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# Efficient finite element approach for structural-acoustic applications including 3D modelling of sound absorbing porous materials

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**Abstract.** *This thesis presents efficient finite element solutions of poro-elasto-acoustic coupled problems using modal-based approaches adapted to non conservative problems, combined with an interpolation method in frequency, based on Padé approximants. The performance of the proposed methods and their combination is demonstrated on several 1D to 3D coupled problems. Conclusions are drawn to provide indications for future research and tests to be conducted in order to further enhance the methodologies proposed in the thesis.*

**Keywords:** Structural-acoustics; Reduced order model; poroelastic materials, Padé approximants, Finite Elements.

## 1 INTRODUCTION

In the context of interior noise reduction, the present work aims at proposing Finite Element (FE) solution strategies for interior structural-acoustic applications including 3D modelling of homogeneous and isotropic poroelastic materials, under time-harmonic excitations, and in the low frequency range. A model based on the Biot-Allard theory [1] is used for the poroelastic materials, which is known to be very costly in terms of computational resources [2]. Reduced models offer the possibility to enhance the resolution of such complex problems. However, their applicability to porous materials remained to be demonstrated.

First, this work presents FE resolutions of poro-elasto-acoustic coupled problems using modal-based approaches both for the acoustic and porous domains. The original modal approach proposed for porous media [3], together with a dedicated mode selection and truncation procedure, are validated on 1D to 3D applications. In a second part, modal-reduced models are combined with a Padé approximant reconstruction scheme in order to further improve the efficiency. Conclusions are then drawn to provide indications for future research and tests to be conducted in order to further enhance the methodologies proposed.

## 2 THE MODAL APPROACH FOR SOUND ABSORBING POROUS MATERIALS

### 2.1 Real-valued modes for isotropic porous materials

The rewriting of the constitutive equations of an isotropic porous material developed by Biot [4], allowed for the convenient FE formulation of a coupled poro-acoustic problem, particularly involving four real-valued, frequency-independent porous matrices  $\mathbf{K}_p^{(1)}$ ,  $\mathbf{K}_p^{(2)}$ ,  $\mathbf{C}_p$  and  $\mathbf{M}_p$ ,

$$\begin{bmatrix} \mathbf{K}_{II} - \omega^2 \mathbf{M}_{II} & \mathbf{K}_{II} - \omega^2 \mathbf{M}_{II} & \mathbf{0} \\ \mathbf{K}_{II} - \omega^2 \mathbf{M}_{II} & \mathbf{K}_{II} - \omega^2 \mathbf{M}_{II} & -\omega^2 \mathbf{A}_{IP} \\ \mathbf{0} & -\mathbf{A}_{IP}^T & \mathbf{K}_p^{(1)} + (\tilde{K}_f - P_0) \mathbf{K}_p^{(2)} + i \omega \tilde{b} \mathbf{C}_p - \omega^2 \mathbf{M}_p \end{bmatrix} \begin{bmatrix} \mathbf{P}_I \\ \mathbf{P}_I \\ \mathbf{U}_p \end{bmatrix} = \begin{bmatrix} \omega^2 \mathbf{U}_{Ib} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (1)$$

where  $\mathbf{P}$  and  $\mathbf{U}_P$  correspond to the primary variables of a poro-acoustic problem, i.e. the acoustic pressure fluctuation in the fluid, and the fluid and solid phases displacement vector in the porous material, respectively. The subscripts  $I$  and  $\bar{I}$  correspond to a convenient separation of the acoustic degrees of freedom (dofs), according to Fig. 1.

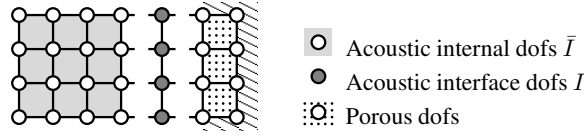


Figure 1: Problem description, separation of the degrees of freedom.

This decomposition allows to establish a real-valued porous eigenvalue problem from the porous coupled mass matrix  $\mathbf{M}_P$ , and the low frequency limit of the porous coupled stiffness matrix  $\mathbf{K}_P^{(1)}$ , i.e.  $(\mathbf{K}_P^{(1)} - \omega^2 \mathbf{M}_P) \phi = \mathbf{0}$ . From the solution of this problem, a truncated modal basis is generated, including the sorted lowest-frequency modes, and allowing, after projection, to reduce the size of the poroelastic unknown vector to a substantially smaller vector of modal coordinates. The convergence of the approach is illustrated in Fig. 3 for a small 3D poro-acoustic problem presented in Fig. 2

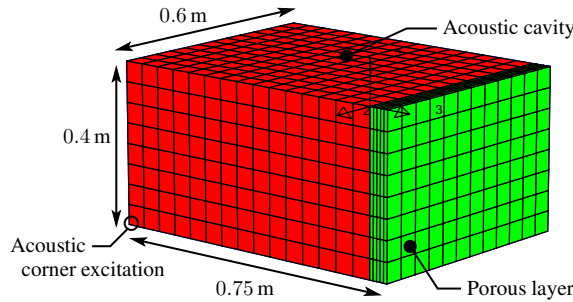


Figure 2: Mesh and dimensions of the 3D poro-acoustic problem.

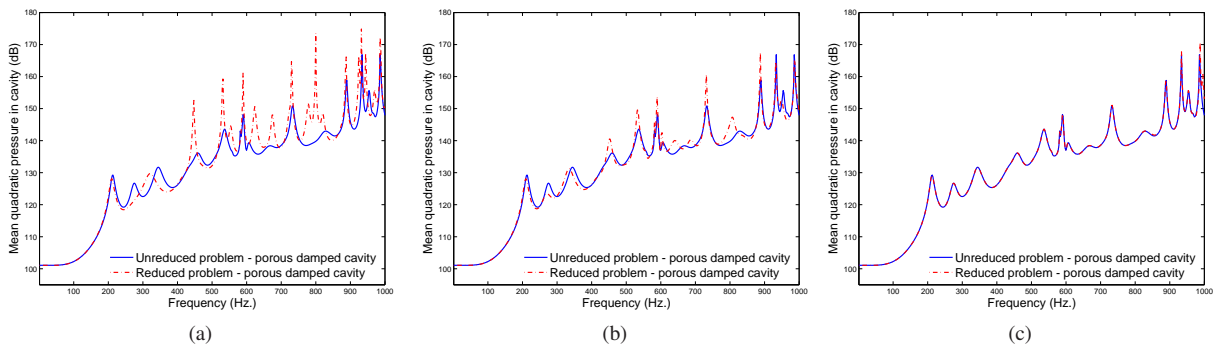


Figure 3: Mean quadratic pressure FRF, convergence of the reduced model: (a) 100, (b) 500, and (c) 800 modes.

While allowing substantial performance improvements –e.g. downsizing the porous domain from 1872 to 800 dofs in this small example–, the relatively large and unevenly contributing modal basis may be further reduced by a selection of the most physically significant modes.

## 2.2 A mode-selection procedure for convergence improvements

A mode sorting and selection procedure is thus proposed, in order to first sort the modes according to their region and significance of contribution in the frequency spectrum, and then, to apply a second truncation of the modal basis. The sorting of the modal basis is done by calculating the modes participation to residual vectors efficiently calculated from poor approximate solutions at several frequencies in the spectrum. This allows to further truncate the basis by

removing the modes with a negligible contribution to these residual vectors. In practice, although this is a problem-dependent outcome, a reduction of the modal basis of up to an order of magnitude has been observed in the few poro-acoustic problems tested.

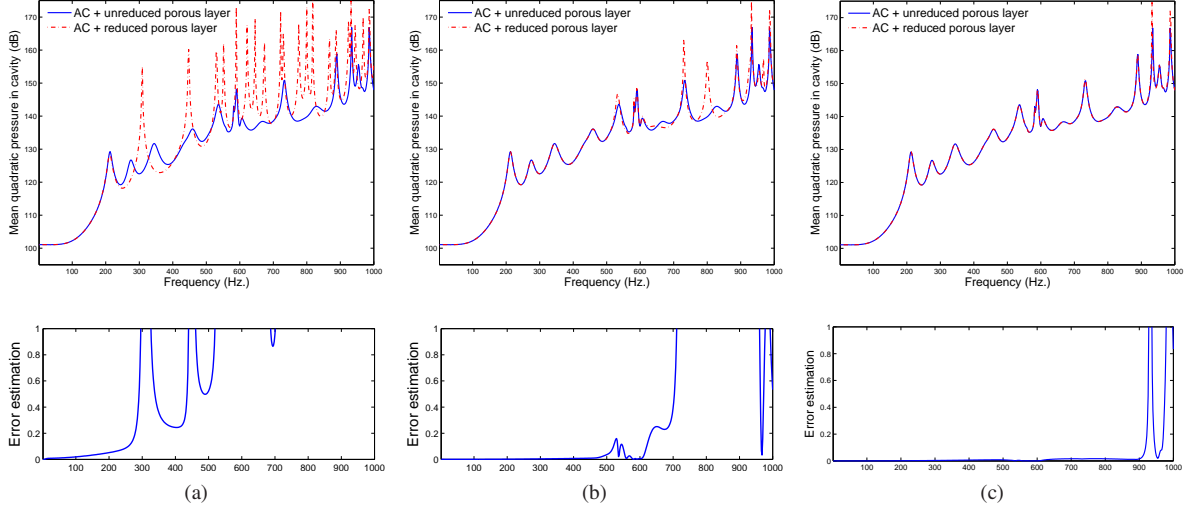


Figure 4: Convergence with modal superposition of selected modes for  $\chi_{\max} = 0.27$ : (a) 2, (b) 58, and (c) 83 modes

Fig. 4 illustrates the improved convergence for the poro-acoustic test case of the previous section, where the number of porous modal coordinates is reduced from 800 to 83.

### 3 FREQUENCY RESPONSE INTERPOLATION USING PADÉ APPROXIMANTS

The use of Padé approximants for the efficient reconstruction of frequency-responses is complementary to the modal approach previously presented. It takes advantage of the reduced number of unknowns in the system while reducing the number of frequencies at which the solution is to be calculated during the frequency sweep. The problems of interest are of the form

$$\mathbf{Z}(\omega)\mathbf{u}(\omega) = \mathbf{F}(\omega), \quad (2)$$

with for the previous test case, and for  $\omega \neq 0$ ,

$$\mathbf{Z}(\omega) = \frac{1}{\omega^2} \mathbf{K}_F + \mathbf{A} + \left( \tilde{K}_F(\omega) - P_0 \right) \mathbf{K}_P^{(2)} + i\omega \tilde{b}(\omega) \mathbf{C}_P - \omega^2 \mathbf{M}_P. \quad (3)$$

Each component of the solution vector may then be calculated at a restricted number of main frequencies –e.g. solution  $\mathbf{u}(\omega_0)$  at  $\omega_0$ –, and expanded using Padé approximants around these frequencies,

$$u(\omega_0 + \Delta\omega) \approx \frac{P_L(\Delta\omega)}{Q_M(\Delta\omega)}. \quad (4)$$

$P_L(\Delta\omega)$  and  $Q_M(\Delta\omega)$  are truncated Taylor series at the orders  $L$  and  $M$  respectively,

$$P_L(\Delta\omega) = \sum_{k=0}^L p_k(\Delta\omega)^k, \text{ et } Q_M(\Delta\omega) = \sum_{k=0}^M q_k(\Delta\omega)^k, \quad (5)$$

whose coefficients can be efficiently determined using a recursive calculation of the solution vector  $L + M$  first frequency derivatives,

$$\mathbf{Z}(\omega_0) \mathbf{u}^{(k)}(\omega_0) = \mathbf{f}^{(k)}(\omega_0) - \sum_{j=0}^{(k-1)} \binom{k}{j} \mathbf{Z}^{(k-j)}(\omega_0) \mathbf{u}^{(j)}(\omega_0), \text{ for } k = 1, \dots, (L + M), \quad (6)$$

and subsequently determining the Padé approximant [4].

The reconstruction procedure thus results in two steps for each main frequency: first, a full-size recursive problem to calculate the solution and its derivatives at the main frequency, and then a succession of  $(L + M + 1)$ -sized problems to be solved for each component of the solution vector. A procedure for the adaptive decomposition of the frequency spectrum in frequency intervals of reconstruction is also proposed, in order to reduce their number to a cost-efficient minimum. Fig. 5 presents the reconstruction of the previously presented poro-acoustic solution

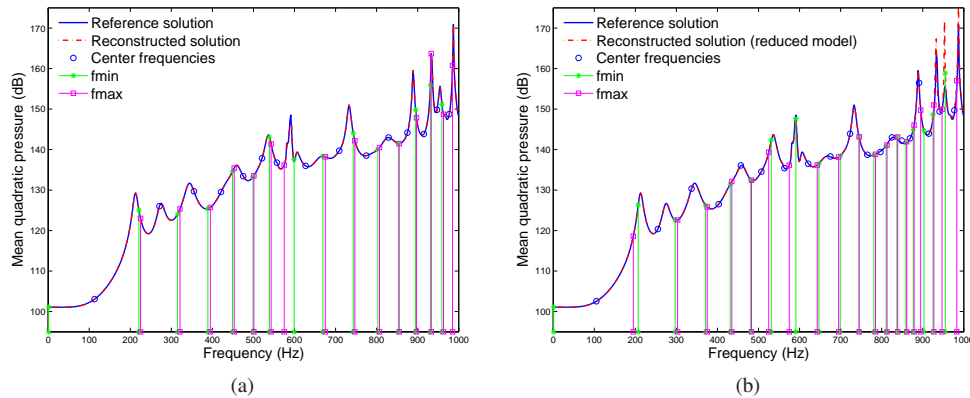


Figure 5: Adaptive reconstruction procedure applied to 3D problem with  $L = 5$ ,  $M = 6$ : (a) from non-reduced problem; (b) from reduced problem.

with a  $(L = 5; M = 6)$  Padé approximant, applied to either the non-reduced problem, or the modal-based reduced problem. Such a combination of the two approaches allows to gain at least an order of magnitude in terms of computational efficiency [4].

## 4 CONCLUSION

In the present work, an original modal approach was proposed for the efficient solution of acoustic problems including porous materials modelled with the Biot-Allard theory. An efficient combination with a Padé-based frequency interpolatory method was also developed, allowing to substantially enhance the computational time for the calculation of frequency responses. More details and applications, as well as several suggestions for further improvements are proposed in Ref.[4], some of which are currently under development.

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