Wave Based Methods to predict the dynamic behaviour of poroelastic materials

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Abstract. *Poroelastic materials are often used as sound and vibration reduction measures in many engineering applications. Their dynamic behaviour is accurately described by the theory of Biot. The Finite Element Method is most commonly used to simulate such materials, however, is only applicable for low-frequency applications due to the associated computational cost. This extended abstract describes the development of a Wave Based Method to efficiently simulate dynamic poroelastic problems for an increased frequency range.*

Keywords: Wave Based Method; poroelastic materials; Biot theory.

1 INTRODUCTION

Poroelastic media consist of a solid phase and a fluid phase contained within its pores, which may exhibit a strong mutual interaction. The most commonly applied mathematical model to describe the dynamic behavior of such materials is the Biot theory [1]. Using a set of two frequency-dependent coupled partial differential equations, this constitutive model predicts the existence of three different fluid-frame coupled wave types: a shear wave and two compressional waves. Often the so-called Johnson-Champoux-Allard theory is used to model thermal and viscous losses [2].

Nowadays, the Finite Element Method (FEM) is most commonly used to predict the behaviour of those materials. Several formulations exist, of which the (u,U) and the (u,p) are the most well-known [2]. However, FE calculations are time-consuming due to the complex and frequency dependent material properties, the high number of unknowns per node and the dense problem discretisations that are needed to capture the short wavelengths in the poroelastic response fields at higher frequencies. As a result, the use of the FEM is practically limited to low-frequency applications.

This extended abstract presents a short overview and the application of an efficient Wave Based Method (WBM) for the solution of the Biot equations. The WBM [3] is a deterministic method, based on an indirect Trefftz approach and has been applied to acoustic and 2D elastic problems. Contrarily to element-based approaches, the problem domain is subdivided into a small number of large, convex subdomains. Within each subdomain, the dynamic field variables are approximated using exact solutions of the dynamic equations. Consequently, the approximations of the dynamic fields may only violate the imposed boundary and interface conditions. These errors are minimised in an integral sense applying a Galerkin weighted residual scheme. The resulting matrices are small, enabling an efficient solution and to increase the attainable frequency range.

In the case of poroelastic materials, the dynamic field variables are approximated using a superposition of three types of *a priori* defined wave functions, corresponding to the three aforementioned poroelastic wave types existing in poroelastic materials. 2D Cartesian and axisymmetric problems are considered.

2 PROBLEM DESCRIPTION

The time-harmonic behaviour of a poroelastic material is described by the coupled Biot equations:

$$N\nabla^2\mathbf{u^s}(\mathbf{r}) + \nabla[(\lambda + \frac{Q^2}{R} + N)e^s(\mathbf{r}) + Qe^f(\mathbf{r})] = -\omega^2(\tilde{\rho}_{11}\mathbf{u^s}(\mathbf{r}) + \tilde{\rho}_{12}\mathbf{u^f}(\mathbf{r})) \quad (1)$$

$$\nabla[Qe^s(\mathbf{r}) + Re^f(\mathbf{r})] = -\omega^2(\tilde{\rho}_{12}\mathbf{u^s}(\mathbf{r}) + \tilde{\rho}_{22}\mathbf{u^f}(\mathbf{r})), \quad (2)$$

where ∇ is the Laplacian operator, $\mathbf{u^s}(\mathbf{r})$ and $\mathbf{u^f}(\mathbf{r})$ are the displacements of the solid and the fluid phase, respectively. The term $e^\bullet(\mathbf{r})$ is the volumetric strain of phase $\bullet$, also known as dilatation and $\mathbf{e^s}(\mathbf{r})$ the symmetric strain tensor of the solid phase. For a description of all parameters involved, the reader is referred to [2]. This theory presents the existence of three wave types in poro-elastic materials: two longitudinal waves and one rotational wave. All waves propagate in the fluid as well as in the solid phase.
To obtain a well-posed problem, three boundary conditions have to be imposed on each point of the boundary. Typically, mechanical, kinematic or mixed boundary conditions can be applied. In addition, coupling conditions between domains of a different nature (e.g. the coupling between an acoustic and a poroelastic domain) can also be imposed [4].

3 THE WAVE BASED METHOD

The WBM is a deterministic prediction method, based on an indirect Trefftz approach. It was originally developed for acoustic problems [3]. Contrarily to element-based techniques, this method partitions the problem domain into a large number of convex subdomains. Convexity of the subdomains was shown to be a sufficient condition for the method to converge. Within each subdomain, the field variables (originally the acoustic pressure) are approximated using a weighted sum of predefined wave functions. These wave functions are exact solutions of the governing partial differential equation. A weighted residual formulation is written on the boundaries and interfaces to build the system of equations. The obtained system of equations is small and can be solved for the unknown contribution factors of each wave function. The method has shown its potential for acoustic, elastic and combined vibro-acoustic problems of a moderate geometrical complexity, however, can also be applied in a hybrid manner together with FEM, SEA and BEM [5].

3.1 General modelling concept

The general modelling concept of the WBM can be summarised in 4 steps, which are detailed in [3, 5]:

1. Partitioning of the considered problem domain into convex subdomains,
2. Selection of a suitable set of wave functions for each subdomain,
3. Construction of the WB system matrices via a weighted residual formulation of the boundary and interface conditions,
4. Solution of the system of equations, yielding the wave function contribution factors and postprocessing of the dynamic variables.

3.2 Application to poroelastic problems

To apply the concept of the WBM for poroelastic problems, exact solutions of the Biot equations (1)-(2) should be used in the approximation set. For this reason, the Biot equations are, in a first step, decoupled into three Helmholtz equations. This can be done by using a strain formulation or a potential formulation, based on the Helmholtz decomposition of a vector field [6]. The three resulting Helmholtz equations represent the three wave types that exist in poroelastic materials. In a similar fashion as for acoustic problems, wave functions can be defined for each of the three strains or potentials, fulfilling the associated Helmholtz equations. As such, the physics of the three wave fields in poroelastic materials is explicitly embedded in the numerical model. The dynamic fields in both phases, such as displacements and stresses, can be expressed in function of the three wave function sets by applying differential operators [6]. The system of equations is again constructed by minimising the errors on the boundaries and interfaces. By solving this system of equations, the unknown wave function contribution factors of each of the wave functions are obtained.
When the problem setting (the material properties, the boundary conditions, the excitation and the problem geometry) is axisymmetric, the problem can be solved by only considering a slice of the original problem. The wave

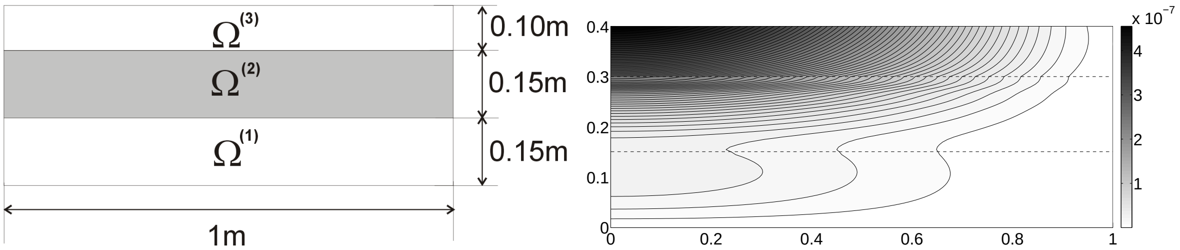
functions are then selected not only to fulfill one of the three resulting Helmholtz equations but also axisymmetry conditions [7].

4 NUMERICAL VALIDATION

In this validation section, two numerical cases are studied: one 2D Cartesian problem and one axisymmetric problem. The aim of this section is to indicate the potential of the WBM as compared to conventional methods.

4.1 2D Cartesian problem case

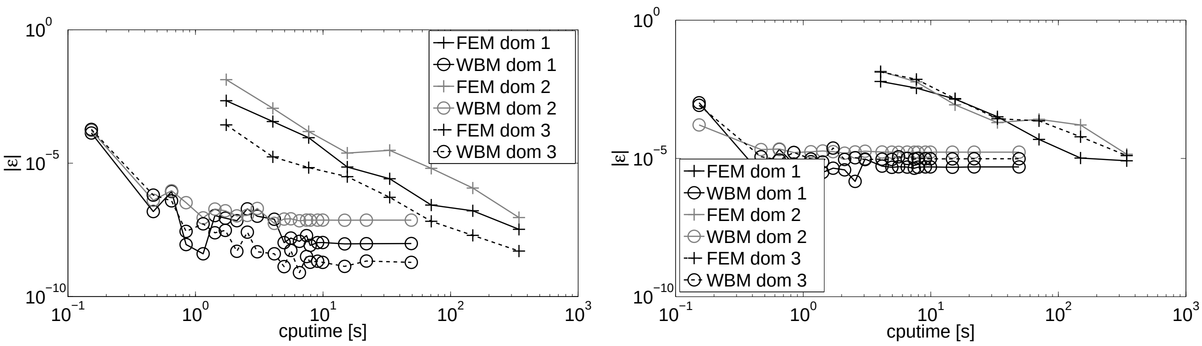
As illustrated in [6], for a variety of materials and boundary conditions, the dynamic behaviour of general 2D poroelastic domains can be accurately represented by the WBM. As an example, a multilayer consisting of three poroelastic layers as shown in Figure 1(a) is considered. The top and the bottom layer contain the same polyurethane foam and the middle layer consists of a carpet material. On all boundaries, except for the top one, sliding edge conditions [4] are imposed. On the top layer an acoustic pressure with the shape $p(x) = 2x^3 - 3x^2 + 1N/m^2$ excites the system.



(a) problem setting (b) relative flux at 600Hz

Figure 1: Left: problem setting, right: contour plot of the relative flux $h^{(\alpha)}(u_y^{f(\alpha)}(\mathbf{r}) - u_y^{s(\alpha)}(\mathbf{r}))[m]$ at 600Hz.

Figure 1(b) shows the contour map of the absolute value of the relative flux at 600Hz in the multilayer. The results are nicely continuous over the domain interfaces, indicated by dashed lines.
Figure 2 shows convergence curves for this validation case obtained with WB and adaptively refined cubic FE models for two poroelastic variables at 200Hz, respectively. The $(\mathbf{u^s},\mathbf{u^f})$-formulation has been applied. Although this formulation does not apply the least possible number of degrees of freedom per node, it gives an indication on the accuracy and calculation time which can be obtained with the WBM. The most detailed FE models are taken as a reference. The averaged relative error in 36 equally distributed postprocessing points in each layer as compared to this reference has been shown with respect to calculation time. A high convergence rate and good accuracies are obtained with the WBM. For this example, the WB convergence curves stagnate at the accuracy of the FE reference.

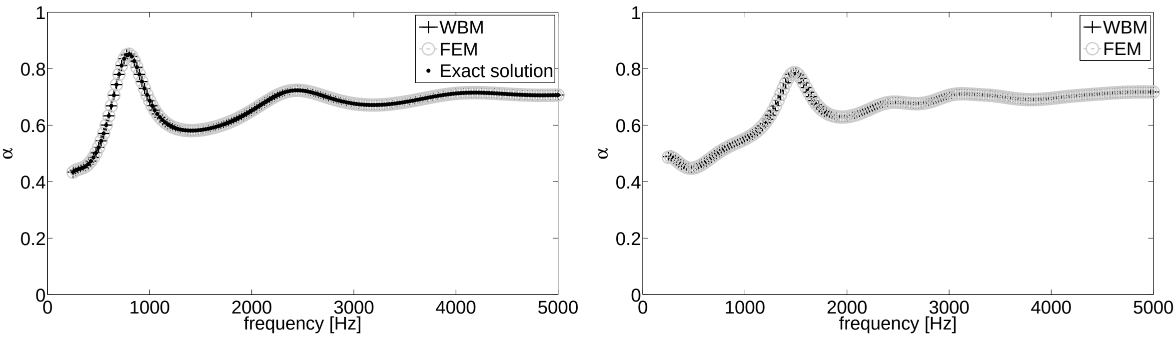


(a) $u_x^f(\mathbf{r})[m]$ (b) $\sigma_{xy}^s(\mathbf{r})[N/m^2]$

Figure 2: Convergence curves of $u_x^f(\mathbf{r})$ and $\sigma_{xy}^s(\mathbf{r})$ at 200 Hz in each of the three subdomains, calculated with the WB potential formulation and the FEM.

4.2 Axisymmetric problem case

Also for axisymmetric acoustic and poroelastic problems, high convergence rates and accuracies are obtained with respect to adaptively refined axisymmetric FE models. By combining axisymmetric acoustic and poroelastic models, for instance a Kundt's tube measurement device can be modelled. As such, the WBM can be applied to verify the effect of mounting conditions of the test sample on the absorption, as illustrated in Figure 3. the problem setting and material properties are given in [7]. The WB curves are on top of the FE results, showing that the same results can be reproduced, but with a much smaller calculation effort. When sliding edge conditions are imposed, also the analytical exact solution [2] is given.



(a) Sliding edge boundary condition

(b) Fixed edge boundary condition

Figure 3: Absorption coefficient of the porous material obtained with a Kundt tube set-up for two types of boundary conditions, as calculated with the WBM and the FEM.

5 CONCLUSIONS

This extended abstract shows the application of the WBM for poroelastic problems. The WBM is based on a Trefftz approach and uses exact solutions of three Helmholtz equations, resulting from the decoupling of the Biot equations. It incorporates *a priori* known information of the problem's physics in the numerical scheme. Two validation cases show the application of the WBM to 2D Cartesian and axisymmetric poroelastic problems. The methods shows high efficiency as compared to the FEM for problems with a moderate geometrical complexity.

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