

Numerical Modelling of Piezoelectric Energy Harvesting Devices

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Abstract. *This paper introduces a monolithic approach that provides simultaneous solution to the coupled system which involves volume-coupled piezoelectric mechanics and a controlling energy harvesting circuit for applications in energy harvesting. The weak form of the governing equations is discretized by space-time finite element method based on mixed velocity-stress/ rate of potential-dielectric displacement setting. The results will be compared to the simple cases with closed-form solution available from literature.*

Keywords: energy harvesting; space-time finite elements; harvesting circuit.

1 MOTIVATION

Research interest in energy harvesting from vibrations has seen a big surge in recent years owing to the recent advances in wireless systems and micro electro-mechanical systems (MEMS). Though there are many such harvesting methods, the research focuses on piezoelectric energy harvesting. Piezoelectric materials have the ability to convert mechanical vibrations into electric charge and vice versa by virtue of its crystalline structure. Thus the motivation of this research is two fold: to make use of the structural vibrations in such a way that the vibrations can effectively be controlled to harvest useful energy to power small-scale energy devices and develop a numerical model to solve the discretized equations in a monolithic framework i.e, a holistic approach that solves the equations in a single algebraic system. The complex coupled system is investigated based on an available and validated simultaneous space-time finite element method [1, 2]. This systems also enables to consider the backward coupling involved between the harvester and the harvesting circuit.

2 MODEL SETUP

The coupled system consists of an elastic beam like structure embedded with thin piezoelectric patches and a suitable energy harvesting circuit that stores and transforms energy generated by the harvester. The behavior of the structure in the time interval $I = (t_a, t_b)$ and occupying the space-time domain $Q_0 = \Omega_0 \times I$ is described in the reference configurations using a *Lagrangian* description to account for large deformations. The harvesting circuit is represented by a resistor element and inductor connected in series or in parallel with the electrodes from the piezoelectric patches. The system is modeled in a 3-dimensional framework to enable straightforward application of constitutive models without reducing them based on assumptions.

3 GOVERNING EQUATIONS: STRONG FORM

This section describes in brief the strong form of the governing equations of the multi-physics problem and the associated boundary conditions along with the linear inverse piezoelectric constitutive law.

3.1 Balance equations

The electrostatic response of a piezoelectric structure coupled to an energy harvesting circuit is governed by the following balance equations

$$\rho_0 \dot{\mathbf{v}} - \nabla_0 \cdot (\mathbf{FS}) - \rho_0 \mathbf{b}_0 = 0 \quad \text{in } Q_0 \quad (1)$$

$$\nabla_0 \cdot \mathbf{D}_0 = 0 \quad \text{in } Q_0 \quad (2)$$

$$V(t) - R\dot{q} - L\ddot{q} = 0 \quad (3)$$

Where (1), (2) and (3) represent the elasto-dynamic behavior of the structure, electrostatic equilibrium of the piezoelectric material represented by Gauss' law and the harvesting circuit represented by Kirchhoff Voltage Law(KVL) respectively. \mathbf{F} represents material deformation gradient, \mathbf{S} is the second Piola-Kirchhoff tensor, \mathbf{v} is the velocity, \mathbf{D} is the dielectric displacement vector, \mathbf{b} is the body force vector and ρ is the mass density. $V(t)$ represents the source voltage at the harvester and q represents the electric charge. R and L are the resistor and inductor components respectively. The subscript 0 represents the quantities described in reference configuration.

3.2 Constitutive Equations

The second Piola-Kirchhoff tensor \mathbf{S} and the dielectric displacement vector \mathbf{D} are related to the Green-Lagrangian strain tensor $\boldsymbol{\varepsilon}$ and the electric field vector \mathbf{E} through linear inverse piezoelectric constitutive relation which is described in a rate formulation.

$$\mathbf{c}\dot{\mathbf{S}} + \mathbf{d}\dot{\mathbf{D}}_0 - \dot{\boldsymbol{\varepsilon}} = 0 \quad (4)$$

$$-\mathbf{d}^\top \dot{\mathbf{S}} + \boldsymbol{\varepsilon} \dot{\mathbf{D}}_0 - \dot{\mathbf{E}}_0 = 0 \quad (5)$$

where \mathbf{c} , \mathbf{d} and $\boldsymbol{\varepsilon}$ are the compliance matrix, piezoelectric constant matrix and dielectric constant matrix respectively. The rate of strain tensor, $\dot{\boldsymbol{\varepsilon}}$ and the rate of electric field vector, $\dot{\mathbf{E}}_0$ are related to the velocity gradient and the rate of electric potential, $\dot{\psi}$ by the following relation

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2} \left(\nabla_0 \mathbf{v} + (\nabla_0 \mathbf{v})^\top + (\nabla_0 \mathbf{u})^\top \nabla_0 \mathbf{v} + (\nabla_0 \mathbf{v})^\top \nabla_0 \mathbf{u} \right) \quad (6)$$

$$\dot{\mathbf{E}}_0 = -\nabla_0 \dot{\psi}; \quad \text{where } \dot{\psi} = \dot{\phi} \quad (7)$$

3.3 Boundary conditions

The piezoelectric body Q_0 , could be subject to either Dirichlet or Neumann mechanical and electrical boundary conditions or a combination of both on its boundary P_0 , where $P_0 = \Gamma_0 \times I$

$$\mathbf{v} = \mathbf{v}_0 \quad \text{on } P_0^g \quad (8a)$$

$$\mathbf{t}_0 = \mathbf{h}_0 \quad \text{on } P_0^h \quad (8b)$$

$$\psi = \psi_0 \quad \text{on } P_0^\psi \quad (8c)$$

$$\mathbf{D}_0 \cdot \mathbf{n}_0 = q \quad \text{on } P_0^q \quad (8d)$$

where \mathbf{v}_0 , \mathbf{h}_0 , ψ_0 , q and \mathbf{n}_0 are specified mechanical velocity, traction, potential rate, surface charge and outward unit normal vector respectively. The boundary $P_0 = P_0^h \cup P_0^q = P_0^\psi \cup P_0^g$. The boundary conditions are considered along with appropriate initial conditions.

4 GOVERNING EQUATIONS: WEAK FORM

The method of weighted residuals is applied to the strong form of the equations. The resulting weak form is discretized both in space and time using space-time finite element method. The integration in time is performed with discontinuous Galerkin method. The temporal axis is included in the finite element

discretization wherein the domain Q^n is divided into N time slabs as $Q^n = \Omega^n \times [t_n, t_{n+1}]$, as can be seen from figure 1, which are solved successively. This leads to numerical efficiency. Additional jump terms are added in the weak form to account for the time-discontinuous approximation of the unknowns since the energy of the discretized system at time slab t_n^- has to be equal to the energy at the beginning of the next time slab t_n^+ . The weak form of the coupled system reads as

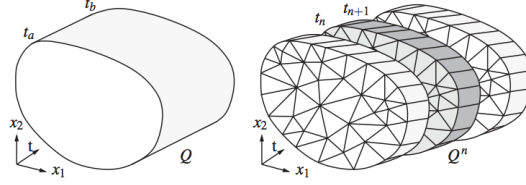


Figure 1: Discretization of the domain in space-time finite element setting [1].

$$\int_{Q_{0,n}} \delta \mathbf{v} \rho_0 \dot{\mathbf{v}} dQ_0 + \int_{Q_{0,n}} \dot{\boldsymbol{\epsilon}}(\mathbf{F}, \delta \mathbf{v}) \mathbf{S} dQ_0 - \int_{Q_{0,n}} \delta \mathbf{v} \rho_0 \mathbf{b}_0 dQ_0 - \int_{Q_{0,n}} \dot{\mathbf{E}}_0(\delta \psi) \mathbf{D}_0 dQ_0 \quad (9a)$$

$$+ \int_{Q_{0,n}} \delta \mathbf{S} \left(\underline{\mathbf{c}} \dot{\mathbf{S}} + \underline{\mathbf{d}} \dot{\mathbf{D}}_0 - \dot{\boldsymbol{\epsilon}}(\mathbf{F}, \mathbf{v}) \right) dQ_0 + \int_{Q_{0,n}} \delta \mathbf{D}_0 \left(-\underline{\mathbf{d}}^\top \dot{\mathbf{S}} + \underline{\boldsymbol{\epsilon}} \dot{\mathbf{D}}_0 - \dot{\mathbf{E}}_0(\psi) \right) dQ_0 \quad (9b)$$

$$+ \int_{\Omega_0} \delta \mathbf{v}(t_n^+) \left(\rho_0 (\mathbf{v}(t_n^+) - \mathbf{v}(t_n^-)) \right) d\Omega_0 \quad (9c)$$

$$+ \int_{\Omega_0} \delta \mathbf{S} \left(\underline{\mathbf{c}} (\mathbf{S}(t_n^+) - \mathbf{S}(t_n^-)) + \underline{\mathbf{d}} (\mathbf{D}_0(t_n^+) - \mathbf{D}_0(t_n^-)) \right) d\Omega_0 \quad (9d)$$

$$+ \int_{\Omega_0} \delta \mathbf{D}_0 \left(-\underline{\mathbf{d}}^\top (\mathbf{S}(t_n^+) - \mathbf{S}(t_n^-)) + \underline{\boldsymbol{\epsilon}} (\mathbf{D}_0(t_n^+) - \mathbf{D}_0(t_n^-)) \right) d\Omega_0 \quad (9e)$$

$$- \int_{P_0^h} \delta \mathbf{v} \mathbf{h}_0 dP_0 - \int_{P_0^q} \delta \psi q dP_0 = 0 \quad \forall \delta \mathbf{v}, \delta \mathbf{S}, \delta \psi, \delta \mathbf{D}_0 \quad (9f)$$

Equations (9a) – (9f) represents the mixed weak form of the piezoelectric continuum under investigation. The weak form consists four fields of unknowns and is formulated in mixed hybrid space-time finite element formulation by static condensation of stresses and dielectric displacement. The harvesting circuit equation can be represented as a function of voltage V , charge q and their first and second rates as $f(\dot{V}, \ddot{V}, V, \dot{q}, \ddot{q}, q)$ and is coupled to the harvester equations either directly or solved in a weak sense weighted by the variation of charge.

5 CONCLUSIONS

The project contributes to the mathematical modelling and numerical discretization of complex multi-physics system in an efficient way by simultaneously solving the various fields involved which facilitates an ideal basis for precise and transient coupling. This may lead to improved convergence and numerical efficiency in comparison with partitioned approaches. This methodology also provides new insights and in-depth understanding on design requirements on such energy harvesting devices in terms of their robustness and efficiency.

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