\Omega_D; \Omega)^d$. The source function $\mathbf{f}(\mathbf{x}, t)$ and Dirichlet boundary function $\mathbf{g}_i(\mathbf{x}, t)$ will be assumed from the FSI effect on an aircraft during flight. Modeling the source function $\mathbf{f}(\mathbf{x}, t)$ and $\mathbf{g}_i(\mathbf{x}, t)$ will be tasks of this research work. Furthermore, $\partial\Omega_D$ and $\partial\Omega_N$ are disjoint, time-independent parts of the boundary of Ω , where homogeneous Dirichlet and Neumann boundary conditions prescribe, respectively. Here, $\partial\Omega_D$ has positive measure. In order to account for the influence of small structures in the underlining differential equations, two scales, \mathbf{x} at the macro-level and \mathbf{y} at the micro-level, will be introduced. The two scales are connected through a scalar quantity (a small adimensionalized parameter) $\xi > 0$, if Ω is periodic by ξ . where, $\rho = \rho^\xi$, $\sigma = \sigma^\xi$, $\mathbb{C}_{ijkl} = \mathbb{C}_{ijkl}^\xi$, $\mathbf{u} = \mathbf{u}^\xi$.

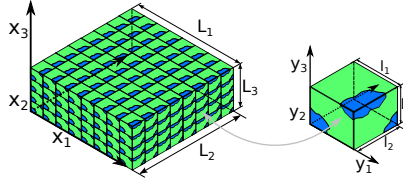


Figure 2: Asymptotic homogenisation, $\vec{y} := \xi \vec{x}$, $0 < \xi \ll 1$. [6]

Micro-level equations of first degree:

$$\frac{\partial}{\partial y_j} \left[\mathbb{C}_{ijkl} \left(I_{kmln} - \frac{\partial \chi_k^{mn}}{\partial y_l} \right) \right] = 0, \quad (3)$$

$$\chi_i^{kl}(y_j = 0) = \chi_i^{kl}(y_j = l_j).$$

This suggests that χ_i^{kl} is periodic and the requirement of periodicity for $\sigma^{(1)}$ is $\mathbb{C}_{ijmn}^* := \mathbb{C}_{ijkl} \left(I_{kmln} - \frac{\partial \chi_k^{mn}}{\partial y_l} \right)$ with boundary condition $\mathbb{C}_{ijmn}^*(y_p = 0) = \mathbb{C}_{ijmn}^*(y_p = l_p)$. For more details, we refer to [4, 6].

3 METHODOLOGY, FLEXIBILITY & ACCURACY

To achieve optimal result on this research work within a limited time frame, we will analyse the incompressible Navier-Stokes and equations of elastodynamics in the ALE framework and present numerical simulation of the FSI effect on an aircraft wing. For the implementation we use the finite element library `deal.II` [2] and the software package `DOPeLib` [3], which is a flexible toolbox providing modularized high-level algorithms that can be used to solve stationary and non-stationary PDE problems as well as optimal control problems constrained by PDEs and Dual-Weighted-Residual approach for goal-oriented error estimation.

4 REMARKS

In this work, we will mainly deal with a sample aircraft wing (3D), where the left end of this airfoil is considered as rigid. Our work on the FSI effect on an aircraft wing (3D) and the elastic wave equation in anisotropic and composite media is still in progress.

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