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► To cite this version:

Andrew P. Bunger, Thierry Menand, Alexander Cruden, Xi Zhang, Henry Halls. Analytical Predictions for a Natural Spacing within Dyke Swarms. *Earth and Planetary Science Letters*, 2013, 375, pp.270-279. <10.1016/j.epsl.2013.05.044>. <hal-00855272>

HAL Id: hal-00855272

<https://hal.science/hal-00855272v1>

Submitted on 5 Sep 2013

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Analytical Predictions for a Natural Spacing within Dyke Swarms

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Abstract

Dykes often grow next to other dykes, evidenced by the widespread occurrence of dyke swarms that comprise many closely-spaced dykes. In giant dyke swarms, dykes are observed to maintain a finite spacing from their neighbors that is tens to hundreds of times smaller than their length. To date, mechanical models have not been able to clarify whether there exists an optimum, or natural spacing between the dykes. And yet, the existence of a natural spacing is at the heart of why dykes grow in swarms in the first place. Here we present and examine a mechanical model for the horizontal propagation of multiple, closely-spaced blade-like dykes in order to find energetically optimal dyke spacings associated with both constant pressure and constant

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influx magma sources. We show that the constant pressure source leads to an optimal spacing that is equal to the height of the blade-like dykes. We also show that the constant influx source leads to two candidates for an optimal spacing, one which is expected to be around 0.3 times the dyke height and the other which is expected to be around 2.5 times the dyke height. Comparison with measurements from dyke swarms in Iceland and Canada lend initial support to our predictions, and we conclude that dyke swarms are indeed expected to have a natural spacing between first generation dykes and that this spacing scales with, and is on the order of, the height of the blade-like dykes that comprise the swarm.

Keywords:

dyke swarms, dyke spacing, fluid-driven cracks, hydraulic fractures

1. Introduction

Dykes represent the dominant mode of magma transport through the Earth's lithosphere, and one striking feature is that they often occur as swarms made of several hundreds of individual, sub-parallel dykes originating from apparently a single source region. At the smallest scale, volcanic dyke systems originate from individual magma chambers, such as the Koolau dyke complex, Oahu, in Hawaii (Walker, 1986), the Spanish Peaks, Colorado (Odé, 1957), and the dyke swarms of Iceland (Gudmundsson, 1983; Paquet et al., 2007). At a larger scale, sheeted dyke complexes form an integral part of the crustal structure at mid-ocean ridges. At the largest scale, one finds giant mafic dyke swarms (Figure 1) that extend over hundreds to several thousands of kilometers in length (Ernst and Baragar, 1992). These giant

13 structures are found not only on Earth, where they are often associated with
14 continental breakup and flood basalts, but also on Mars and Venus (Halls
15 and Fahrig, 1987; Ernst et al., 2001). The width of these swarms is assumed
16 to reflect the lateral extend of their feeding source, usually thought to be
17 mantle plumes (e.g. Ernst et al., 2001).

18 Yet, in spite of their ubiquity, dyke swarms have been studied rather de-
19 scriptively. As a result, field data that could inform about the mechanics and
20 dynamics of dyke swarms remain scarce. The crustal dilation that is induced
21 or accommodated by a swarm is sometimes recorded at different locations
22 within that swarm (e.g. Walker, 1986; Hou et al., 2010), but most field studies
23 record only the strike and dip of the dykes, along with their length and thick-
24 ness distributions. Length distributions seem to be power-law (e.g. Paquet
25 et al., 2007, and references therein), whereas thickness distributions have
26 been variously described as power-law (e.g. Gudmundsson, 1995), negative-
27 exponential or log-normal (e.g. Jolly and Sanderson, 1995; Jolly et al., 1998).

28 Comparatively, data on dyke spacing are rarely reported. Jolly and
29 Sanderson (1995) demonstrate log-normal distribution of the dyke spacing
30 within the Mull Swarm, Scotland, and from this infer the existence of char-
31 acteristic length scale that is best described by the median or geometric mean
32 of the spacing. In a similar study, Jolly et al. (1998) examine the geometry of
33 clastic dykes in the Sacramento Valley, California. In this case the authors
34 interpret the dyke spacing to follow a power-law distribution, although it
35 should be noted that their discrimination between power-law and log-normal
36 behavior seems it was not carried out formally but rather relied on visual
37 assessment and is therefore prone to misinterpretation (e.g. Clauset et al.,

2009). Hence, the limited available data provide sufficient motivation to pursue model-derived insight into whether or not a characteristic length scale is expected to exist related to dyke spacing, and if so, what are its physical origins and significance.

The mechanics of dyke propagation and prediction of spacing between cracks in rocks have both received significant attention over the past few decades. On the one hand, the growth of a single dyke has been analyzed in a variety of combinations of geometry and boundary conditions (e.g. Lister, 1990; Mériaux and Jaupart, 1998; Roper and Lister, 2005; Taisne and Jaupart, 2009; Taisne et al., 2011). On the other hand, both analytical (e.g. Hobbs, 1967) and numerical (e.g. Narr and Suppe, 1991; Bai and Pollard, 2000; Olson, 2004) approaches have been applied for the purpose of predicting the spacing between opening mode cracks in layered rocks. But, while there has been a number of mainly industry-driven contributions aimed at understanding crack patterns and driving pressure associated with the growth of multiple hydraulic fractures (e.g. Germanovich et al., 1997; Zhang et al., 2007; Olson, 2008; Jin and Johnson, 2008; Olson and Dahi-Taleghani, 2009; Zhang et al., 2011; Roussel and Sharma, 2011; Bunger et al., 2012; Vermilyen and Zoback, 2011; Weng et al., 2011), the issue of optimal spacing between fluid-driven cracks for geometries and boundary conditions that are relevant to dyke propagation has not been addressed.

In this paper we ask whether there is evidence from mechanical analysis that dyke swarms should form with a particular inter-dyke spacing. This question is at the heart of the issue of why dykes should form swarms at all. If mechanical models predict a natural spacing that tends to zero or

63 infinity, then it remains fundamentally unclear why there is a widespread
64 morphology wherein many distinct dykes maintain a finite separation over
65 tens to thousands of kilometers of growth.

66 Whether mechanical analysis can identify a finite characteristic spacing
67 for dyke swarms is not apparent at the outset. There is a temptation to view
68 the problem in terms of fracture mechanics alone. But if we do this, we im-
69 mediately discover the well-known fact that closely-spaced pressurized cracks
70 exert compressive stresses on each other that reduce the stress intensity that
71 drives the fracturing process (e.g. Benthem and Koiter, 1973). Viewed this
72 way, it is unclear how dykes in a swarm can grow to be a hundred times
73 longer than the spacing between them.

74 One potential resolution to this issue is to suggest that the dykes must
75 form sequentially, with one dyke propagating after the next to eventually
76 form the observed dyke swarm morphologies. It seems reasonable that this
77 should be a part of the answer. However, crosscutting relationships observed
78 in the field indicate that contemporaneous as well as successive dyke em-
79 placement can be observed within the same swarm (Burchardt et al., 2011).
80 Moreover, the analysis of Bunger (2013) shows that multiple, simultane-
81 ously growing fluid-driven cracks can propagate to a length that is much
82 greater than their separation provided that the fluid driving them is suffi-
83 ciently viscous — which is to say that the energy dissipated in viscous flow
84 greatly exceeds the energy dissipated through breakage of the rock — and
85 provided that their growth in height is constrained so that they are much
86 longer than they are high and hence grow in the well-known blade-like geom-
87 etry (e.g Perkins and Kern, 1961; Nordgren, 1972; Rubin and Pollard, 1987;

88 Lister, 1990; Adachi and Peirce, 2008). Here we examine the mechanical evi-
89 dence for a natural, or optimal spacing within dyke swarms by extending the
90 method that has been previously developed by Bunger (2013) in order to ac-
91 count for both the asymptotic limits of widely and closely spaced swarms of
92 blade-shaped dykes under both constant pressure and constant influx source
93 conditions.

94 **2. Dyke Propagation Model**

95 We consider a model for an array of equally-spaced blade-like dykes that
96 are propagating horizontally through brittle host rock, as sketched in Figure
97 2. This model is justified for large dyke swarms that grow to be many
98 times greater in length than the thickness of the crust. Examples include
99 the Mackenzie swarm, the Matachewan swarm, the Grenville swarm, and the
100 Abitibi swarm, all in Canada, the Yakust swarm in Siberia, and the Central
101 Atlantic reconstructed swarm (Ernst et al., 1995, and references therein).

102 For the sake of simplicity, we assume the swarm is characterized by a
103 single spacing h between adjacent dykes (Figure 2), and we investigate how
104 this spacing h affects the propagation of the dykes. In this regard, we neglect
105 the details of the source geometry and the radial propagation of dykes near
106 the source and instead focus on the parallel propagation in a regime that is
107 taken to persist after an early time, source geometry dominated period of
108 growth. Subject to this geometric limitation, details of dyke initiation and
109 early growth wherein the dyke length R is not substantially greater than the
110 height H will not be considered. Practically, the model is valid when R is at
111 least 3 to 5 times greater than H (Adachi and Peirce, 2008). When this is the

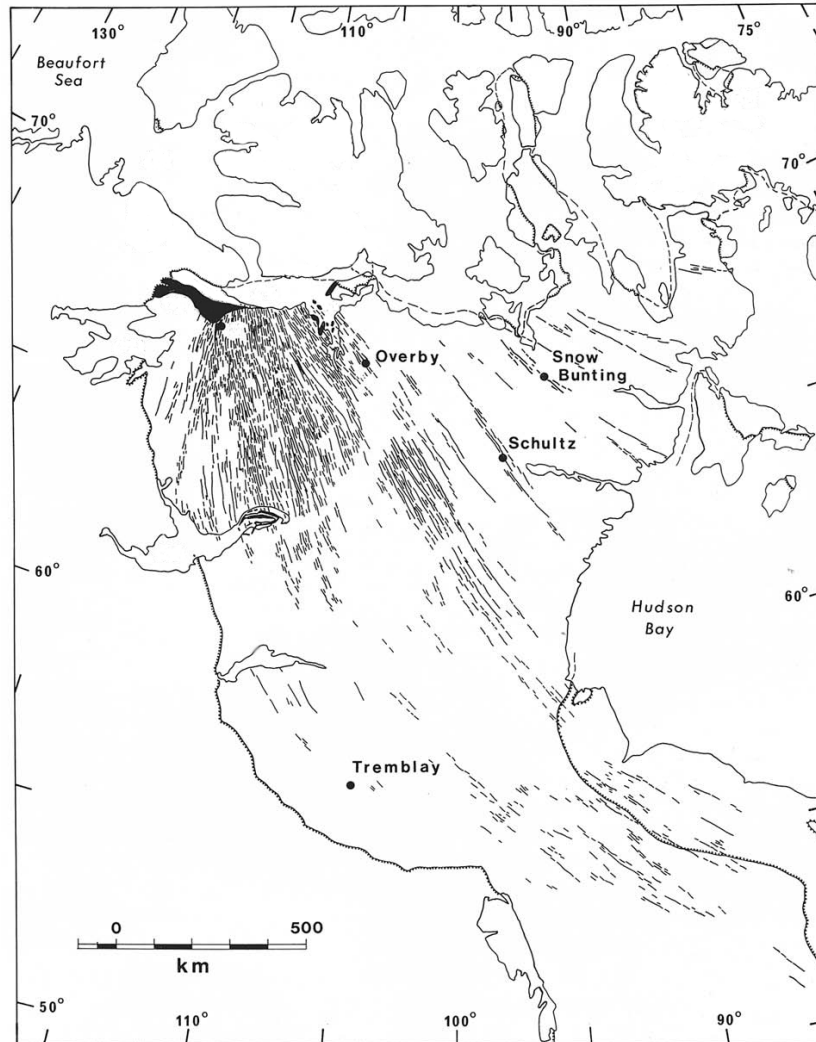


Figure 1: The 1270 Ma giant Mackenzie mafic dyke swarm in the northwestern Canadian Shield (after LeCheminant and Heaman (1989)), whose dykes extend over more than 2,000 km with an average thickness of 30 m (Fahrig, 1987).

case, it is valid to assume (Nordgren, 1972): 1) fluid flow to be unidirectional and along the x direction in Figure 2, that is, parallel to the direction of dyke propagation, and 2) pressure to be uniform within each vertical $y - z$ planar cross section of the hydraulic fracture with the pressure and thickness related according to a local, plane strain condition. The elasticity relation between net pressure ($p = p_f - \sigma_o$ for minimum in situ stress σ_o and total magma pressure p_f) and thickness (w) along center line of the dyke ($y = 0$) is thus given by

$$w(x, t) = \alpha_1 H \frac{p(x, t) - \sigma_I}{E'}, \quad (1)$$

where $E' = E/(1 - \nu^2)$ for Young's modulus E and Poisson's ratio ν , and σ_I is the compressive stress exerted on the dyke by its neighbors, which is approximated for the widely-spaced case $H \ll h \ll R$ as (Benthem and Koiter, 1973; Bungler, 2013)

$$\sigma_I = p \frac{3H^2}{8h^2} (1 + O(h/H)^{-2}), \quad (2)$$

where the classical “Big O” notation is used to indicate the limiting behavior of the series. Similarly for the closely spaced case $h \ll H \ll R$ (Supplementary Section 1) the interaction stress is approximated by

$$\sigma_I = p \left(1 - \frac{4h}{H} + O(h/H)^3 \right). \quad (3)$$

Also, $\alpha_1(H/h)$ is a factor that accounts for interaction where

$$\alpha_1(H/h) \sim \begin{cases} 2, & H/h \ll 1 \\ 0.35, & H/h \gg 1 \end{cases}$$

with the large spacing limit ($H/h \ll 1$) readily available from the solution for a single, pressurized crack in plane strain (Sneddon, 1946), and the small

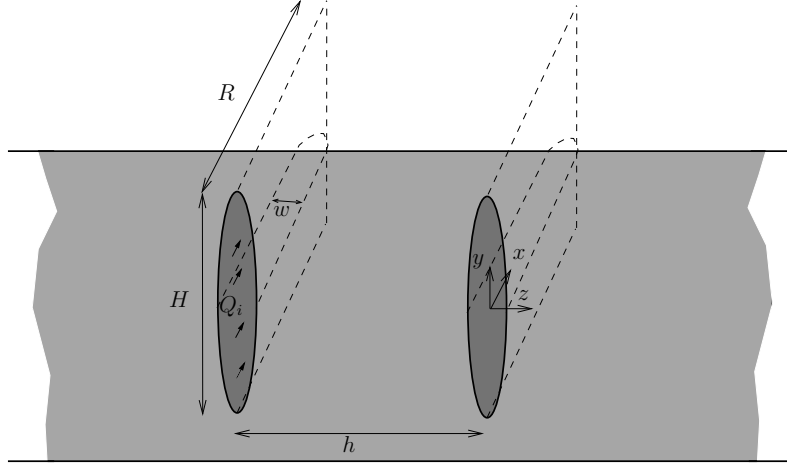


Figure 2: Sketch of the model geometry, showing two members of an infinite array of blade-like dykes.

spacing ($H/h \gg 1$) limit determined numerically, as detailed in Supplementary Section 2.

Assuming the magma is incompressible, fluid continuity, which comprises the second governing equation, is given by (Nordgren, 1972)

$$\alpha_2 H \frac{\partial w}{\partial t} + \frac{\partial q}{\partial x} = 0, \quad (4)$$

where $q(x, t)$ is the volume rate of flow through a cross section, once again w is the opening along $y = 0$, and $\alpha_2(H/h)$ is a factor that behaves like

$$\alpha_2(H/h) \sim \begin{cases} \frac{\pi}{4}, & H/h \ll 1 \\ 1, & H/h \gg 1 \end{cases}$$

with the large spacing limit ($H/h \ll 1$) arising from the area of an elliptical cross section ($\pi w H/4$) and the small spacing ($H/h \gg 1$) limit coinciding with a rectangular cross-section, which is taken as an approximation of the

139 cross section of the dyke in this case, as demonstrated in Supplementary
140 Section 2.

141 The third governing equation is the Poiseuille equation relating the fluid
142 flux to the fluid pressure gradient. This equation results from solution of
143 the Navier-Stokes equations for laminar flow of a Newtonian fluid subjected
144 to no-slip boundary conditions at the boundaries of the channel and where
145 the thickness of the flow channel is much less than its length. The result is
146 (Nordgren, 1972)

$$q = -\alpha_3 \frac{Hw^3}{\mu'} \frac{\partial p}{\partial x}, \quad (5)$$

147 where $\mu' = 12\mu$ and μ is the dynamic viscosity of the magma, and

$$\alpha_3(H/h) \sim \begin{cases} \frac{3\pi}{16}, & H/h \ll 1 \\ 1, & H/h \gg 1 \end{cases}$$

148 where the large spacing limit ($H/h \ll 1$) arises from integrating the flux
149 over an elliptical cross section and the small spacing ($H/h \gg 1$) limit arises
150 from integrating the flux over an approximately rectangular cross section. It
151 should be noted, however, that in the present work we are concerned with
152 orders of magnitude so that what is important is not the precise values of
153 α_1 , α_2 , α_3 , but rather that we have confirmed these to be order one.

154 The leading edge of the dyke requires a condition governing its propaga-
155 tion. However, one of the well-known deficiencies of the approach of Perkins
156 and Kern (1961) and Nordgren (1972) to modeling blade-like hydraulic frac-
157 tures is that the stresses are not well-defined in the near-tip region, therefore
158 precluding a well-defined propagation condition. A recent asymptotic analy-
159 sis of the full elasticity equation by Adachi and Peirce (2008) provides a way

160 forward, however a fluid-driven blade-like crack model has yet to be devel-
 161 oped. But a lack of such a model is not important for our analysis provided
 162 that we assume that the energy dissipated by flow of the viscous fluid is much
 163 larger than the energy that is dissipated by rock fracture (after e.g. Lister,
 164 1990; Lister and Kerr, 1991). It follows that if we are in viscosity dominated
 165 conditions, the scaling and energy relations that are subsequently derived
 166 will not depend on this moving boundary condition at the dyke tip.

167 Finally, assuming that the behavior for $R \gg H$ (long blade-like dykes)
 168 does not depend on the details of the initial conditions, these can be ne-
 169 glected for now. The system of equations is thus completed by homogeneous
 170 boundary conditions on the thickness and magma flux at the leading edge

$$x = R : \quad w = 0, \quad q = 0. \quad (6)$$

171 and the magma source condition, which is discussed in the following section.

172 **3. Magma Source Condition**

173 The source is idealized as a time varying volume of magma ($V(t)$) that is
 174 characterized by a compressibility C_m that describes the change in pressure
 175 associated with a given change in stored magma volume. The source is
 176 overpressurized relative to the minimum component of the in situ stress σ_o
 177 by a time dependent amount

$$p_o(t) = p_o(0) + \frac{V_r(t) - V_d(t)}{V(0)C_m}. \quad (7)$$

178 Hence, $p_o(0)$ and $V(0)$ are the source overpressure and volume at the start
 179 of dyke growth and V_d and V_r are the total volume injected into dykes and

180 added to the source region through recharge processes, respectively. Giant
 181 dyke swarms are usually thought to be fed by mantle plumes (Ernst et al.,
 182 2001), and so the recharge processes envisaged here would be the supply of
 183 magma from the tail to the head of these mantle plumes.

184 The total volume is thus given by $V(t) = V(0) + V_r(t) - V_d(t)$. Letting
 185 $Q(t) = q(0, t)$ be the volumetric flow rate out of the source and into the
 186 dykes, and $Q_r(t)$ be the recharge rate of the source region, we have

$$p_o(t) = p_o(0) + \frac{1}{V(0)C_m} \int_0^t (Q_r - Q) dt. \quad (8)$$

187 This description of the source leads naturally to consideration of two
 188 limiting cases. The first is for an infinitely large and compressible source,
 189 where we are left with a constant pressure condition

$$x = 0 : \quad p = p_0 = p_o(0), \quad V(0)C_m \rightarrow \infty. \quad (9)$$

190 Obviously, for $Q_r \neq Q$, this boundary condition is associated with time being
 191 sufficiently small so that the second term on the right hand side of Eq. (8)
 192 vanishes relative to p_0 .

193 On the other hand, the small, incompressible source limit is most clearly
 194 represented by differentiating Eq. (8) with respect to time to obtain

$$Q = Q_r - V(0)C_m \frac{dp_o(t)}{dt}, \quad (10)$$

195 where it is clear, then, that the source boundary condition is

$$x = 0 : \quad q = Q_r, \quad V(0)C_m \rightarrow 0, \quad (11)$$

196 which is a condition of constant influx if we further assume $Q_r(t) = Q_o$,
 197 a constant. Furthermore, it is apparent from Eq. (10) that the constant

influx condition is associated with large time if dp_o/dt decays with time – for example if $p_o \sim t^b$ for $b < 1$.

We note, however, that dyke flow rates Q are usually several orders of magnitude greater than their source recharge rates Q_r . For instance, studies of long-term magma supply rate at Kilauea, Hawaii (Swanson, 1972) and Krafla, Iceland (Johnsen et al., 1980) give $Q_r \sim 1 - 5 \text{ m}^3 \text{ s}^{-1}$. Estimates of dyke flow velocities are in the range 0.1 - 1 m/s (Brandsdóttir and Einarsson, 1979; Peltier et al., 2007; Ayele et al., 2009; White et al., 2011), which would amount to average volumetric flow rates $Q \sim 10^2 - 10^4 \text{ m}^3 \text{ s}^{-1}$ for horizontally propagating dykes that are 1 m wide and 1-10 km high. This range of values reflects the requirement that dykes need to propagate fast enough through the Earth’s crust to avoid death by solidification: continued magma flow in dykes requires a minimum dyke width hence magma flow rate for the advective supply of heat by flowing magma to be able to offset the heat conducted away by the colder host rocks (Bruce and Huppert, 1989; Petford et al., 1993). This range of values agrees with the volumetric flow rates estimated for the 1783-1785 Laki eruption in Iceland ($100 - 9000 \text{ m}^3 \text{ s}^{-1}$, Thordarson and Self, 1993), the magmatic activity in Hawaii in the 1970s ($1 - 700 \text{ m}^3 \text{ s}^{-1}$, Wright and Tilling, 1980; Duffield et al., 1982), the September 1984 eruption of Krafla, Iceland ($10 - 10^3 \text{ m}^3 \text{ s}^{-1}$, Tryggvason, 1986), or the 2003 magmatic activity at Piton de la Fournaise, Réunion Island ($10 - 700 \text{ m}^3 \text{ s}^{-1}$, Peltier et al., 2007). Some of these volumetric-flow-rate estimates are eruption rates and are observed to decline with time, whereas dyke intrusions might involve more constant rates (e.g. Peltier et al., 2007; Traversa et al., 2010). Moreover, one could argue that volumetric flow rates Q for giant

dyke swarms would be even greater than these reported values due to the larger average thickness of their dykes. This being the case, Q would have to be derived mainly from the stored volume, hence the infinitely large and compressible source, Eq. (9) is probably applicable to many, if not most, dyke swarms.

4. Energy Considerations

For a compressible magma source, the elastic strain energy (\mathcal{E}) is increased by the work done on the magma source by the recharge (W_r) and work done on the source by the in situ stress (W_{so}), and it is decreased by the work done by the magma source on the array of dykes (W_{df}). Energy conservation thus requires

$$\dot{\mathcal{E}} = \dot{W}_r + \dot{W}_{so} - \dot{W}_{df}, \quad (12)$$

where the overdot indicates the time derivative and, following Lecampion and Detournay (2007), it is easy to show that $\dot{W}_r = Q_r p_f$, $\dot{W}_{df} = Q p_f$, and $\dot{W}_{so} = \sigma_o(Q - Q_r)$. Hence

$$\dot{\mathcal{E}} = (Q_r - Q) p. \quad (13)$$

For the infinitely compressible source, that is, when $p = p_0$ at the inlet according to Eq. (9), maximizing the rate of decrease in stored elastic energy in the magma source corresponds to maximizing Q (when Q_r is a constant). The first of two energy conjectures, then, is that dyke systems associated with infinitely compressible sources will energetically favor configurations that maximize $-\dot{\mathcal{E}}$, and therefore growth geometry that maximizes the magma

influx rate to the dykes Q will be considered advantageous. What's more, if $Q_r \ll Q$, as indicated by field data, we have $-\dot{\mathcal{E}} \sim Qp$ so that it makes sense to focus on quantifying what we will call the “net dyke propagation work rate”, $\dot{W}_d = Qp$.

On the other hand, for an incompressible source, fluid can neither be stored nor mobilized from storage, hence $Q = Q_r$ (Eq. 11). So it is obvious that $\dot{\mathcal{E}} \equiv 0$ and therefore we cannot consider the change in strain energy of the source as we did when it was compressible. In this case, we follow Bungler (2013) and consider the rate of work done on the dykes $\dot{W}_{df} = Qp_f$. The second energy conjecture is that dyke swarms associated with incompressible sources will energetically favor configurations that minimize \dot{W}_{df} , and therefore growth geometry that minimizes the pressure required to drive growth at a fixed rate of influx ($Q(t) = Q_o$) will be considered advantageous. Furthermore, when the in situ stress σ_o is a constant, the minimum of \dot{W}_{df} coincides with the minimum of $\dot{W}_d = Qp$, so that once again it is sensible to focus on quantifying the dyke propagation work rate, \dot{W}_d .

Ongoing studies are required to better understand the conditions under which these conjectures are valid. When the overall geometry of a dyke swarm is relatively simple, they seem reasonable. However, when the dyke patterns become more complicated, the energy conjectures may not always hold. For example, mine-through mapping of hydraulic fracture growth through rock masses that contain natural fractures has shown the hydraulic fracture path can offset as it grows through some of the discontinuities so that the final fracture is not planar, but rather follows a stair-like morphology (Jeffrey et al., 2009). The available 2D modeling (Jeffrey et al., 2009) shows that

268 these offsets lead to an increase in the wellbore pressure relative to the case
 269 of planar growth for a given injection rate. This implies that the pattern of
 270 hydraulic fracture growth does not always result in a final configuration that
 271 would be predicted from global, equilibrium energy considerations. Instead,
 272 the morphology, or pattern of hydraulic fractures, appears to be determined
 273 by local interaction laws that determine the evolution of the system to at-
 274 tain a final configuration that cannot in general be predicted from simply
 275 considering global, equilibrium energy minimization.

276 These caveats aside, it is prudent to investigate a relatively simple dyke
 277 swarm geometry as a starting point from which we can understand if, in fact,
 278 the mathematical model implies the existence of an energetically optimal
 279 spacing between the dykes and to determine how this spacing depends on
 280 the nature of the source.

281 **5. Approximating the Energy Rate**

282 We consider a uniform array of blade-like dykes originating from the same
 283 source and maintaining a constant spacing and equal lengths as they grow.
 284 In the absence of a fully coupled model that accounts for all of the mechanical
 285 interactions among the dykes, a straightforward method for estimating the
 286 “input power” \dot{W}_d based on scaling relationships can be used. Following
 287 Bungler (2013), the input power required to propagate a swarm of N growing
 288 dykes can be expressed as

$$\dot{W}_d = \sum_{i=1}^N \dot{W}^{(i)}, \quad \dot{W}^{(i)} = \dot{U}^{(i)} - \dot{W}_I^{(i)} + D_c^{(i)} + D_f^{(i)}. \quad (14)$$

289 Which is to say that the input power to each dyke increases the strain energy
 290 in the host rock $\dot{U}^{(i)}$, overcomes the work that is done on that dyke by the
 291 stresses induced by the others $\dot{W}_I^{(i)}$, or is dissipated either through rock frac-
 292 ture $D_c^{(i)}$ or viscous flow of the magma $D_f^{(i)}$. Recalling that our consideration
 293 is limited here to viscosity dominated hydraulic fractures, we only consider
 294 cases wherein $D_c^{(i)} \ll D_f^{(i)}$. Hence the contribution of $D_c^{(i)}$ to Eq. (14) can
 295 be neglected for the present study (see Bunger (2013) for a more thorough
 296 discussion).

297 For the case of a uniform array of dykes that are at the onset of interaction
 298 such that $h \gg H$, Bunger (2013) shows that

$$\begin{aligned}\dot{U}^{(i)} &\approx \frac{LPXH}{t}, & \dot{W}_I^{(i)} &\approx -\frac{LPXH}{t} \left(\frac{H^2}{h^2} + O(H/h)^4 \right), \\ D_f^{(i)} &\approx \frac{X^3 P^2 H}{L\mu'} \left(1 + \frac{H^2}{h^2} + O(H/h)^4 \right).\end{aligned}\quad (15)$$

299 Here L , P , and X are characteristic quantities that estimate the dyke length,
 300 the magma over pressure, and the dyke thickness, respectively. The form of
 301 Eq. (15), then, clearly shows that $\dot{W}_I^{(i)}$ is negligible as $h/H \rightarrow \infty$, that is,
 302 for very widely spaced dykes, and its importance is greater for smaller dyke
 303 spacing. Before moving on to obtain $\{L, P, X\}$ from the governing equations,
 304 let us also present the approximations for the terms in Eq. (14) for the case
 305 of closely spaced dykes ($h \ll H$),

$$\begin{aligned}\dot{U}^{(i)} &\approx \frac{LPXH}{t}, & \dot{W}_I^{(i)} &\approx -\frac{LPXH}{t} \left(1 + \frac{h}{H} + O(h/H)^2 \right), \\ D_f^{(i)} &\approx \frac{X^3 P^2 H}{L\mu'} \left(1 + \frac{h}{H} + O(h/H)^2 \right).\end{aligned}\quad (16)$$

306 The governing equations (Eqs. 1-11) directly lead to appropriate expres-
 307 sions for L , P , and X . A useful technique (after Detournay (2004)) is to

308 substitute

$$w = X\Omega, \quad p = P\Pi, \quad R = L\gamma, \quad (17)$$

309 whereupon the objective becomes to define $\{X, P, L\}$ such that the dimen-
 310 sionless quantities $\{\Omega, \Pi, \gamma\}$ are all of order one ($O(1)$). For example, in
 311 the case of an infinitely compressible source with widely-spaced dykes, the
 312 inlet boundary condition (Eq. 9) tells us that $\Pi = O(1)$ if we take $P = p_0$.
 313 Then, substituting into the elasticity equation (Eq. 1), we can ensure that
 314 $O(\Omega) = O(\Pi)$ (and hence $\Omega = O(1)$) by taking $X = HP/E'$. Finally, the
 315 characteristic dyke length is obtained by first substituting the Poiseuille equa-
 316 tion (Eq. 5) into the continuity equation (Eq. 4) along with aforementioned
 317 values of P and X . The characteristic length L is then chosen so that the
 318 two terms of the continuity equation are guaranteed to be of the same order,
 319 which is to set the group of parameters that appears after the substitution
 320 to one. The result is $L = Hp_0^{3/2}t^{1/2}/(E'\mu^{1/2})$.

321 The procedure can be repeated for each of the four limiting regimes that
 322 come from the widely and closely spaced limits for infinitely compressible and
 323 incompressible sources, respectively. This scaling procedure is both straight-
 324 forward and it has been discussed at length in a number of prior contributions
 325 (see Detournay (2004) for a review), hence the details are omitted. The re-
 326 sulting characteristic quantities are summarized in Table 1. Substituting
 327 these quantities into the appropriate choice of Eq. (15) or (16) and summing
 328 according to Eq. (14) provides a rapid way of estimating the total input
 329 power required to sustain the growth of a swarm of dykes.

Source Condition	Spacing	X	P	L
$p = p_0$	$h \gg H$	$\frac{Hp_0}{E'}$	p_0	$\frac{Hp_0^{3/2}t^{1/2}}{E'\mu^{1/2}}$
$p = p_0$	$h \ll H$	$\frac{hp_0}{E'}$	p_0	$\frac{hp_0^{3/2}t^{1/2}}{E'\mu^{1/2}}$
$q = Q_o$	$h \gg H$	$\left(\frac{Q_i^2\mu't}{E'H}\right)^{1/5}$	$\left(\frac{E'^4Q_i^2\mu't}{H^6}\right)^{1/5}$	$\left(\frac{E'Q_i^3t^4}{H^4\mu'}\right)^{1/5}$
$q = Q_o$	$h \ll H$	$\left(\frac{hQ_i^2\mu't}{E'H^2}\right)^{1/5}$	$\left(\frac{E'^4Q_i^2\mu't}{h^4H^2}\right)^{1/5}$	$\left(\frac{E'Q_i^3t^4}{hH^3\mu'}\right)^{1/5}$

Table 1: Scaling factors that estimate the dyke thickness X , magma net pressure P , and dyke length L for the four limiting regimes, where the $q = Q_o$, $h \gg H$ case comes from Nordgren (1972).

6. Constant Pressure Limit

For the constant inlet pressure limiting case the applicable energy conjecture is that the dyke configuration that maximizes the rate of work done by the magma source on the dyke swarm will be energetically advantageous (Section 4). By this statement, searching for an optimum spacing between the dykes is synonymous with searching for a spacing that maximizes \dot{W}_d (Eq. 14).

Because we are limiting consideration to a uniform array of dykes, the summation in Eq. (14) can be expressed simply as $\dot{W}_d = N\dot{W}^{(i)}$, where $\dot{W}^{(i)}$ is the input power required to propagate one dyke in the array. Furthermore, it is not physically reasonable to let the width of the swarm grow unconstrained as would be the case if h and N were both unconstrained. Rather, natural dyke swarms are usually observed to cover a zone of some finite width (Halls and Fahrig, 1987; Ernst et al., 2001; Paquet et al., 2007). For example, this finite width, Z , can be considered to be on the order of the lateral extent of the magmatic source feeding the swarm. This being the

346 case, the swarm width Z , the number of dykes N , and their spacing h are re-
 347 lated as $h = Z/(N - 1)$, which for $N \gg 1$ can be approximated as $h \approx Z/N$,
 348 so that $\dot{W}_d \approx (Z/h)\dot{W}^{(i)}$. Taking the approximations from Eq. (15) and
 349 characteristic quantities from Table 1, the input power for the widely-spaced
 350 ($h \gg H$) regime is

$$\dot{W}_d \approx \frac{H^3 p_0^{7/2} Z}{h E'^2 \mu^{1/2} t^{1/2}} (1 + O(H/h)^2). \quad (18)$$

351 On the other hand, for the closely-spaced ($h \ll H$) regime, the approxima-
 352 tions from Eq. (16) lead to

$$\dot{W}_d \approx \frac{h H p_0^{7/2} Z}{E'^2 \mu^{1/2} t^{1/2}} (1 + O(h/H)). \quad (19)$$

353 These two expressions hold a number of important insights regarding
 354 the behavior of the problem under consideration. Firstly we can see that
 355 \dot{W}_d decreases with time for a fixed initial number of dykes N_0 . This is an
 356 intriguing result because it means that at some time it will be advantageous,
 357 that is, in the sense of causing an increase in \dot{W}_d , to initiate new dykes in
 358 the spaces between the initial dykes. And after some time with these two
 359 generations of dykes growing, it could become advantageous again to initiate
 360 a third generation of dykes growing in the spaces between the existing dykes.

361 It is important to realize, then, that field observations, especially in the
 362 vicinity of the source, can be expected to show a dyke spacing that is less
 363 than the predictions from our analysis. Also, calculations of median or mean
 364 dyke spacings across an entire swarm will be smaller than what is predicted
 365 here. So to summarize: 1) the subsequent analysis in this paper provides
 366 an estimate of the spacing between dykes in the first generation, and 2)

the dependence of \dot{W}_d on t as shown in Eqs. (18) and (19) suggests that subsequent generations can be expected to form leading to hierarchical sets of dykes within the swarm. Clearly a simulator of dyke swarm growth that is able to capture this complex behavior, and especially the point at which the system prefers to initiate new, infilling dykes rather than to continue growing the original array of dykes, would be a highly valuable tool for further investigation of this anticipated phenomenon.

It is also useful to identify a characteristic work rate (\dot{W}_d^*) that emerges when $h \approx H$ given by

$$\dot{W}_d^* = \frac{H^2 p_0^{7/2} Z}{E'^2 \mu'^{1/2} t^{1/2}}. \quad (20)$$

Recalling that $\dot{W}_d \approx p_0 Q$, we can therefore estimate the total rate of influx to the swarm from the magma source when $h \approx H$ as

$$Q \approx \frac{H^2 p_0^{5/2} Z}{E'^2 \mu'^{1/2} t^{1/2}}, \quad h \approx H. \quad (21)$$

By integrating Q with respect to time we can obtain an estimate of the volume of the swarm, V , given by

$$V \approx \frac{H^2 p_0^{5/2} Z t^{1/2}}{E'^2 \mu'^{1/2}}, \quad h \approx H. \quad (22)$$

Note that the factor of 2 that arises from the integration of Q has been dropped because it is spurious in light of the fact that these quantities are intended to estimate order of magnitude, not to provide precise predictions.

Most importantly, though, Eqs. (18) and (19) provide insight into the dependence of \dot{W}_d on the spacing h . As a visual approach, we have normalized both expressions by \dot{W}_d^* (Eq. 20) and plotted the resulting normalized input

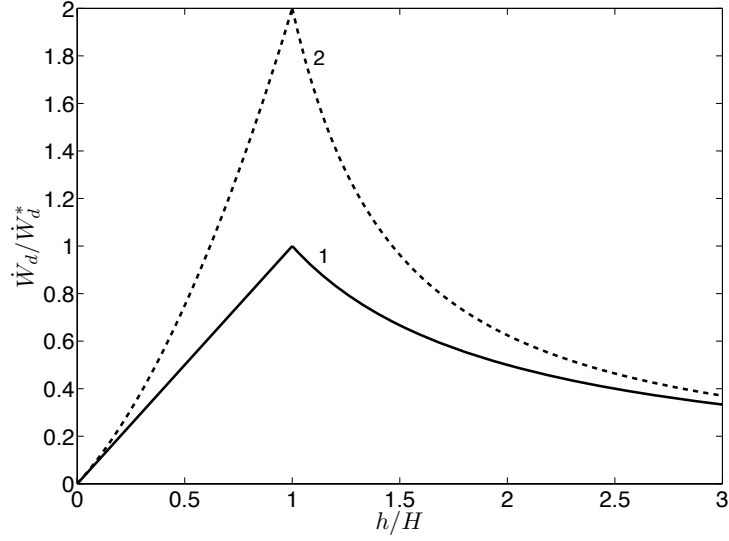


Figure 3: Normalized input power \dot{W}_d to the dyke swarm for the case of constant pressure at the source, where the numerical label indicates the number of terms retained in the asymptotic series.

power as a function of h/H in Figure 3. This result, and indeed direct inspection of Eqs. (18) and (19), shows that \dot{W}_d increases with decreasing h for $h \gg H$ and decreases for decreasing h for $h \ll H$, with suggestion of a sharp peak at $h \approx H$. Therefore, we conclude that a dyke swarm that is driven by a constant pressure source will have an optimum (first generation) dyke spacing of $h \approx H$.

7. Constant Influx Limit

For the constant influx limiting case the applicable energy conjecture is that the dyke configuration that minimizes the rate of work done by the magma source on the dyke swarm will be energetically advantageous (Section

4). By this statement, searching for an optimum spacing between the dykes is synonymous with searching for a spacing that minimizes \dot{W}_d (Eq. 14). Also we recall that the constant influx limit is probably not as widely applicable to dyke swarms as the constant pressure limit (Section 3).

Nonetheless, for the limiting case of constant total influx Q_o that is partitioned equally among all of the dykes, the approximations from Eq. (15) and characteristic quantities from Table 1 lead to an estimate for the input power for the widely-spaced ($h \gg H$) regime as

$$\dot{W}_d \approx \left(\frac{h^2 E'^4 \mu' Q_o^7 t}{H^6 Z^2} \right)^{1/5} (1 + O(H/h)^2). \quad (23)$$

On the other hand, for the closely-spaced ($h \ll H$) regime, the approximations from Eq. (16) lead to

$$\dot{W}_d \approx \left(\frac{E'^4 \mu' Q_o^7 t}{h^2 H^2 Z^2} \right)^{1/5} (1 + O(h/H)). \quad (24)$$

To leading order $\dot{W}_d \approx Q_o P$ in both cases, with P from Table 1. And so we see that P , and hence \dot{W}_d , increases with time. Recalling that the energy conjecture for the constant influx case is that the system will favor configurations that minimize \dot{W}_d , this increasing behavior with time once again opens the possibility that subsequent generations of dykes could be initiated in the spaces between the primary dykes.

As in the case of the constant pressure source, the most interesting implication of Eqs. (23) and (24) has to do with the spacing that optimizes (in this case minimizes) \dot{W}_d . And here we have a somewhat more complicated situation than for the constant pressure source. By introducing and

416 normalizing by a characteristic power

$$\dot{W}_d^* = \left(\frac{E'^4 \mu' Q_o^7 t}{H^4 Z^2} \right)^{1/5}, \quad (25)$$

417 it is apparent that for the constant pressure source, the leading order term
 418 of the widely-spaced approximation (Eq. 18) goes like H/h with subsequent
 419 terms going like $(h/H)^{1-2n}$ for $n = 1, 2, \dots$. Which is to say that the lead-
 420 ing order term and all subsequent correction terms show \dot{W}_d increases with
 421 decreasing h/H . The converse is true for the closely-spaced approximation
 422 (Eq. 19), with the important point being that the leading order term and
 423 all subsequent correction terms in the series indicate that \dot{W}_d decreases with
 424 decreasing h . This shows that both expansions can be pushed all the way to
 425 $h = H$ without a change in the sign of the derivative of \dot{W}_d with respect to
 426 h/H .

427 The behavior of both series is fundamentally different for the constant
 428 influx limiting case. Starting with the widely-spaced approximation (Eq.
 429 23), we see that the leading order term of the series goes like $(h/H)^{2/5}$. But
 430 the next term in the series goes like $(h/H)^{-8/5}$ with subsequent terms going
 431 like $(h/H)^{(-10n+2)/5}$ for $n = 2, 3, \dots$. So the leading order term indicates that
 432 \dot{W}_d decreases (which is considered advantageous in this case) with decreasing
 433 h/H for $h \gg H$. However, as $h \rightarrow H$ the subsequent terms in the series
 434 become important and will at some point change the sign of $d\dot{W}_d/d(h/H)$.
 435 The situation is similar for the closely-spaced approximation (Eq. 24), so that
 436 we also expect the sign of $d\dot{W}_d/d(h/H)$ to change in the range $0 < h/H < 1$.

437 Figure 4 shows the behavior of both the widely and closely spaced ap-
 438 proximations of \dot{W}_d (Eqs. 23 and 24), normalized by the characteristic power
 439 \dot{W}_d^* . Four curves are graphed for each approximation. These are labeled with

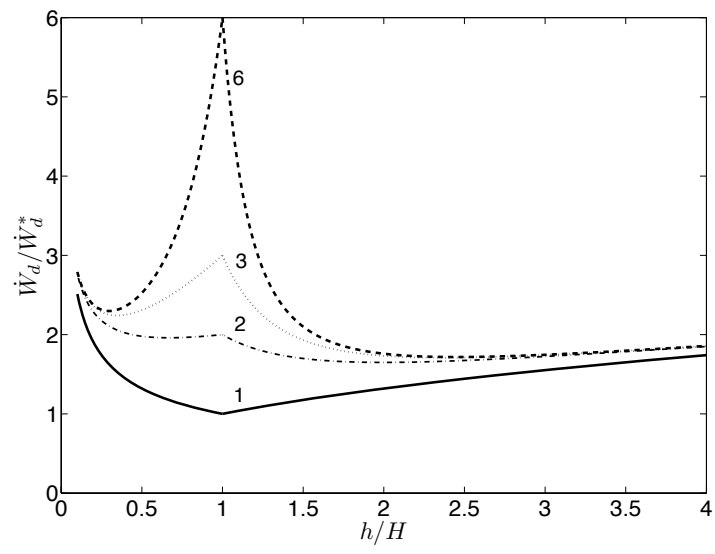


Figure 4: Normalized input power \dot{W}_d to the dyke swarm for the case of constant influx from the source, where the numerical label indicates the number of terms retained in the asymptotic series.

440 a number that indicates the number of terms $(M + 1)$ retained in the series
 441 $\dot{W}_d/\dot{W}_d^* \approx \sum_{n=0}^M (h/H)^{(-10n+2)/5}$ corresponding to Eq. (23), or the series
 442 $\dot{W}_d/\dot{W}_d^* \approx \sum_{n=0}^M (h/H)^{(5n-2)/5}$ corresponding to Eq. (24). Per the relevant
 443 energy conjecture (Section 4), in this case we are looking for minima rather
 444 than maxima in these curves and, as expected we observe two local minima,
 445 one in the range $0 < h/H < 1$ and one for $h/H > 1$.

446 It is important to be clear that Figure 4 represents an approximation
 447 to the behavior of \dot{W}_d . From it we can see clearly that the model predicts
 448 two local minima and we can be confident that they will be $O(1)$ and in
 449 the ranges $0 < h/H < 1$ and $h/H > 1$. However, we cannot precisely pre-
 450 dict the values of h/H that minimize \dot{W}_d nor can we be sure which of the
 451 local minima will be the global minimum. This is because the actual large
 452 and small h/H expansions embodied in Eqs. (23) and (24) have the form
 453 $\dot{W}_d/\dot{W}_d^* \approx \sum_{n=0}^M a_n (h/H)^{(-10n+2)/5}$ and $\dot{W}_d/\dot{W}_d^* \approx \sum_{n=0}^M b_n (h/H)^{(5n-2)/5}$, re-
 454 spectively, where a_n and b_n are $O(1)$ quantities that must be determined from
 455 a solution to the governing equations (Eqs. 1-11) that enables computation
 456 of the energy integrals defined by Bungert (2013) (for example see Supplemen-
 457 tary Section 3). Here we have simply taken $a_n = 1$ and $b_n = 1$. In this coarse
 458 approximation, the widely-spaced local minimum appears as the global min-
 459 imum, and its location is $h/H = 2$ for the 2 term series and it moves towards
 460 $h/H \approx 2.5$ when many terms are included in the series. On the other hand,
 461 the location of the closely spaced local minimum is $h/H = 2/3$ for the 2 term
 462 series and it moves towards $h/H \approx 0.3$ when many terms are included in the
 463 series.

464 The striking conclusion is that there exist two local minima in the input

power \dot{W}_d , both of which could represent optimal spacings for dyke growth under conditions of constant influx (if indeed the constant influx condition is relevant to some field cases). Further analysis is required to pinpoint the locations of the minima, but we roughly expect them to be around $h/H \approx 2.5$ and $h/H \approx 0.3$. Further analysis is also required to determine which of these is the global minimum.

8. Field Comparisons

Our model predicts the optimal first-generation dyke-spacing that dyke swarms will tend to develop. By “first-generation” we mean the spacing of the first set of dykes that grow into the host rock. These will naturally arrest at some point and additional dykes will fill in between them. However, we expect from this model, based on the constant pressure inlet conditions (as argued in Section 3), that the first-generation will be the thickest dykes and these will have a spacing that is commensurate with the dyke height H . The model provides also an estimate of how the volume of the swarm will increase with time (Eq. 22). Both predictions can be tested against field observations.

8.1. Iceland

We first devote our attention to the magmatic activity that took place at Krafla in the late 1970s, and to the Tertiary Alftafjörður dyke swarm in eastern Iceland.

According to Sigurdsson (1987), the Krafla rifting episode involved the repeated horizontal injection of fairly similar dykes, whose height ranged between 2 km and 5 km (with an average of 2.8 km) and which propagated at an average velocity of 0.5 m/s over distances of 10 km to 30 km from a

489 magmatic source with an estimated overpressure of about 10 MPa. The total
490 volume of magma that was evacuated from the magma chamber during the
491 whole event has been estimated to be 1–2 km³ (Sigurdsson, 1987).

492 Eq. (22) provides an estimate of a dyke-swarm volume as a function of
493 time. Conversely, we can use this equation to estimate the time required to
494 emplace a swarm of a particular volume. Taking the average values provided
495 by Sigurdsson (1987) along with a dyke swarm width of 10 km, values for the
496 Young’s modulus $E = 10$ GPa and the Poisson’s ratio $\nu = 0.25$, and assuming
497 a magma viscosity of 100 Pa s, Eq. (22) predicts an injection duration for a
498 1-km³ dyke swarm of about 7 h. This is in the same order of magnitude as
499 the duration of dyke injections at Krafla, which was estimated to last about
500 25 h based on the monitoring of their seismic activity (Sigurdsson, 1987).

501 Paquet et al. (2007) studied the Tertiary Alftafjörður dyke swarm in East-
502 ern Iceland where they measured the dyke-thickness distribution within the
503 swarm at two different locations. They observed a clustering of dykes with a
504 characteristic spacing of 1.5 km to 2.5 km, which seems to have been deter-
505 mined visually. Additionally, a Fast Fourier Transform analysis gives a mode
506 of 2.5 km. Importantly, these spacing values are reported to correspond to
507 the distribution of the thickest dykes, which would reflect the first generation
508 of dykes and hence those we expect to be consistent with our model. If one
509 takes the average dike height given by Sigurdsson (1987) at Krafla of 2.8
510 km as representative of horizontally-propagating dykes throughout Iceland,
511 then the study of Paquet et al. (2007) suggests that the Tertiary Alftafjörður
512 dyke swarm developed a characteristic dyke-spacing comparable to the aver-
513 age height of its dykes, as suggested by our model.

514 8.2. *Canada*

515 In crustal-scale giant radiating dyke swarms (Halls and Fahrig, 1987) it is
516 reasonable to assume that individual dykes traverse the entire thickness of the
517 crust ($H \approx 30\text{-}40$ km) or a significant portion of the crust. Here we focus on
518 constraining the spacing of first-generation dykes in the 1270 Ma Mackenzie
519 (Figure 1) and the 2470-2450 Ma Matachewan dyke swarms, Canada.

520 Dykes in the Mackenzie swarm converge towards a common origin, at-
521 tributed to the head of the mantle plume that supplied magma to the dykes,
522 north of Coppermine in the Canadian Arctic archipelago (Figure 1). The
523 swarm radiates across the northern half of the Canadian Shield with a fan
524 angle close to the origin of 100 degrees, covering an area of 2.1×10^6 km² and
525 extending up to 2,400 km along strike (Ernst and Buchan, 2001). In more
526 distal southeastern parts of the swarm, >1000 km from the origin, the dyke
527 pattern is more linear and attributed to a transition from propagation within
528 a radial plume-related stress regime to a regional stress regime (Ernst and
529 Buchan, 2001; Hou et al., 2010). Magnetic fabric analysis indicates a second
530 transition from vertical to horizontal magma flow regimes occurring 500-600
531 km from the swarm center, probably associated with the outer boundary of
532 the plume head (Ernst and Baragar, 1992). Mackenzie dykes range in thick-
533 ness from 1 m to 150 m, with a mean of 30 m (Fahrig, 1987). The mean
534 thickness increases from ~ 18 m, 400 km from the swarm center to ~ 33 m
535 more than 600 km out (Baragar et al., 1996). Likewise, the mean spacing
536 between dykes increases from ~ 6.7 km about 500 km from the swarm center
537 to ~ 25 km approximately 2100 km to the southeast in northwestern On-
538 tario (Hou et al., 2010). A recent compilation of Proterozoic intrusions in

northwest Ontario confirms the mean spacing of distal Mackenzie dykes to be 27 km with a range from 7.8 to 93 km (Stott and Josey, 2009). However, spacing between the most continuous dykes is typically between 35 and 65 km.

The systematic outward increase in both mean dyke thickness and spacing from the swarm center can be explained by a corresponding decrease in the number of second- and higher-generation dykes. We therefore suggest that the thickness and spacing of Mackenzie dykes at the distal fringes of the swarm in northwestern Ontario are characteristic of the first-generation dykes. Assuming the dykes propagated horizontally over a height approximately equal to the thickness of the crust then $h \approx H$, in agreement with the model prediction under the constant pressure inlet condition.

The 2490-2450 Ma Matachewan dyke swarm of central Ontario is well characterized by aeromagnetic mapping, and paleomagnetic, geochemical, geochronological and petrologic studies (West and Ernst, 1991; Bates and Halls, 1991; Halls et al., 1994; Percival et al., 1994; Phinney and Halls, 2001). The Matachewan swarm fans northwards from a center located in Lake Huron and covers an area of 250,000 km² (Halls et al., 1994). Dykes can be traced for more than 1000 km northwards from the center across a fan angle of ~ 45 degrees and they occur in three sub-swarms now offset and uplifted differentially by the ca. 2000 Ma Kapuskasing structure (West and Ernst, 1991). Geothermobarometric analysis indicates that the dykes exposed at the surface today were emplaced at paleodepths of 10 to 21 km (Percival et al., 1994). A study of dyke geochemistry concluded that their petrogenesis was a two stage process involving lower-crustal fractionation and assimilation of

564 plume head-derived melts, followed by later compositional modification in
 565 mid-crustal, 15-20 km deep magma chambers (Phinney and Halls, 2001).
 566 This contrasts with the Mackenzie dyke swarm, which appears to have been
 567 extracted directly from a plume head, and suggests that propagation of the
 568 Matachewan dykes may have been confined to the top 20 to 25 km of crust.

569 The average width of Matachewan dykes outside of the Kapuskasing zone
 570 is 10 to 20 m, but there is a strong tendency for dykes to become fewer in
 571 number and thicker moving away from the swarm center (Bates and Halls,
 572 1991; Halls et al., 1994). For example, towards the northern end of the M2
 573 sub-swarm, >40% of dykes have widths in the range 25 to 55 m, whereas in
 574 the southern part of the same sub-swarm only ~20% of dykes are wider than
 575 25 m (Halls et al., 1994). Based on aeromagnetic interpretation by West
 576 and Ernst (1991), the mean spacing between Matachewan dykes ~500 km
 577 north of the swarm center in all three sub swarms is 4.2 ± 2.4 km. However,
 578 this is likely sampling second- and high-order dykes. Moving out to the
 579 distal fringes of the swarm, the spacing between continuous dykes with the
 580 strongest magnetic anomalies is 19 to 32 km in the northern part of the M2
 581 sub-swarm and between 12.4 and 16.5 km in the northwest part of the M3
 582 sub-swarm. As noted by Halls et al. (1994), there is a correlation between the
 583 widest dykes and the strongest magnetic anomalies, hence we consider this to
 584 be a reasonable estimate of the spacing between first-generation dykes in the
 585 Matachewan swarm. If the interpretation above that these dykes propagated
 586 within the mid- to upper crust is correct, then this observation is consistent
 587 with the predicted $h \approx H$ relationship. The lower spacing in the western M3
 588 swarm may indicate a slightly shallower source magma chamber than the

589 M2 swarm and a correspondingly lower height of dyke propagation within
590 the upper crust.

591 9. Conclusions

592 Analysis of the work rates associated with driving dyke swarms, coupled
593 with scaling analysis that gives rise to estimates of the dyke pressure, thick-
594 ness, and length, allows us to search for an optimal dyke spacing. To this
595 point it has been a mystery, from a mechanical perspective, as to why multiple
596 dykes would grow in close, but apparently not *too* close, proximity to one an-
597 other, thus forming the morphology described as a dyke swarm. Now we can
598 see that, in fact, the mechanical model for a uniform array of horizontally-
599 propagating blade-like dykes implies that an intermediate spacing, on the
600 order of the height of the dykes themselves, is energetically optimal. What's
601 more, we have found that the optimal spacing depends on the nature of the
602 magma source condition, with the constant pressure source condition giving
603 rise to an optimal spacing of $h \approx H$, while the constant magma influx source
604 condition gives rise to two candidates, one near $h \approx 2.5H$ and one near
605 $h \approx 0.3H$, the former of which tentatively appears as the global minimum
606 based on a coarse analysis.

607 We have also shown that in the case of the constant pressure source, the
608 total flow rate of magma into the dyke swarm decreases with time. Similarly,
609 for the case of constant influx from the source, the pressure required to
610 propagate the dyke swarm increases with time. Both of these behaviors
611 suggest that at some point the system will prefer to initiate new generations
612 of dykes rather than continuing to propagate only the primary generation.

Hence we anticipate that the dyke spacing will actually be more dense than what is predicted by the optimal spacing, especially in the vicinity of the source.

Dyke swarms in both Iceland and Canada demonstrate spacing between the thickest dykes, which we interpret to be the first generation of growth and which is the set of dykes to which our model is applicable, that scales with and is of the same order as the dyke height. Hence these comparisons with field data lend preliminary support to our analysis.

Acknowledgement

Support for AB and XZ has been provided by the Commonwealth Scientific and Industrial Research Organisation (CSIRO) through its Petroleum and Geothermal Research Innovative Science Funds.

References

- Adachi, J.I., Peirce, A.P., 2008. Asymptotic analysis of an elasticity equation for a finger-like hydraulic fracture. *J. Elasticity* 90, 43–69.
- Ayele, A., Keir, D., Ebinger, C., Wright, T.J., Stuart, G.W., Buck, W.R., Jacques, E., Ogubazghi, G., Sholan, J., 2009. September 2005 mega-dike emplacement in the Manda-Harraro nascent oceanic rift (Afar depression). *Geophysical Research Letters* 36, L20306.
- Bai, T., Pollard, D.D., 2000. Fracture spacing in layered rocks: a new explanation based on the stress transition. *J. Struct. Geol.* 22, 43–57.

- 634 Baragar, W.R.A., Ernst, R.E., Hulbert, L., Peterson, T., 1996. Longitudi-
635 nal petrochemical variation in the Mackenzie dyke swarm, northwestern
636 Canadian Shield. *J. Petrology* 37, 317–359.
- 637 Bates, M.P., Halls, H.C., 1991. Broad-scale Proterozoic deformation of the
638 central Superior Province revealed by paleomagnetism of the 2.45 Ga Mat-
639 achewan dyke swarm. *Can. J. Earth Sci.* 28, 1780–1796.
- 640 Ben-Jacob, E., Levine, H., 2001. The artistry of nature. *Nature* 409, 985–986.
- 641 Benthem, J.P., Koiter, W.T., 1973. Asymptotic approximations to crack
642 problems, in: Sih, G.H. (Ed.), *Methods of analysis and solutions of crack*
643 *problems: Recent developments in fracture mechanics; Theory and meth-*
644 *ods of solving crack problems.* Noordhoff International Publishing, Leiden,
645 pp. 131–178.
- 646 Brandsdóttir, B., Einarsson, P., 1979. Seismic activity associated with the
647 September 1977 deflation of the Krafla central volcano in North-Eastern
648 Iceland. *J. Volcanol. Geotherm. Res.* 6, 197–212.
- 649 Bruce, P. M., Huppert, H. H., 1989. Thermal controls of basaltic fissure
650 eruptions. *Nature* 342, 665–667.
- 651 Bunger, A.P., 2013. Analysis of the Power Input Needed to Propagate Mul-
652 tiple Hydraulic Fractures. *Int. J. Solids Struct.* 50, 1538–1549.
- 653 Bunger, A.P., Zhang, X., Jeffrey, R.G., 2012. Parameters effecting the in-
654 teraction among closely spaced hydraulic fractures. *Soc. Pet. Eng. J.* 17,
655 292–306.

- 656 Burchardt, S., Tanner, D.C., Troll, V.R., Krumbholz, M., Gustafsson, L.E.,
657 2011. Three-dimensional geometry of concentric intrusive sheet swarms in
658 the Geitafell and the Dyrfjöll volcanoes, eastern Iceland. *Geochemistry*
659 *Geophysics Geosystems* 12.
- 660 Clauset, A., Shalizi, C.R., Newman, M.E.J., 2009. Power-law distribution in
661 empirical data. *SIAM Review* 51, 661–703.
- 662 Detournay, E., 2004. Propagation regimes of fluid-driven fractures in imper-
663 meable rocks. *Int. J. Geomechanics* 4, 1–11.
- 664 Duffield, W.A., Christiansen, R.L., Koyanagi, R.Y., Peterson, D.W., 1982.
665 Storage, migration, and eruption of magma at Lilauea volcano, Hawaii,
666 1971-1972. *J. Volcanol. Geotherm. Res.* 13, 273–307.
- 667 Ernst, R.E., Baragar, W.R.A., 1992. Evidence from magnetic fabric for
668 the flow pattern of magma in the Mackenzie giant radiating dyke swarm.
669 *Nature* 356, 511–513.
- 670 Ernst, R.E., Buchan, K.L., 2001. The use of mafic dike swarms in identi-
671 fying and locating mantle plumes, in: Ernst, R.E., Buchan, K.L. (Eds.),
672 *Mantle Plumes: Their Identification Through Time*. Geological Society of
673 America, Boulder, Colorado, pp. 247–265. Special Paper 352.
- 674 Ernst, R.E., Grosfils, E.B., Mège, D., 2001. Giant Dike Swarms: Earth,
675 Venus, and Mars. *Annu. Rev. Earth Planet. Sci.* 29, 489–534.
- 676 Ernst, R.E., Head, J.W., Parfitt, E., Grosfils, E., Wilson, L., 1995. Giant
677 radiating dyke swarms on Earth and Venus. *Earth and Environmental*
678 *Science Transactions of the Royal Society of Edinburgh* 39, 1–58.

- 679 Fahrig, W.F., 1987. The tectonic settings of continental mafic dyke swarms:
680 failed arm and early passive margin, in: Halls, H.C., Fahrig, W.F. (Eds.),
681 Mafic Dyke Swarms. Geol. Assoc. Can. Spec. Paper 34, pp. 331–348.
- 682 Germanovich, L.N., Ring, L.M., Astakhov, D.K., Shlyopobersky, J., Mayer-
683 hofer, M.J., 1997. Hydraulic fracture with multiple segments II: Modeling.
684 Int. J. Rock Mech. Min. Sci. 34. Paper 098.
- 685 Gudmundsson, A., 1983. Form and dimensions of dykes in eastern Iceland.
686 Tectonophysics 95, 295–307.
- 687 Gudmundsson, A., 1995. Infrastructure and mechanics of volcanic systems
688 in Iceland. J. Volcanol. Geotherm. Res. 64, 1–22.
- 689 Halls, H.C., Fahrig, W.F. (Eds.), 1987. Mafic Dyke Swarms. Geol. Assoc.
690 Can. Spec. Pap. 34.
- 691 Halls, H.C., Palmer, H.C., Bates, M.P., Phinney, W.C., 1994. Constraints of
692 the nature of the Kapuskasing structural zone from the study of Proterozoic
693 dyke swarms. Can. J. Earth Sci. 31, 1182–1196.
- 694 Hobbs, D.W., 1967. The formation of tension joints in sedimentary rocks:
695 an explanation. Geological Magazine 104, 550–556.
- 696 Hou, G., Kusky, T.M., Want, C., Wang, Y., 2010. Mechanics of the giant
697 radiating Mackenzie dyke swarm: A paleostress field modeling. J. Geophys.
698 Res. 115, B02402.
- 699 Jeffrey, R.G., Bunger, A.P., Lecampion, B., Zhang, X., Chen, Z.R., van
700 As, A., Allison, D., Beer, W.D., Dudley, J.W., Siebrits, E., Thiercelin,

701 M., Mainguy, M., 2009. Measuring hydraulic fracture growth in naturally
702 fractured rock, in: Proceedings SPE Annual Technical Conference and
703 Exhibition, New Orleans, Louisiana, USA. SPE 124919.

704 Jin, Z.H., Johnson, S.E., 2008. Magma-driven multiple dike propagation and
705 fracture toughness of crustal rocks. *J. Geophys. Res.* 113, B03206.

706 Johnsen, G.V., Björnsson, A., Sigurdsson, S., 1980. Gravity and elevation
707 changes caused by magma movement beneath the Krafla Caldera, North-
708 east Iceland. *J. Geophys.* 47, 132–140.

709 Jolly, R.J.H., Cosgrove, J.W., Dewhurst, D.N., 1998. Thickness and spatial
710 distributions of clastic dykes, northwest Sacramento Valley, California. *J.*
711 *Struct. Geol.* 20, 1663–1672.

712 Jolly, R.J.H., Sanderson, D.J., 1995. Variation in the form and distribution
713 of dykes in the Mull swarm, Scotland. *J. Struct. Geol.* 17, 1543–1557.

714 Lecampion, B., Detournay, E., 2007. An implicit algorithm for the propaga-
715 tion of a plane strain hydraulic fracture with fluid lag. *Computer Meth.*
716 *Appl. Mech. Eng* 196, 4863–4880.

717 LeCheminant, A., Heaman, L., 1989. Mackenzie igneous events, Canada:
718 Middle Proterozoic hotspot magmatism associated with ocean opening.
719 *Earth Planet. Sci. Lett* 96, 38–48.

720 Lister, J.R., 1990. Buoyancy-driven fluid fracture: similarity solutions for the
721 horizontal and vertical propagation of fluid-filled cracks. *J. Fluid Mech.*
722 217, 213–239.

- 723 Lister, J.R., Kerr, R.C., 1991. Fluid-mechanical models of crack propagation
724 and their application to magma transport in dykes. *J. Geophys. Res.* 96,
725 10049–10077.
- 726 Mériaux, C., Jaupart, C., 1998. Dike propagation through an elastic plate.
727 *J. Geophys. Res.* 103, 18,295–18,314.
- 728 Narr, W., Suppe, J., 1991. Joint spacing in sedimentary rocks. *J. Struct.*
729 *Geol.* 11, 1037–1048.
- 730 Nordgren, R., 1972. Propagation of vertical hydraulic fractures. *J. Pet. Tech.*
731 253, 306–314. (SPE 3009).
- 732 Odé, H., 1957. Mechanical analysis of the dike pattern of the Spanish Peaks
733 area, Colorado. *Bulletin of the Geological Society of America* 68, 567–576.
- 734 Olson, J.E., 2004. Predicting fracture swarms – the influence of subcritical
735 crack growth and the crack-tip process zone on joint spacing in rock, in:
736 Cosgrove, J.W., Engelder, T. (Eds.), *The initiation, propagation, and ar-*
737 *rest of joints and other fractures.* Geological Society, London. volume 231,
738 pp. 73–87.
- 739 Olson, J.E., 2008. Multi-fracture propagation modeling: Applications to
740 hydraulic fracturing in shales and tight gas sands, in: *Proceedings 42nd*
741 *US Rock Mechanics Symposium*, San Francisco, CA, USA. ARMA 08-327.
- 742 Olson, J.E., Dahi-Taleghani, A., 2009. Modeling simultaneous growth of
743 multiple hydraulic fractures and their interaction with natural fractures,
744 in: *Proceedings SPE Hydraulic Fracturing Technology Conference and*
745 *Exhibition*, The Woodlands, Texas, USA. SPE 119739.

746 Paquet, F., Dauteuil, O., Hallot, E., Moreau, F., 2007. Tectonics and magma
747 dynamics coupling in a dyke swarm of Iceland. *Journal of Structural Ge-*
748 *ology* 29, 1477–1493.

749 Peltier, A., Staudacher, T., Bachèlery, P., 2007. Constraints on magma trans-
750 fers and structures involved in the 2003 activity at Piton de La Fournaise
751 from displacement data. *Journal of Geophysical Research* 112, B03207.

752 Percival, J.A., Palmer, H.C., Barnett, R.L., 1994. Quantitative estimates of
753 emplacement level of postmetamorphic mafic dykes and subsequent erosion
754 magnitude in the southern Kapuskasing uplift. *Can. J. Earth Sci.* 31, 1218–
755 1226.

756 Perkins, T., Kern, L., 1961. Widths of hydraulic fractures. *J. Pet. Tech.*,
757 *Trans. AIME* 222, 937–949.

758 Petford, N., Kerr, R. C., Lister, J. R., 1993. Dike transport of granitoid
759 magmas. *Geology* 21, 845–848.

760 Phinney, W.C., Halls, H.C., 2001. Petrogenesis of the Early Proterozoic Mat-
761 achewan dykes swarm, Canada, and implications for magma emplacement
762 and subsequent deformation. *Can. J. Earth Sci.* 38, 1541–1563.

763 Roper, S.M., Lister, J.R., 2005. Buoyancy-driven crack propagation from an
764 over-pressured source. *J. Fluid Mech.* 536, 79–98.

765 Roussel, N.P., Sharma, M.M., 2011. Optimizing fracture spacing and se-
766 quencing in horizontal-well fracturing. *SPE Production & Operations* ,
767 173–184SPE 127986.

768 Rubin, A.M., Pollard, D.D., 1987. Origin of blade-like dikes in volcanic rift
769 zones. U.S. Geol. Surv. Prof. Pap. , 1449–1470.

770 Sigurdsson, H., 1987. Dyke injection in Iceland: A review, in: Halls, H.C.,
771 Fahrig, W.F. (Eds.), Mafic Dyke Swarms, pp. 55–64.

772 Sneddon, I.N., 1946. The distribution of stress in the neighborhood of a crack
773 in an elastic solid. Proc. Roy. Soc. London A 187, 229–260.

774 Stott, G.M., Josey, S.D., 2009. Proterozoic mafic (diabase) dikes and other
775 Post-Archean intrusions of northwestern Ontario north of latitude 49°30'.
776 Ont. Geol. Surv. Prelim. Map P3606, scale 1:1,000,000.

777 Swanson, D.A., 1972. Magma supply rate at Kilauea volcano, 1952–1971.
778 Science 175, 169–170.

779 Taisne, B., Jaupart, C., 2009. Dike propagation through layered rocks. J.
780 Geophys. Res. 114, B09203.

781 Taisne, B., Tait, S., Jaupart, C., 2011. Conditions for the arrest of a vertical
782 propagating dyke. Bull. Volcanology 73, 191–204.

783 Thordarson, T., Self, S., 1993. The Laki (Saftár Fires) and Grímsvötn erup-
784 tions in 1783–1785. Bull. Volcanology 55, 233–263.

785 Traversa, P., Pinel, V., Grasso, J.R., 2010. A constant influx model for dike
786 propagation: Implications for magma reservoir dynamics. J. Geophys. Res.
787 115, B01201.

- 788 Tryggvason, E., 1986. Multiple magma reservoirs in a rift zone volcano:
789 ground deformation and magma transport during the September 1984
790 eruption of Krafla, Iceland. *J. Volcanol. Geotherm. Res.* 28, 1–44.
- 791 Vermilyen, J.P., Zoback, M.D., 2011. Hydraulic fracturing, microseismic mag-
792 nitudes, and stress evolution in the Barnett Shale, Texas, USA, in: Pro-
793 ceedings SPE Hydraulic Fracturing Technology Conference and Exhibition,
794 The Woodlands, Texas, USA. SPE 140507.
- 795 Walker, G.P.L., 1986. Koolau dike complex, Oahu: Intensity and origin of
796 a sheeted-dike complex high in a Hawaiian volcanic edifice. *Geology* 14,
797 310–313.
- 798 Weng, X., Kresse, O., Cohen, C., Wu, R., Gu, H., 2011. Modeling of
799 hydraulic-fracture-network propagation in a naturally fractured formation.
800 *SPE Production & Operations* 26, 368–380.
- 801 West, G.F., Ernst, R.E., 1991. Evidence from aeromagnetism on the configu-
802 ration of Matachewan dykes and the tectonic evolution of the Kapuskasing
803 Structural Zone, Ontario, Canada. *Can. J. Earth Sci.* 28, 1797–1811.
- 804 White, R.S., Drew, J., Martens, H.R., Key, J., Soosalu, H., Jakobsdóttir,
805 S.S., 2011. Dynamics of dyke intrusion in the mid-crust of Iceland. *Earth*
806 *Planet. Sci. Lett.* 304, 300–312.
- 807 Wright, T.L., Tilling, R.I., 1980. Chemical variation in Kilauea eruption
808 1971–1974. *Am. J. Sci.* 280-A, 777–793.
- 809 Zhang, X., Jeffrey, R.G., Bunger, A.P., Thiercelin, M., 2011. Initiation and

- 810 growth of a hydraulic fracture from a circular wellbore. *Int. J. Rock Mech.*
811 *Min. Sci.* 48, 984–995.
- 812 Zhang, X., Jeffrey, R.G., Thiercelin, M., 2007. Deflection and propagation
813 of fluid-driven fractures at frictional bedding interfaces: A numerical in-
814 vestigation. *J. Struct. Geol.* 29, 396–410.

Analytical Predictions for a Natural Spacing within Dyke Swarms: Supplementary Material

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1. Near-Field Interaction Stress

Here we present the asymptotic form of the interaction stress for the closely-spaced limit $h \ll H \ll R$. Proceeding in the same way as Bunger (2012), we begin with the expression for the normal traction σ_z (compression positive) induced on a plane $z = \pm h$ due to a crack located at $z = 0$, $-H/2 < y < H/2$ and subjected to an internal pressure p_o is given by (Sneddon, 1946)

$$-\frac{\sigma_z}{p_o} = \text{Re}Y + \zeta \text{Im}Y', \quad (1)$$

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where Re and Im indicate the real and imaginary parts, respectively, Y is the Westergaard stress function

$$Y = \frac{\mathbf{z}}{\sqrt{\mathbf{z}^2 - 1}} - 1, \quad (2)$$

and the $'$ denotes the derivative with respect to the complex coordinate

$$\mathbf{z} = \eta + i\zeta, \quad (3)$$

with $i = \sqrt{-1}$ and where $\eta = 2y/H$ and $\zeta = 2h/H$. Taking the Taylor Series of Eq. (2) for $\zeta \ll 1$ and substituting into Eq. (1) gives

$$\sigma_I = p \left(1 - \frac{2\zeta}{(1 - \eta^2)^{3/2}} + O(h/H)^3 \right). \quad (4)$$

Considering the stress along $\eta = 0$ leads directly to Eq. (3, Main Text). Note that the influence of the η (y) dependence of the interaction stress in the near-field case on the opening at the center w is compensated using the variable coefficient α_1 (Eq. 1, Main Text), which is determined numerically in 2.

2. Calculations for Interacting Cracks

Calculation of the cross sections of multiple interacting cracks was carried out using the MineHF implementation (Zhang et al., 2007) of the displacement discontinuity method (Crouch and Starfield, 1983). Because we consider cross sections of blade-like cracks, the pressure is taken to be uniform (e.g. Nordgren, 1972). We also take the pressure to be equal in each crack in the array. For these calculations, $p_f = 7$ MPa, $\sigma_o = 6$ MPa, $E' = 52.5$ GPa, $H = 2$ m, and the spacing h is varied between 20 m and 0.1 m. Each

crack was discretized with 80 elements, and numerical experiments with 50 elements confirm mesh insensitivity at this discretization. The crack tip is captured using a square root element and the other elements are linear displacement discontinuities. We use the central crack in an array of $N = 13$ cracks in each case we present.

Figure S1 shows that the cracks transition from an elliptical shape when widely-spaced to the closely-spaced case wherein it takes a shape that increases from the central portion to the vicinity of the tip where it rapidly decreases to zero. For the modified Poiseuille equation (Eq. 5, Main Text with $\alpha_3 = 1$) we assume a rectangular cross section in the closely-spaced limit.

Figure S2 shows the transition from the elasticity relationship $w = 2Hp/E'$ when interaction can be neglected to $w \approx 0.35H(p - \sigma_I)/E'$ with σ_I given by Eq. (3, Main Text) when the cracks are closely spaced. This calculation is the basis for the value of α_1 in Eq. (1, Main Text).

Figure S3 shows the transition from the area given by an ellipse when widely-spaced to a scenario where the area exceeds by 10% that which would be obtained from a rectangular crack opening when $H/h = 20$. Because the present work is aimed at approximation, we take the area to be equal to wH for the purpose of the continuity equation (Eq. 4, Main Text).

3. Closely-Spaced Power Factors

Following Bungler (2012), the rate of work of the interaction stress (shown here for a single blade-like wing in contrast to the reference which considers

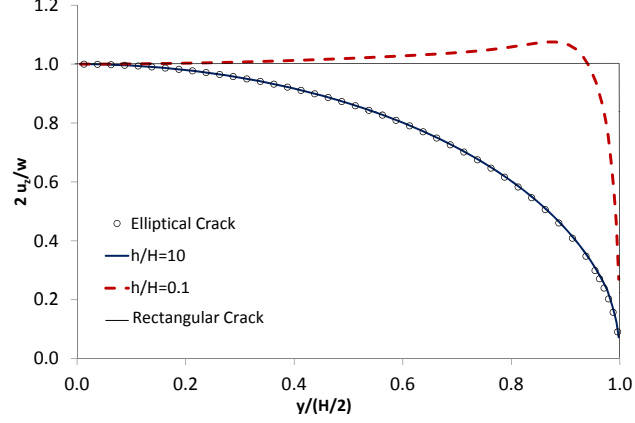


Figure S1: Opening profile for widely-spaced and closely-spaced cracks, where the $y > 0$ half of the crack is presented by symmetry and here we have used the central crack in an array of $N = 13$. Here $u_z(y)$ is the displacement of each crack face and $w = 2u_z(0)$.

49 a hydraulic fracture with two wings) is given by

$$\dot{W}_I = -\frac{\pi}{4}H \int_0^R \sigma_I \frac{\partial w}{\partial t} dx. \quad (5)$$

50 Substituting Eq. (3, Main Text) for the near-field stress ($h \ll H$) and the
 51 scaling from Eq. (17, Main Text) leads to

$$\dot{W}_I = -\frac{HLPX}{t} \frac{\pi}{4} \gamma \int_0^1 \frac{t}{X} \frac{\partial \Omega}{\partial t} \Pi \left(1 - \frac{4h}{H} + O(h/H)^2 \right) d\rho. \quad (6)$$

52 Hence it is clear that \dot{W}_I is approximated according to Eq. (16, Main Text)
 53 provided that the characteristic quantities $\{L, X, P\}$ are chosen such that
 54 $\{\gamma, \Omega, \Pi\}$ are all $O(1)$.

55 Similarly, following Bungier (2012), the expression for the fluid dissipation

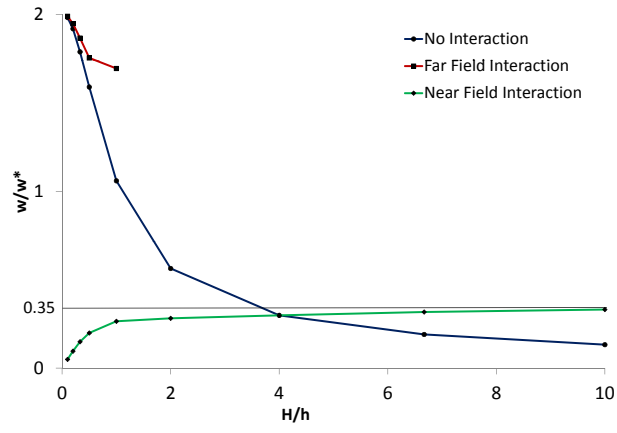


Figure S2: Relationship between w and w^* determined from elasticity as a function of H/h , where for “no interaction” $w^* = Hp/E'$ and $w^* = H(p - \sigma_I)/E'$ otherwise, with σ_I from Eq. (2, Main Text) for the “far field interaction” ($h/H \gg 1$) case and from Eq. (3, Main Text) for the “near field interaction” ($h/H \ll 1$) case.

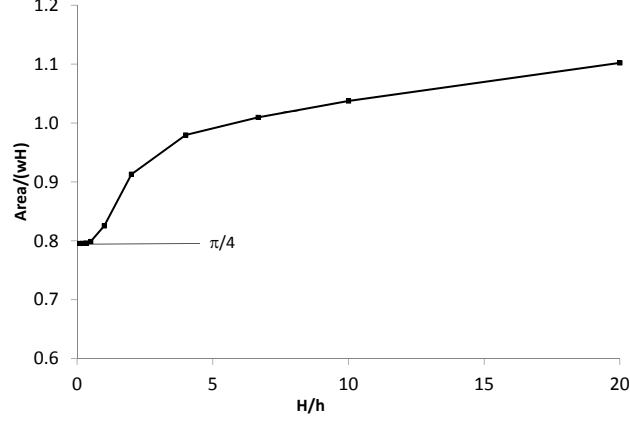


Figure S3: Dependence of the crack opening area normalized by wH on H/h showing tendency to $\pi/4$ for the elliptical profile for widely-spaced cracks and to tend to a value that is a bit greater than 1 for closely-spaced cracks.

is given by

$$D_f = \frac{3\pi H}{32 \mu'} \int_0^R w^3 \left(\frac{\partial p}{\partial x} + \frac{\partial \sigma_I}{\partial x} \right)^2 dx. \quad (7)$$

Again, substituting Eq. (3, Main Text) for the near-field stress ($h \ll H$) and the scaling from Eq. (17, Main Text) leads to

$$D_f = \frac{HX^3P^2}{L\mu'} \frac{3\pi}{8\gamma} \int_0^1 \Omega^3 \left(\frac{\partial \Pi}{\partial \rho} \right)^2 \left(1 - \frac{2h}{H} + O(h/H)^2 \right)^2 d\rho. \quad (8)$$

And so it is again clear that D_f is approximated according to Eq. (16, Main Text) provided that the characteristic quantities $\{L, X, P\}$ are chosen such that $\{\gamma, \Omega, \Pi\}$ are all $O(1)$.

62 References

- 63 Bungier, A.P., 2012. Analysis of the Power Input Needed to Propagate Mul-
64 tiple Hydraulic Fractures. Technical Report EP128743. CSIRO Earth Sci-
65 ence and Resource Engineering. Melbourne, Australia. Submitted to In-
66 ternational Journal of Solids and Structures.
- 67 Crouch, S., Starfield, A., 1983. Boundary Element Methods in Solid Me-
68 chanics. Unwin Hyman, London.
- 69 Nordgren, R., 1972. Propagation of vertical hydraulic fractures. J. Pet. Tech.
70 253, 306–314. (SPE 3009).
- 71 Sneddon, I.N., 1946. The distribution of stress in the neighborhood of a crack
72 in an elastic solid. Proc. Roy. Soc. London A 187, 229–260.
- 73 Zhang, X., Jeffrey, R.G., Thiercelin, M., 2007. Deflection and propagation
74 of fluid-driven fractures at frictional bedding interfaces: A numerical in-
75 vestigation. J. Struct. Geol. 29, 396–410.