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## Accelerating CFD via local POD plus Galerkin projection

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**Abstract.** Various methods involving local proper orthogonal decomposition (POD) and Galerkin projection are presented aiming at accelerating the numerical integration of nonlinear, time dependent, dissipative problems. The approach combines short runs with a given computational fluid dynamics (CFD) solver and low dimensional models constructed by appropriate POD modes, which adapt to the dynamics. Application to the approximation of transients in laminar fluid flows and bifurcation diagrams is discussed in terms of flexibility and efficiency.

**Keywords:** computational fluid dynamics; proper orthogonal decomposition; reduced order models.

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### 1 INTRODUCTION

Numerical solvers are paramount in the analysis of both scientific and industrial problems, e.g. in the conceptual design of many engineering systems in the aerospace sector involving fluid flows. A major difficulty is related to the huge computational resources often required by standard CFD approaches, which both delays cycles and increases costs. Analysis of flow instabilities and transitions is another relevant issue that may involve quite expensive numerical simulations, especially when the dynamics exhibit complex time dependent bifurcations. Therefore, new paradigms providing a satisfactory computational efficiency are highly needed.

Reduced order models based on POD have been developed during the last decades for a variety of applications [1, 2, 3] since they exhibit a good balance between reasonable accuracy and satisfactory efficiency. These models are constructed in three steps, namely by (i) CFD-computing some flow snapshots, (ii) extracting the most energetic POD modes from the snapshots, and (iii) projecting the governing equations onto the resulting POD manifold.

In this work, various ideas coming from a new approach to POD-based model reduction techniques are introduced and exploited to speed up the simulation of transient and asymptotic dynamics of unsteady dissipative fluid flows.

### 2 SIMULATION OF TRANSIENTS IN LAMINAR FLOWS

Relying on the continuous dependence of the POD manifold on time and some *a priori* estimates of the truncation error in a low dimensional approximation, the method sketched in Figure 1 is presented [4, 5].

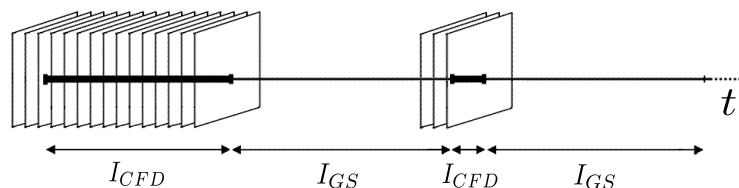


Figure 1: Sketch of the adaptive reduced order model to approximate temporal transients.

The proposed approach combines, in interspersed time intervals, short runs with a given CFD solver (providing some snapshots) and the time integration of a Galerkin system (GS). The latter is constructed by expanding the solution of the problem into appropriate POD modes (computed from the snapshots) and Galerkin-projecting some evolution equations onto that linear basis, which yields a reduced system of equations for the time dependent mode amplitudes. The POD manifold is completely calculated from the outset and only updated as time proceeds according to the dynamics. Updating is performed *on demand*, when the GS approximation fails, by suitably mixing ‘old’ and ‘new’ POD modes. In addition, a second ‘instrumental’ GS retaining a few more modes is time integrated, which allows to detect possible high-order modes truncation instabilities. The adaptive procedure turns out to be robust and efficient, also thanks to a non-standard inner product based on a limited number of mesh points (carefully selected in the CFD grid) that is used to perform POD plus Galerkin projection.

This is illustrated considering the two-dimensional *lid-driven cavity problem*, which describes the circulating flow in a square domain whose upper wall is moved by an external, horizontally directed forcing [6]. The driving velocity is allowed to depend on time, leading to nontrivial large-time dynamics at moderate values of the Reynolds number. Moreover, the flow snapshots are computed by a fairly rough but fast CFD code, which creates a demanding setting for the method. Results for values of the Reynolds number up to 800 show how periodic and quasi-periodic transient dynamics in the cavity can be approximated with accuracy comparable to that of the CFD solver in a much shorter CPU time. Specifically, acceleration factors up to 10 are obtained.

### 3 APPROXIMATION OF BIFURCATION DIAGRAMS

When the governing equations depend on some parameters, the POD modes computed for specific parameter values may be appropriate for other (different) values also. In other words, the POD manifold is quite robust and weakly dependent on the problem parameters. This heuristic property, together with the observation that transients may contain richer information on the dynamics than attractors, can be successfully exploited in bifurcation problems. Thus, the following method is proposed to efficiently compute the bifurcation diagrams of a dissipative system in a given bifurcation parameter span [7].

- (1) Apply POD to a set of snapshots associated with an orbit of the system computed by a time dependent CFD solver, for a fixed value of the bifurcation parameter  $\mu$ , a generic initial condition, and a (non-small) time interval.
- (2) Select the  $n$  most energetic POD modes, Galerkin-project the governing equations onto these modes, and construct the bifurcation diagram of the resulting GS.
- (3) Validate the results by repeating calculations with a larger value of  $n$  and comparing the outputs.

In order to illustrate the performance of the reduced order model, the one-dimensional *complex Ginzburg-Landau equation* (on the spatial domain  $(0, 1)$ , with homogeneous Neumann boundary conditions) is considered, which is a well-known paradigm of a simple equation showing intrinsically complex (e.g., quasi-periodic and chaotic) dynamics [8]. An example of the obtained results is shown in Figure 2. In this case, 100 snapshots are computed at  $\mu = 20$  and the 8 most energetic POD modes are retained. The bifurcation diagrams in  $0 < \mu \leq 300$  (here,  $\mu$  is the linear growth coefficient of the equation) provided by a CFD solver (top) and the constructed GS (bottom) are almost identical. The apparent discrepancies in  $55.6 < \mu < 75.3$  and  $143.6 < \mu < 234.2$  are artifacts of symmetry-breaking that may be encountered when computing bifurcation diagrams using two different methods, however close to each other the approximations are. Namely, in these intervals the CFD solver and the GS follow two branches that are reflection-symmetric between each other, which is appreciated by a cross (left-right) comparison between top and bottom plots in Figure 2. Approximation of the involved attractors (omitted here) is also quite good.

Flexibility of the strategy is fairly high, as similar results can be obtained by (i) fixing different values of  $\mu$  for the POD basis construction, (ii) extending the bifurcation parameter span to larger values of  $\mu$ , or (iii) using a rough CFD solver or even a slightly different equation to compute the necessary snapshots. Finally, simulations based on the low dimensional model turn out to be much computationally faster (e.g., 10 times faster for the case in Figure 2) than those performed by the used CFD solver.

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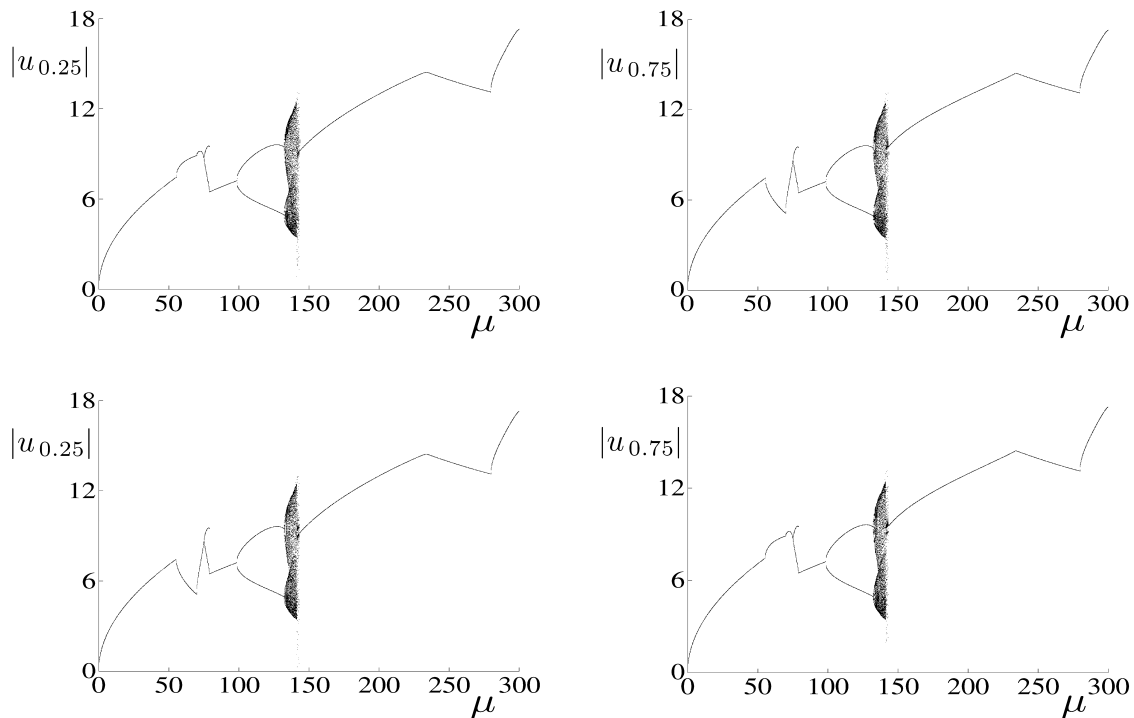


Figure 2: Forward bifurcation diagrams for the complex Ginzburg-Landau equation in  $0 < \mu \leq 300$ , showing the values of  $|u_{0.25}| \equiv |u(0.25, t)|$  (left plots) and  $|u_{0.75}| \equiv |u(0.75, t)|$  (right plots) at the outward intersections of the orbits with an appropriate Poincaré hypersurface, as obtained by a CFD solver (top plots) and the constructed GS (bottom plots).

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