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# Adaptive numerical simulation of contact problems. Resolving local effects at the contact boundary in space and time.

M. Walloth <sup>a,\*</sup>, R. Krause <sup>b</sup>, A. Veerer <sup>c</sup>

<sup>a</sup> Technische Universität Darmstadt, Fachbereich Mathematik, AG Numerik und Wissenschaftliches Rechnen  
Dolivostraße 15, D-64293 Darmstadt

<sup>b</sup> Institute of Computational Science, University of Lugano  
Via Giuseppe Buffi 13, CH-6900 Lugano

<sup>c</sup> Dipartimento di Matematica, Università degli Studi di Milano  
Via C. Saldini 50, I-20133 Milano

\*mwalloth@uni-bonn.de

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**Abstract.** *This talk is concerned with the space discretization of static and the space and time discretization of dynamic contact problems. In particular, we derive a new efficient and reliable residual-type a posteriori error estimator for static contact problems and a new space-time connecting discretization scheme for dynamic contact problems in linear elasticity. The methods enable the efficient resolution of local effects at the contact boundary in space and time.*

**Keywords:** static and dynamic contact problems; residual-type a posteriori error estimator; space-time connecting discretization.

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## 1 INTRODUCTION

In many engineering applications, the accurate simulation of contact problems in linear elasticity is of great importance. Due to the non-linear and non-smooth character of contact problems special care has to be taken to the choice of the discretization method. Especially the resolution of local effects at the contact boundary in space and time affects the accuracy of the solution considerably. In this talk we present a new a posteriori error estimator for adaptive refinement of the finite element discretization in space and we present a new time discretization method which implicitly resolves the local impact times at each contacting node. In fact, these methods significantly improve the quality of the numerical solution for given computational resources.

## 2 RESIDUAL-TYPE A POSTERIORI ERROR ESTIMATORS

For the discretization in space of contact problems we use linear finite elements. In order to reduce the computational costs without sacrificing the accuracy of the numerical solution, an adaptive mesh generation is highly advantageous. It is usually implemented by an iterative process that, with the help of a so-called a posteriori error estimator, determines regions with less regular or even singular behaviour of the unknown exact solution. For this purpose, the estimator should be reliable, i.e., give an upper bound of the error and efficient, i.e. give a lower bound of the error, at least up to so-called oscillation terms of higher order.

For linear elliptic boundary value problems, where no constraints are imposed, the standard residual a posteriori error estimator is reliable and efficient. This estimator is very common due to its easy computation. The proofs of upper and lower bound are based on the equivalence  $\|\mathcal{R}_m^{\text{lin}}\|_{-1} \lesssim \|u - u_m\|_1 \lesssim \|\mathcal{R}_m^{\text{lin}}\|_{-1}$  between the linear residual and the error. This relation is disturbed if constraints are imposed as, e.g., in contact problems or the closely related obstacle problems. Therefore, the linear residual overestimates the error and the standard residual error estimator is not efficient. However, in the literature one can find residual-type a posteriori error estimators which extend the

concept of the standard residual estimator to contact and obstacle problems. We analyze the basic ideas of selected residual-type estimators for contact and obstacle problems found in the literature [4, 5, 1, 12, 11] and we explain the difficulties arising in their construction.

We propose and analyze a new residual-type a posteriori error estimator for the linear finite element solution of the Signorini problem [9]. We prove its reliability and efficiency for two- and three-dimensional simplicial meshes if the gap function is discrete. Even for arbitrary, non-discrete gap functions the reliability is proven. A key ingredient of our approach is the so-called Galerkin functional. It is a modification of the residual with respect to the corresponding linear problem with the help of a suitable approximation of the contact force and, thus, may be seen as the residual of a linear auxiliary problem. The estimator contributions addressing the nonlinearity are related to the contact stresses, the complementarity condition, and the approximation of the gap function. If no actual contact occurs the error estimator coincides with the standard residual error estimator for linear elliptic problems.

Our theoretical findings are supported by intensive numerical studies. The adaptively refined grids and the relevance of the different error estimator contributions are analyzed by means of different illustrative numerical experiments in  $3D$ . We also quantitatively investigate the convergence of the error estimator by comparing to the case of uniformly refined grids. Furthermore, for selected examples in  $2D$  and even in  $3D$  where the contact stresses are known analytically, we compare the numerically computed contact stresses on adaptively refined grids with the exact contact stresses. Interestingly, although the proofs of upper and lower bound are given for simplicial meshes, the numerical studies show also very good performance of the new residual-type a posteriori error estimator for more general meshes consisting of hexahedra, tetrahedra, prisms, and pyramids.

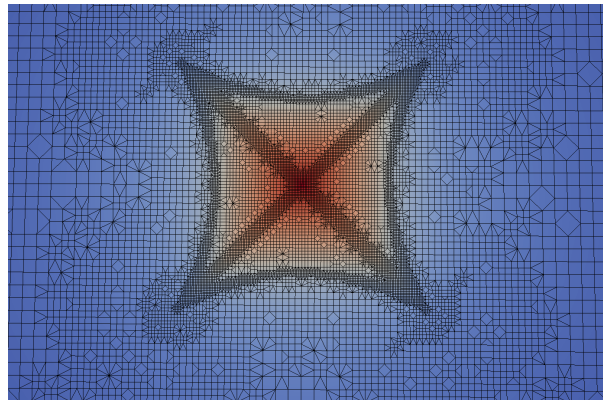


Figure 1: adaptively refined contact boundary if the tip of a pyramid indents

### 3 TIME DISCRETIZATION SCHEMES FOR DYNAMIC CONTACT PROBLEMS

In the numerical simulation of dynamic contact problems the construction of a suitable time discretization scheme is of crucial importance. Due to the non-smooth character of dynamic contact problems, classical time discretization schemes cannot be applied in a straightforward way. Therefore, in the literature, several methods have been proposed which adapt classical time discretization schemes to the case of contact problems. Since up to now no existence results for the hyperbolic system of dynamic contact problems are available, the quality of time discretization schemes of contact problems is measured by means of physical properties of the time-discretized system. Besides the displacements, velocities and contact stresses, the energy and momentum conservation and the persistency condition are relevant.

In this work we focus on modifications of the classical Newmark scheme. The classical Newmark scheme is very common in continuum mechanics as it is of second order consistency and conserves the energy in the unconstrained case. Unfortunately, in the case of contact constraints, the classical Newmark scheme evokes oscillations in the contact stresses, the displacements and the velocities at the contact boundary and even energy blow-ups may occur which spoil the accuracy of the solution. We give a deeper insight into the causes of these instabilities. Further, we present and compare selected modifications of the Newmark scheme which can be found in the literature, see [10, 6, 2, 7, 3]. Each of this methods has its advantage concerning the stability of contact stresses or the course of energy. We illustrate these properties by means of numerical examples in  $3D$ .

We present a new space-time connecting discretization scheme for dynamic contact problems which is based on the

Newmark scheme. It avoids oscillations in the contact stresses, is provably dissipative and allows for a physically motivated update of the velocities. We find out that by means of the predictor step used in [2] the impact times of the single nodes can be computed implicitly. By means of these impact times, the change in the velocity in the moment of impact can be taken into account. This is not possible in classical time discretization schemes. Further, we show that this new time discretization scheme as well as other methods proposed in the literature, e.g., [6, 2] are elements of a family of modified Newmark schemes depending on matrix-valued parameters. We discuss the influence of different choices of the matrix-valued parameters on the course of energy and the behavior of the contact stresses and velocities. Numerical examples in 3D complement the theoretical analysis.

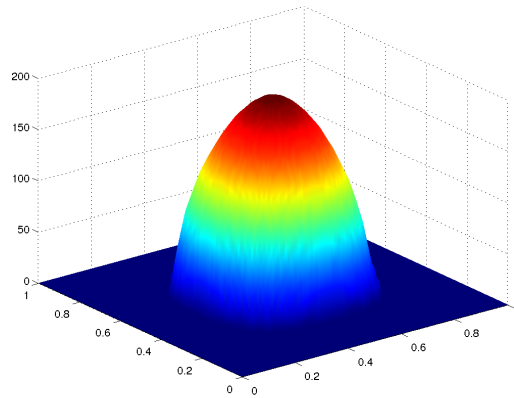


Figure 2: Signorini problem: contact stresses

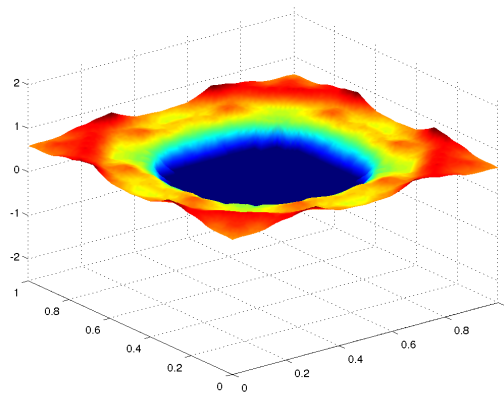


Figure 3: Signorini problem: velocity at the contact boundary

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