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# Uncertainty Quantification in Flight Dynamics Modelling

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**Abstract.** *The modelling of dynamic effects for flight dynamics analysis typically makes use of dynamic derivatives. A single value for the derivative is used and is assumed to be independent of the conditions for which it is calculated. This is however not the case. This work looks to quantify the uncertainty introduced as a result for an arbitrary manoeuvre within the flight envelope. The manoeuvres can then be simulated with a result that will have a quantifiable level of confidence.*

**Keywords:** flight dynamics; dynamic derivatives; uncertainty

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## 1 INTRODUCTION

Flight dynamics modelling requires taking account of unsteady effects introduced by a manoeuvring aircraft. This is typically done by modifying the database of static data with a dynamic derivative term. This stability derivative model was originally proposed by Bryan in [2]. Despite many alternatives available to account for unsteady effects, this model is widely used for industrial applications due to its simplicity.

Dynamic derivatives describe how the aircraft loads and moments are affected by the rates of the motion. The values are obtained from a forced oscillatory motion with the force and moment history being post-processed using a method as presented in [3]. The forced oscillation tests can be done in wind tunnels, using semi-empirical methods or using high-fidelity computational fluid dynamics (CFD) which has the benefit of being able to extract more detail from the model. The value of the dynamic derivative is affected by the flight parameters chosen such as oscillatory frequency, Mach number, oscillatory amplitude and mean incidence. Previous work has looked at the effect of these for example in [1] and [4].

When a manoeuvre simulation is carried out, a single value for the dynamic derivative is usually taken which is assumed to be independent of the flight parameters. This is however not the case. It therefore becomes necessary to look at some way of quantifying the error introduced by this assumption. Uncertainty quantification is widely used in Engineering, particularly for civil applications. Although the dynamic derivative model is widely used, it has never been the subject of a mathematical uncertainty analysis. This work looks to apply methods which will combine the typical parameters encountered in manoeuvres for a given aircraft configuration with the variation in the dynamic derivative value as a result of a change in the input parameters. This gives a defined confidence level in the solution for an arbitrary manoeuvre.

## 2 FORMULATION

### 2.1 Dynamic Derivatives

For the application of the dynamic derivative model, the load and moment coefficients are considered as consisting of a steady and an unsteady component expanded as shown in equation (1) for a pitching motion.

$$C_j(t) = C_{j_0}(t) + \bar{C}_{j_q} q(t) \frac{c}{U_\infty}, \quad (1)$$

The  $j$  subscript represents the force or moment of interest (i.e. L,D,M), the zero subscript term is the steady value at time  $t$  and the  $q(t)$  term is the rate of change of incidence at time  $t$ . The dynamic stability derivative term  $\dot{C}_{j_q}$  in this example represents how the force or moment coefficient changes with the pitch rate. The first Fourier coefficients of the time history of the response correspond to the values of the stability derivatives.

In practice, a single value is used for the dynamic derivative which, as shown in literature, varies with the flight parameters. This introduces a level of uncertainty in the results obtained from this model when replaying any manoeuvre.

## 2.2 Uncertainty Framework

The framework proposed here to quantify the uncertainty arising in the aforementioned flight dynamics modelling approach has a number of distinct steps. The first of which is to obtain a statistical distribution of the flight parameters experienced during manoeuvres such as that shown in figure 1(a). This curve fits a normal distribution and can be modified to have reduced frequency along the  $x$  axis rather than load factor to produce that shown in figure 1(b).

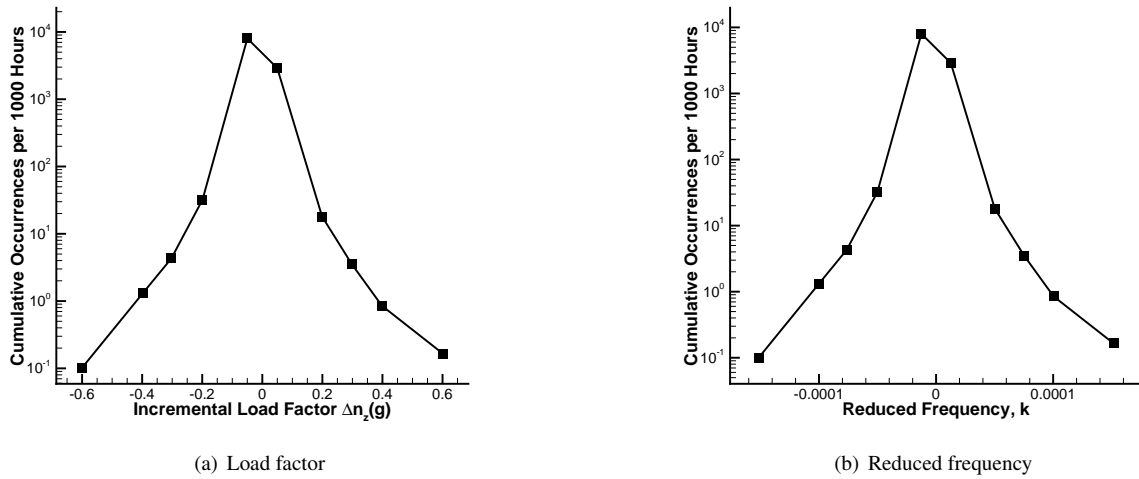


Figure 1: Manoeuvre data for A320 during cruise between 29,500 and 39,500 feet

This data has been taken from the technical report [7] during 10,066 total flights and 20,525.28 hours spent in the cruise phase for the Airbus A320 aircraft. Once this distribution is obtained, a number of calculations can be run for the dynamic derivative terms through the range of input parameters so that the variation can be obtained within a 99.99% confidence interval (i.e. within 4 standard deviations from the mean) or lower if allowed by the design specifications, to a value representative of a manoeuvre expected in service.

The next step is that the designer chooses a value for each of the dynamic derivative terms for the simulation and defines the manoeuvre to be run. This manoeuvre will potentially have varying rates and as such, the dynamic derivative value should change. Making use of the previously calculated variation of this term with the input parameters, a differencing can take place between the valued used and that which represents the actual aircraft state. This difference can be stored to later be used to define the uncertainty in the loads and moments at each point within a manoeuvre. These steps can be compiled into an algorithm as:

1. Obtain statistical distribution of flight parameters during manoeuvres
2. Range of derivative values calculated within a given confidence interval
3. Derivative values are chosen and manoeuvre is defined
4. Manoeuvre data is used to compute derivative values at each step
5. Differencing of derivative values between that used and that from previous step
6. Differences used to compute uncertainty in final loads and moments

This framework can simply be extended to more than a single input as will be used here.

### 3 CALCULATIONS

The test case chosen here is that of a NACA 0012 aerofoil to simplify the problem and allow a fundamental demonstration of the method. The CFD solver used is the University of Liverpool PML code [6] which is an implicit meshless compressible RANS code. The point cloud used is shown in figure 2.

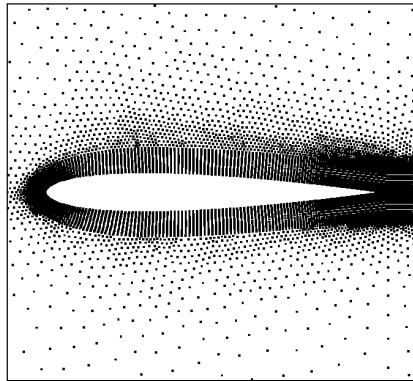


Figure 2: NACA 0012 point distribution

As previously mentioned, the dynamic derivative value varies with the flight parameters chosen. This is shown in figures 3(a) and 3(b) which have been obtained for the aerofoil test case. In order to speed up the calculation of the derivative values, the Harmonic Balance approach [5] has been used. This allows direct computation of the periodic state resulting from the forced oscillation in calculating these terms and can offer a speed up of up to 10 against a time-accurate solver.

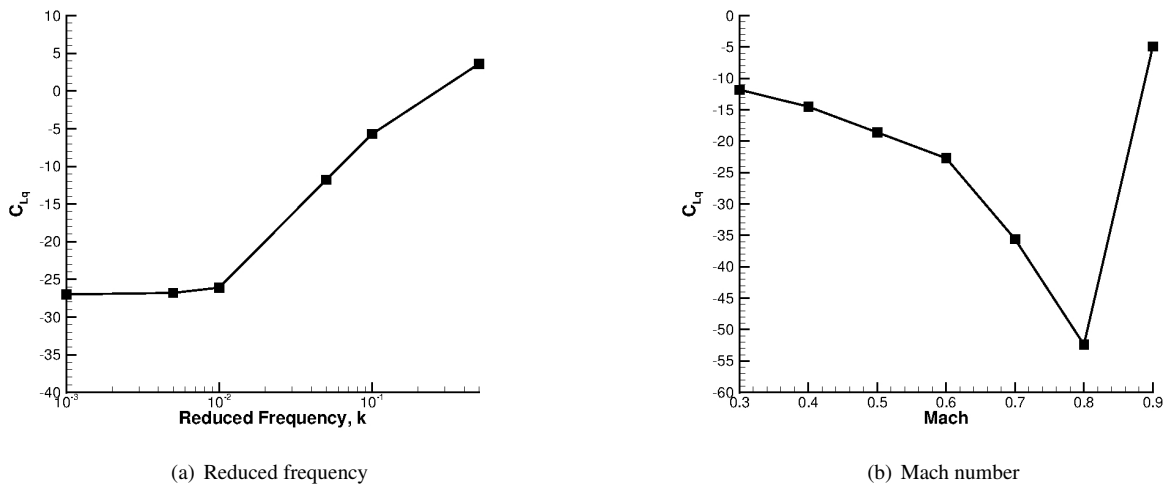


Figure 3: Lift damping derivative variation with flight parameters

The variation with reduced frequency is particularly important for manoeuvre simulation as this is likely to vary considerably from one manoeuvre to another. The derivative value does stay largely constant through frequencies where the aircraft performs most of its manoeuvres but on occasion, extreme values will be reached and the uncertainty must be considered during the design phase. The Mach number variation, although more extreme, is not as important due to the manoeuvre speed remaining fairly steady throughout. It does however become a problem if the designer uses a dynamic derivative value calculated for Mach 0.3 which is used to assess a manoeuvre at Mach 0.8

as the damping is underestimated.

### 3.1 Manoeuvres

A number of manoeuvres will be used to demonstrate this framework including a ramp manoeuvre at low and high rates and an obstacle avoidance at high mach number where nonlinearities will dominate and result in uncertainties in the solution. Plots will be made showing the load and moment histories through the manoeuvres with the upper and lower bounds applied from the framework presented here.

For this framework, only the variation with reduced frequency will be taken into account due to the availability of corresponding flight data and due to this varying throughout manoeuvres. Further parameters may be considered depending on time available.

## 4 CONCLUSIONS

A method to quantify the uncertainty for flight dynamics modelling using the dynamic stability derivatives approach has been described. The final presentation will show results of the method in practice for the aerofoil test case performing a number of different manoeuvres. A complete description of the framework will also be provided covering how it can be implemented into industrial practices.

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