



**HAL**  
open science

# Finite element modeling of the electromechanical coupling in electroactive polymers

Mahmood Jabareen

► **To cite this version:**

Mahmood Jabareen. Finite element modeling of the electromechanical coupling in electroactive polymers. 2nd ECCOMAS Young Investigators Conference (YIC 2013), Sep 2013, Bordeaux, France. hal-00855859

**HAL Id: hal-00855859**

**<https://hal.science/hal-00855859>**

Submitted on 30 Aug 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Finite element modeling of the electromechanical coupling in electroactive polymers

M. Jabareen<sup>a,\*</sup>

<sup>a</sup> Faculty of Civil and Environmental Engineering, Technion - Israel Institute of Technology  
Haifa, Israel

\*cvjmah@tx.technion.ac.il

---

**Abstract.** *Due to the large strain capability, electroactive polymers (EAPs) are promising materials for several engineering applications, for example: they can be used as actuators and sensors in cameras, pumps, valves, etc. Generally speaking, the mode of action in EAPs is based on Coulomb forces generated by electric field that causes the polymer membrane to contract in the thickness direction. In this contribution, a mixed finite element formulation was developed for modeling the electromechanical coupling and for simulation concerning electroactive polymers.*

**Keywords:** Electroactive polymers; Electromechanical coupling; Finite deformation; Mixed finite element formulation

---

## 1 INTRODUCTION

Electroactive polymers (EAPs) can be considered as functional materials that respond to electrical stimulation with a significant change in shape or size. They can be used as actuators in active structures, in particular when large deformations are required. Compared with other types of actuator technologies, EAP actuators generate much larger strain (in order of 10% to 100%) than electroactive ceramics and respond faster than shape memory alloys. Because of the potential applications of EAPs, the theory of non-linear electroelasticity experiences a renaissance in recent years leading to fundamental theoretical aspects related to the coupling phenomena in active elastomers and numerical modeling based on the theory of finite elasticity. Specifically, Dorfmann and Ogden [3, 4] developed constitutive equations for finite deformations of the electroelastic elastomers. Moreover, in order to allow effective design of EAP actuated mechanisms with large induced strains, adequate analytical and numerical tools are necessary for predicting the behavior of EAP actuators as well as for simulating their response.

Among the electric EAP materials, the dielectric elastomers (DE) is one of the most promising with outstanding properties. Experimental work was conducted to characterize the dielectric elastomers and it was found that they are highly rate-dependent ([5], [6], [8], [9], [11], [12], [13]). Specifically, Hossain et al. [5] presented a comprehensive mechanical characterization of VHB 4910, using different standard experiments such as: single-step relaxation tests, multi-step relaxation tests, and loading-unloading cyclic tests.

Regarding the computational aspect, Landis [7] presented a new finite element formulation as a solution to electro-mechanical problems in the small deformations regime. Variational approaches were formulated and implemented in the finite element method by Vu et al. [10]. Recently, Ask et al. [1] presented a mixed finite element formulation and applied it in the inverse motion problem. Bueschel et al. [2] applied the standard Bubnov-Galerkin approach for the simulation of dielectric elastomers including the time dependent nature of the elastomers.

The objective of this contribution is to develop a mixed finite element formulation for modeling the electromechanical coupling in electroactive polymers. In the current abstract, the hyperelastic response of electroactive polymers was considered, while the inelastic response due to viscoelasticity is under development.

## 2 FINITE ELEMENT FORMULATION

Generally speaking, polymers have a bulk modulus which is much higher than the shear modulus. Therefore, polymers can be considered as nearly incompressible materials and the use of the standard Bubnov-Galerkin approach will lead to volumetric locking. Hence, element technologies have to be used in order to overcome the aforementioned locking. One of such technologies is the mixed formulation, at which the volumetric measure of the deformation will be evaluated in an average manner, while the distortional part will be valid point wise. Particularly, the functional of the total potential energy is decomposed into an internal part due to the strain energy function and to external part due to external tractions and electrical fluxes, such that:

$$\Pi = \int_{\Omega_0} [\widehat{W}(\overline{\mathbf{C}}, \mathbf{E}) + \bar{p}(J - \bar{J})] d\Omega_0 - \int_{\Gamma_0} [\mathbf{u} \cdot \overline{\mathbf{T}} + \phi \overline{Q}] d\Gamma_0. \quad (1)$$

where  $\overline{\mathbf{C}}$  is the modified right Cauchy-Green deformation tensor,  $\mathbf{E}$  is the electric field and they are given by

$$\begin{aligned} \overline{\mathbf{C}} &= \overline{\mathbf{F}}^T \overline{\mathbf{F}}, \quad \overline{\mathbf{F}} = \left(\frac{\bar{J}}{J}\right)^{1/3} \mathbf{F}, \quad \mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}, \\ \mathbf{E} &= -\frac{\partial \phi}{\partial \mathbf{X}}, \end{aligned} \quad (2)$$

where  $\overline{\mathbf{F}}$  is the modified deformation gradient. Taking the variation of (1), yields

$$\delta \Pi = \int_{\Omega_0} \left[ \overline{\mathbf{S}} : \delta \mathbf{E} + (J - \bar{J}) \delta \bar{p} + \left( \frac{\overline{\mathbf{S}} : \overline{\mathbf{C}}}{3\bar{J}} - \bar{p} \right) \delta \bar{J} - \mathbf{D} \cdot \delta \mathbf{E} \right] d\Omega_0 - \int_{\Gamma_0} [\overline{\mathbf{T}} \cdot \delta \mathbf{u} + \overline{Q} \delta \phi] d\Gamma_0. \quad (3)$$

Also,  $\overline{\mathbf{S}}$  and  $\overline{\mathbf{S}}$  are stress quantities,  $\delta \mathbf{E}$  is the variation of the Green-Lagrange strain tensor,  $\mathbf{D}$  is the electric displacement vector, and  $\delta \mathbf{E}$  is the variation of the electric field defined by

$$\begin{aligned} \overline{\mathbf{S}} &= 2 \frac{\partial \widehat{W}(\overline{\mathbf{C}}, \mathbf{E})}{\partial \overline{\mathbf{C}}}, \quad \overline{\mathbf{S}} = \left(\frac{\bar{J}}{J}\right)^{1/3} \overline{\mathbf{S}} - \frac{1}{3} (\overline{\mathbf{S}} : \overline{\mathbf{C}}) \overline{\mathbf{C}}^{-1} + \bar{p} J \overline{\mathbf{C}}^{-1}, \quad \delta \mathbf{E} = \frac{1}{2} (\delta \mathbf{F}^T \mathbf{F} + \mathbf{F}^T \delta \mathbf{F}), \\ \mathbf{D} &= \frac{\partial \widehat{W}(\overline{\mathbf{C}}, \mathbf{E})}{\partial \mathbf{E}}, \quad \delta \mathbf{E} = -\frac{\partial \delta \phi}{\partial \mathbf{X}}. \end{aligned} \quad (4)$$

It can be shown that the average pressure,  $\bar{p}$  and the average dilatation,  $\bar{J}$  can be determined in the element level, such that

$$\bar{p} = \frac{1}{\Omega_0} \int_{\Omega_0} \frac{\overline{\mathbf{S}} : \overline{\mathbf{C}}}{3\bar{J}} d\Omega_0, \quad \bar{J} = \frac{1}{\Omega_0} \int_{\Omega_0} J d\Omega_0. \quad (5)$$

Finally, the strain energy function  $\widehat{W}(\overline{\mathbf{C}}, \mathbf{E})$  is defined by (see [10])

$$\begin{aligned} \widehat{W}(\overline{\mathbf{C}}, \mathbf{E}) &= W(\overline{\mathbf{C}}) - \frac{1}{2} \epsilon_0 \bar{J} \overline{\mathbf{C}}^{-1} : (\mathbf{E} \otimes \mathbf{E}) + c_1 \mathbf{I} : (\mathbf{E} \otimes \mathbf{E}) + c_2 \overline{\mathbf{C}} : (\mathbf{E} \otimes \mathbf{E}), \\ W(\overline{\mathbf{C}}) &= \frac{1}{2} K (\bar{J} - 1) + \mu (\bar{\alpha}_1 - 3), \quad \bar{\alpha}_1 = \bar{J}^{-2/3} \overline{\mathbf{C}} : \mathbf{I}, \end{aligned} \quad (6)$$

where  $\epsilon_0$  is the electric permittivity of vacuum,  $K$  and  $\mu$  are the bulk and shear moduli, respectively.

Following the aforementioned mixed formulation and specifically using Eqs. (5) and (4), a brick element with eight nodes and four degrees of freedom at each node (three nodal displacements,  $\hat{\mathbf{u}}$ , and one nodal electric potential,  $\hat{\phi}$ ) was developed and implemented in Abaqus 6.10.3 (a finite element package) for the numerical simulations of electroactive polymers.

## 3 NUMERICAL EXAMPLE

In order to verify the correctness of the implementation, the patch test is recalled. In this test we consider a plate that is meshed by seven general elements (highly distorted elements, see figure 1a) and subjected to electric potential at the bottom and top faces. Since the deformation is homogeneous (see figure 1b) and the electric potential varies linearly through the thickness of the plate, it is possible to formulate an analytical solution of this problem.

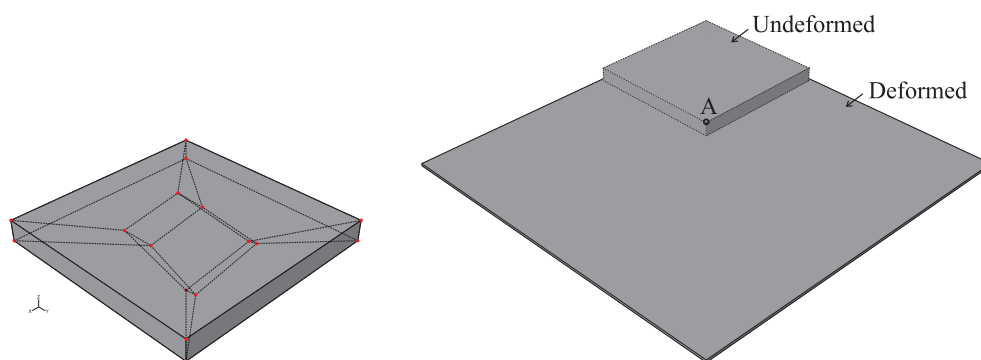


Figure 1: Geometry of the plate, with seven elements; The undeformed and deformed configurations.

Specifically, figure 2a shows the vertical displacement at point A (see figure 1b) versus the applied electric potential using the developed element. This solution was compared to the analytical solution of the electromechanical coupling equations, and it can be seen that both numerical and analytical solutions coincide. Moreover, the change in volume was calculated and plotted in figure 2b. It can be seen that using the mixed formulation with a high bulk modulus enforces the volume to remain constant.

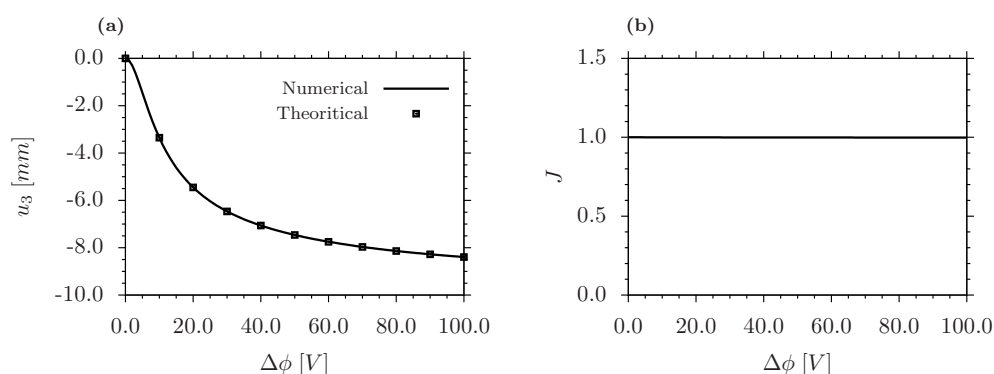


Figure 2: The vertical displacement at point A versus the applied electric field; (b) The volume change versus the applied electric field.

## 4 CONCLUSION

In this contribution, a mixed finite element formulation was presented for modeling the electromechanical coupling in electroactive polymers. The developed finite element was implemented in the finite element package Abaqus for the numerical study. The correctness of the implementation was verified through homogeneously deformed problem (patch-test). Moreover, the developed formulation was applied in series of problems (that will be presented in the conference), and it was found that it is robust and can be confidently used for simulating electromechanical coupling in electroactive polymers. The extension to viscoelasticity is under development.

## REFERENCES

- [1] Ask, A., Denzer, R., Menzel, A., Ristinmaa, M.. Inverse-motion-based form finding for quasi-incompressible finite electroelasticity. *International Journal for Numerical Methods in Engineering*, doi: 10.1002/nme.4462, 2013.
- [2] Bueschela, A., Klinkel, S., Wagner, W.. Dielectric elastomers-numerical modeling of nonlinear visco-electroelasticity. *International Journal for Numerical Methods in Engineering*, **93**: 834856, 2013.
- [3] Dorfmann, A., Ogden, R.W.. Nonlinear electroelasticity. *Acta. Mech.*, **4(3-4)**: 434-444, 2005.
- [4] Dorfmann, A., Ogden, R.W.. Nonlinear electroelastic deformations. *Journal of Elasticity*, **82(2)**: 99-127, 2006.
- [5] Hossain, M., Vu, D.K., Steinmann, P.. Experimental study and numerical modelling of VHB 4910 polymer. *Computational Materials Science*, **59**: 65-74, 2012.
- [6] Kofod, G.. Dielectric Elastomer Actuators. *Ph.D. Thesis*, Riso-R-1286, Denmark.

- 
- [7] Landis, C.M.. A new finite-element formulation for electromechanical boundary value problems. *International Journal for Numerical Methods in Engineering*, **55(5)**: 613-628, 2002.
  - [8] Plante, J.S., Dubowsky, S., Large-scale failure modes of dielectric elastomer actuators. *International Journal of Solids and Structures*, **43**: 7727-7751, 2006.
  - [9] Sommer-Larsen, P., Kofod, G., Shridhar, M., Benslimane, M., Gravesen, P.. Performance of dielectric elastomer actuators and materials. Proc. SPIE, **4695**: 158-166, 2002.
  - [10] Vu, D.K., Steinmann, P., Possart, G.. Numerical modelling of non-linear electroelasticity. *International Journal for Numerical Methods in Engineering*, **70(6)**: 685-704, 2007.
  - [11] Wissler, M., Mazza, E.. Modeling of a pre-strained circular actuator made of dielectric elastomers. *Sensors and Actuators A*, **120**: 184-192, 2005.
  - [12] Wissler, M., Mazza, E.. Modeling and simulation of dielectric elastomer actuators. *Smart Materials and Structures*, **14**: 1396-1402, 2005.
  - [13] Zhang, X., Lowe, C., Wissler, M., Jahne, B., Kovacs, G.. Dielectric elastomers in actuator technology. *Advanced Engineering Materials*, **7(5)**: 361-367, 2005.