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## Anisotropic transport and diffusion of elastic waves in random media

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**Abstract.** *We discuss the influence of material anisotropy on the possible depolarization and diffusion of elastic waves in randomly heterogeneous media. Anisotropy is considered at two levels. The first one is related to the constitutive law of random materials, which may be handled by a random matrix theory for the elasticity tensor. The second level is related to the correlation structure of these random materials. Since the propagation of waves in such complex media cannot be described by deterministic models, a probabilistic framework shall be adopted based on a radiative transfer model in the so-called mesoscopic regime, when the wavelength and the correlation length are comparable.*

**Keywords:** Wave propagation; Elasticity; Random media; Anisotropic media; Meso-scale modeling.

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Electromagnetic, acoustic or elastic waves in a homogeneous medium have a fairly well identified behavior, even in the presence of a smooth boundary as a free surface for example. This is no longer the case in a heterogeneous or random medium. Wave propagation in heterogeneous media is the subject of intense, multidisciplinary researches in order to exhibit, on the one hand, some new phenomena such as anisotropic diffusion (strong localization) or coherent back-scattering effects (weak localization), and to take advantage, on the other hand, of these effects in applications such as time reversal techniques. This issue is less studied in continuum mechanics, beside some early works in seismology and surface geophysics [1]. However it is essential in the understanding of high-frequency wave propagation phenomena in elastic media at the mesoscopic and macroscopic scales, for applications in structural-acoustics, room acoustics, or structural health monitoring by ultrasonic techniques, among others.

Introducing the wavelength  $\lambda$  and a characteristic length of the heterogeneities  $l$ , such as a correlation length, the microscopic, mesoscopic and macroscopic regimes of wave propagation are classically defined by  $\delta = \lambda/l > 1$ ,  $\delta \approx 1$ , and  $\delta < 1$  respectively [2]. In the microscopic regime the relevant evolution model is the wave equation, requiring a priori a precise knowledge of the shape and location of the heterogeneities. The limit case  $\delta \gg 1$  where the mechanical parameters of the medium vary rapidly can be treated by homogenization techniques or an effective medium theory [3, 4]. Both approaches extend to the case of randomly distributed inclusions of which number increases indefinitely at a constant volume fraction in a homogeneous background medium. In the mesoscopic regime, the upscaled evolution model is the transport equation, or radiative transfer equation in a high-frequency random medium where the correlation length is comparable to the wavelength [5]. It corresponds to a situation of strong interaction between waves and random heterogeneities which cannot be addressed by usual homogenization and multi-scale techniques. The characteristic homogenized parameters of this medium are the scattering mean free paths and transport velocities, which are derived from the microscopic regime. At last in the macroscopic regime the relevant evolution model is the diffusion equation. It is characterized by a diffusion operator which is derived from the mesoscopic parameters. In this regime all interferences of waves with the heterogeneities are smoothed out and angularly averaged wave speckles are considered.

The purpose of the research presented in this talk is to assess the influence of material anisotropy on the mesoscopic regime of elastic waves in randomly heterogeneous media. Anisotropy should be considered at two levels. The first one is related to the constitutive law of random materials, which may be handled by a random matrix theory for the

elasticity tensor [6, 7]. The second level is related to the correlation structure of these random materials, which is referred to as anisotropy in the dedicated literature. Since the propagation of waves in such complex media cannot be described by deterministic models, a so-called kinetic framework shall be adopted. It is based on a radiative transfer model coupling all different polarization modes of the elastic waves. The former is derived from the elastic wave equation in anisotropic media by a classical micro-local analysis [8]. More specifically we shall present our work on developing formal models for the consideration of anisotropy in the collision kernels of the radiative transfer equations, in order to establish the corresponding multiple scattering, diffusive (macroscopic) limit if any. These models describe the evolution of the wave energy density in the phase space position  $\times$  wave vector. They also allow to track the corresponding energy flows and thus the energy transfers between different sub-regions. Some numerical examples shall be presented as well for some selected material symmetries.

The objective of the theory is to develop probing techniques for anisotropic heterogeneous media such as composite structures in the aerospace industry, biological tissues in medical imaging, or textured layers of the Earth crust, for example.

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