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Elasto-viscoplastic modeling of soft biological tissues: An application to facial tissue

E. Mtanes ^{a,*}, J. Weickenmeier ^b, M. Jabareen ^a

^a Faculty of Civil and Environmental Engineering, Technion - Israel Institute of Technology
Haifa, Israel

^b Department of Mechanical and Process Engineering, Swiss Federal Institute of Technology
Zurich, Switzerland

*elitans@tx.technion.ac.il

Abstract. *Soft biological tissues are characterized by a highly nonlinear, time dependent mechanical response. Constitutive model equations capturing this material behavior often make use of evolution equations given in rate form (see [4]). In this contribution a new perspective of the mixed finite element formulation was developed together with a strongly objective integration scheme of the evolution equations. The capability of the developed finite element formulation and the strongly objective integrator was demonstrated by an indentation example.*

Keywords: Finite deformation; Soft tissue; Viscoplasticity; Mixed finite element formulation

1 INTRODUCTION

Understanding the mechanical behavior of facial soft tissue has become an important aspect in problems concerning computational simulation of surgical intervention, the evaluation of optimal surgical and suturing procedures, and the design of prosthetics. Many different constitutive model formulations have been introduced in recent years that describe the highly nonlinear, time dependent force deformation characteristics of soft biological tissues. The first fully three-dimensional finite strain viscoelastic model was introduced by Simo (1987) [6]. Further developments in the direction of fibre-reinforced, i.e. anisotropic, composite models specifically thought to predict the material response of soft tissues, followed thereafter, see [3] for example.

Alongside the advancements in modeling, the experimental characterization of time dependent material behavior of facial tissues allowed to validate models due to data available for pre-conditioning effects, stress relaxation at constant strain, hysteresis loops in cyclic loading and unloading. Rubin and Bodner [4] introduced a nonlinear model for dissipative soft tissue response which considers changing material response through history dependent state variables that are governed by evolution equations. The elastic part of this model was adopted by Barbarino et al. [1, 2] for simulations concerning an anatomically based finite element model of the face. Specifically, the model was used to understand the mechanical response of facial soft tissues under different loading conditions and the predictions of the model were compared to ex-vivo and in-vivo experimental data.

2 CONSTITUTIVE EQUATIONS AND STRESS UPDATE ALGORITHM

Rubin and Bodner [4] developed three dimensional constitutive equations that are valid for finite elasto-viscoplastic deformations and which produce reasonable agreement with experimental data of facial tissue. Specifically, Rubin and Bodner [4] considered the tissue as a composition of an elastic component and a dissipative component. In particular, the strain energy function W is given by

$$W = \frac{\mu_0}{2q} (e^{aq} - 1), \quad (1)$$

where μ_0 and q are material parameters, and the function $g = g(J, \beta_1, \lambda_I, \alpha_1)$ was decoupled into four parts such that

$$\begin{aligned} g(J, \beta_1, \lambda_I, \alpha_1) &= g_1(J) + g_2(\beta_1) + g_3(\lambda_I) + g_4(\alpha_1), \\ g_1(J) &= 2m_1(J - 1 - \ln(J)), \quad g_2(\beta_1) = m_2(\beta_1 - 3), \\ g_3(\lambda_I) &= \frac{m_3}{m_4} \sum_{I=1}^{N_{fib.}} (\lambda_I - 1)^{2m_4}, \quad g_4(\alpha_1) = \alpha_1 - 3, \\ J &= \det(\mathbf{F}), \quad \lambda_I = \sqrt{\mathbf{F}\mathbf{M}_I \cdot \mathbf{F}\mathbf{M}_I}, \quad \beta_1 = \mathbf{b}'_e : \mathbf{I} \quad \alpha_1 = \mathbf{b}'_{de} : \mathbf{I}, \end{aligned} \quad (2)$$

where the function $g_1(J)$ accounts for total volume dilatation, $g_2(\beta_1)$ accounts for the distortional deformation of the isotropic matrix, $g_3(\lambda_I)$ accounts for the stretch of the I 'th fiber family and $g_4(\alpha_1)$ accounts for the elastic distortional deformation of the dissipative component of the tissue. The tensor \mathbf{F} is the deformation gradient and the unit vectors \mathbf{M}_I characterize the orientations of fibers. Furthermore, the evolution equations of the tensorial measure of the elastic distortion of the elastic and dissipative components, respectively, $\{\mathbf{b}'_e, \mathbf{b}'_{de}\}$ are

$$\begin{aligned} \dot{\mathbf{b}}'_e &= \mathbf{l}\mathbf{b}'_e + \mathbf{b}'_e\mathbf{l}^\top - \frac{2}{3}(\mathbf{d} : \mathbf{I})\mathbf{b}'_e, \\ \dot{\mathbf{b}}'_{de} &= \mathbf{l}\mathbf{b}'_{de} + \mathbf{b}'_{de}\mathbf{l}^\top - \frac{2}{3}(\mathbf{d} : \mathbf{I})\mathbf{b}'_{de} - \Gamma\mathbf{a}_d. \end{aligned} \quad (3)$$

Note that the inelastic deformation is determined by the scalar Γ and the tensor \mathbf{a}_d . The tensor \mathbf{a}_d causes the tensorial measure of the elastic distortion of the dissipative component, \mathbf{b}'_{de} , to evolve toward unity. Moreover, the contribution $\Gamma\mathbf{a}_d$ is subtracted from the elastic distortion rate $\dot{\mathbf{b}}'_{de}$ and therefore the term $\Gamma\mathbf{a}_d$ weakens it in the direction of the tensor \mathbf{a}_d . In particular, Rubin and Bodner [4] suggested the following specific form for the scalar Γ and the tensor \mathbf{a}_d

$$\Gamma = (\Gamma_1 + \Gamma_2\dot{\epsilon}) \exp\left(-\frac{1}{2}\left(\frac{\beta}{\beta_{de}}\right)^{2n}\right), \quad \mathbf{a}_d = \mathbf{b}'_{de} - \frac{3}{\mathbf{b}'_{de} : \mathbf{I}}\mathbf{I}, \quad (4)$$

where $\{\Gamma_1, \Gamma_2, n\}$ are material parameters, $\dot{\epsilon}$ is the effective total distortional deformation rate, β_{de} is a measure of elastic distortional deformation associated with the dissipative component, and β is the hardening variable associated with fluid flow through the cells of the tissue. In particular, the specific forms of $\{\dot{\epsilon}, \beta_{de}, \beta\}$ are given by

$$\dot{\epsilon} = \sqrt{\frac{2}{3}dev(\mathbf{d}) : dev(\mathbf{d})}, \quad \beta_{de} = \sqrt{\frac{3}{2}dev(\mathbf{b}'_{de}) : dev(\mathbf{b}'_{de})}, \quad \dot{\beta} = \frac{r_1r_3 + r_2\dot{\epsilon}}{r_3 + \dot{\epsilon}}\Gamma\beta_{de} - r_4\beta^{r_5}, \quad (5)$$

where the constants r_1, \dots, r_5 are material parameters. Note that the first part on the right hand side of Eq. (5c) causes β to grow, while the second part is responsible for material recovery. Now, using the Clausius-Duhem inequality, it can be shown that the Cauchy stress tensor is given by

$$\boldsymbol{\sigma} = \frac{\mu_0 e^{qq}}{J} \left[m_1(J - 1)\mathbf{I} + m_2 dev(\mathbf{b}'_e) + m_3 \sum_{I=1}^{N_{fib.}} \frac{\langle \lambda_I - 1 \rangle^{2m_4 - 1}}{\lambda_I} (\mathbf{F}\mathbf{M}_I) \otimes (\mathbf{F}\mathbf{M}_I) + dev(\mathbf{b}'_{de}) \right]. \quad (6)$$

It can be seen from Eq. (6) that the stress tensor depends on the tensors $\{\mathbf{b}'_e, \mathbf{b}'_{de}\}$, which are given by evolution equations. Therefore, it is necessary to develop a strongly objective integrator (see [5]) for the evolution equations of the tensors $\{\mathbf{b}'_e, \mathbf{b}'_{de}\}$ in the sense, that the tensors $\{\mathbf{b}'_e, \mathbf{b}'_{de}\}$ at time increment t_{n+1} (i.e. $\mathbf{b}'_e(t_{n+1})$ and $\mathbf{b}'_{de}(t_{n+1})$) have the same invariance properties under superposed rigid body motions as the exact tensors $\{\mathbf{b}'_e, \mathbf{b}'_{de}, \beta\}$. In this regard the solution of Eq. (3) and Eq. (5c) read

$$\begin{aligned} \mathbf{b}'_e(t_{n+1}) &= \mathbf{F}'_r(t_{n+1})\mathbf{b}'_e(t_n)\mathbf{F}'_r{}^\top(t_{n+1}), \\ \mathbf{b}'_{de}(t_{n+1}) &= \mathbf{F}'_r(t_{n+1})\mathbf{b}'_{de}(t_n)\mathbf{F}'_r{}^\top(t_{n+1}) - \Delta t\Gamma(t_{n+1})\mathbf{a}_d(t_{n+1}), \\ \beta(t_{n+1}) &= \beta(t_n) + \Delta t \left[\frac{r_1r_3 + r_2\dot{\epsilon}(t_{n+1})}{r_3 + \dot{\epsilon}(t_{n+1})}\Gamma(t_{n+1})\beta_{de}(t_{n+1}) - r_4\beta(t_{n+1})^{r_5} \right], \end{aligned} \quad (7)$$

where $\mathbf{F}'_r = (\det(\mathbf{F}_r))^{-1/3} \mathbf{F}_r$ and \mathbf{F}_r is the relative deformation gradient that maps the motion from time increment t_n to time increment t_{n+1} . Moreover, using Eqs. (4a) and (7b) it is possible to show, that

$$\text{dev}(\mathbf{b}'_{de}(t_{n+1})) = \frac{1}{1 + \Delta t \Gamma(t_{n+1})} \text{dev}(\mathbf{F}'_r(t_{n+1}) \mathbf{b}'_{de}(t_n) \mathbf{F}_r'^T(t_{n+1})). \quad (8)$$

Finally, the scalar Γ and the hardening variable β can be obtained by solving the nonlinear equations given by Eqs. (4b) and (7c). For the numerical study, a new perspective of the mixed finite element formulation was developed in which the constitutive equations of soft tissues and the strongly objective integration algorithm were implemented. It was found that the developed finite element formulation is very suitable for viscoplastic models that are described through evolution equations. Most significantly however, it turned out that the reference configuration of the body became irrelevant for the calculations obtained by the developed finite element formulation.

3 NUMERICAL EXAMPLE

The following example demonstrates the capability of the constitutive equations described in the previous section together with the developed finite element formulation to capture the nonlinear behavior of soft tissues and the possible stress relaxation. Specifically, consider a block of SMAS (superficial musculoaponeurotic system) subjected to indentation by a smooth rigid sphere (see Figure 1a). The total length of $2L_1$, total depth $2L_2$, total height L_3 of the block, and the radius R of the spherical punch are specified by

$$L_1 = L_2 = L_3 = 1.0 \text{ m}, \quad R = 0.5 \text{ m}. \quad (9)$$

The bottom surface of the block ($X_3 = 0$) remains in contact and slides freely on the plane $X_3 = 0$ and the remaining surfaces of the block are traction free, except for the points of contact between the rigid spherical punch and the deformed top surface of the block ($X_3 = L_3$). Moreover, due to symmetry only one quarter of the block is analyzed and is modeled by $\{15, 15, 15\}$ eight nodes brick elements. The rigid punch can only move vertically with displacement $u_3 = \delta$ in the \mathbf{e}_3 direction. The prescribed vertical displacement δ of the spherical punch is given in Figure 2a. In particular, the prescribed vertical displacement is ramped to $\delta = -0.25 \text{ m}$ and then kept constant for the remaining simulation time.

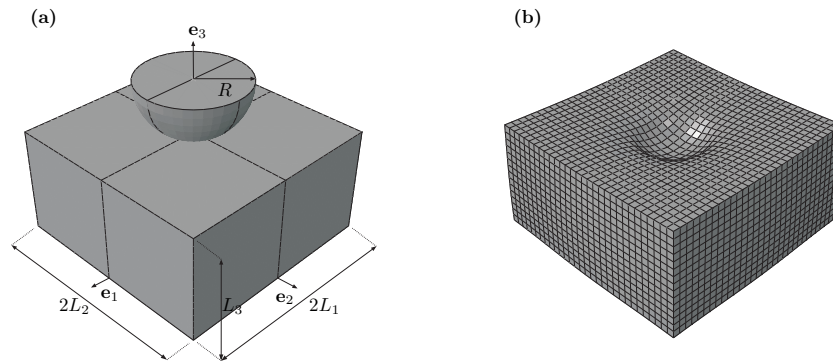


Figure 1: Indentation of a smooth rigid sphere into a SMAS block. (a) Reference geometry; (b) Deformed shape

The normalized vertical reaction force at the spherical punch (F_3/F_0 , $F_0 = -341.8 \text{ kN}$) versus the normalized prescribed δ/δ_0 , $\delta_0 = -0.25 \text{ m}$ is given in Figure 2b. It can be seen that the response is highly nonlinear. Moreover, Figure 2c shows the variation of the normalized vertical reaction force. It can be seen that in the ramping stage, the reaction force increases, while during the second stage, at which the prescribed displacement was kept constant, the vertical reaction force decreases. This example shows the capability of the model to represent stress relaxation in soft tissues. Finally, the deformed shape is given in Figure 2b.

4 CONCLUSION

In face of the significant challenges with respect to the numerical implementation of elasto-viscoplastic constitutive equations describing the highly nonlinear, time dependent mechanical response of soft tissues, a newly developed mixed finite element formulation with a strongly objective time integrator algorithm is presented. The proposed

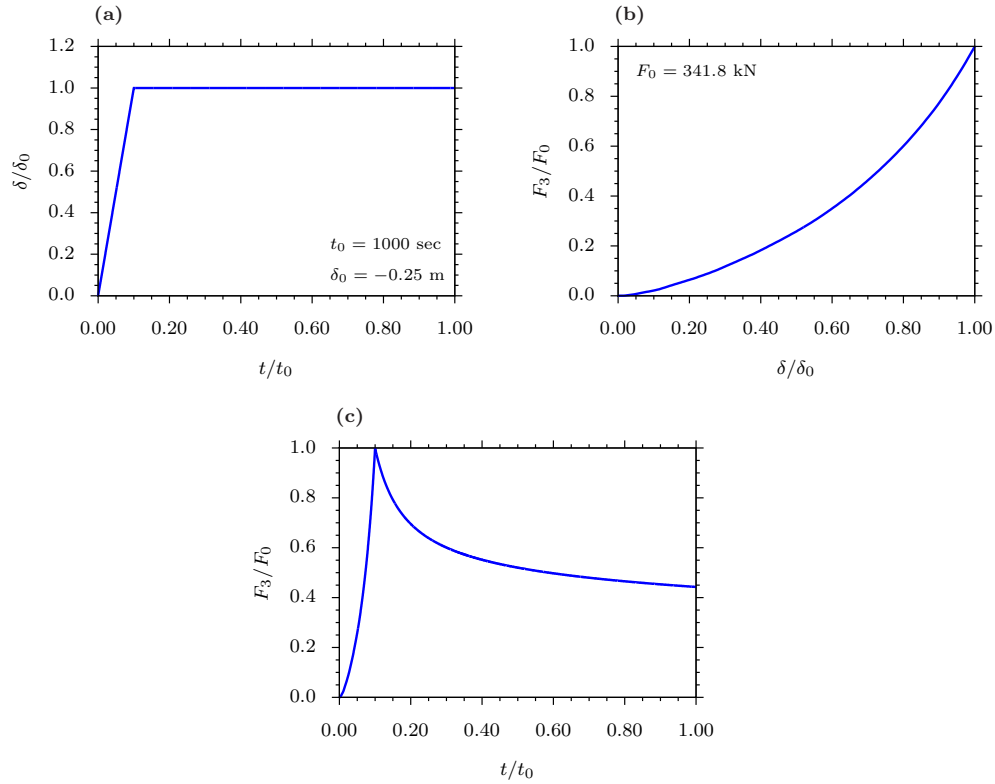


Figure 2: (a) Normalized vertical displacement history; (b) Load-displacement cure; (c) Normalized vertical reaction force history

numerical scheme allows to compute the history dependent state variables, governing the time dependent material mechanics. Specifically, the obtained response of the indentation example demonstrates the capability of the developed model.

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