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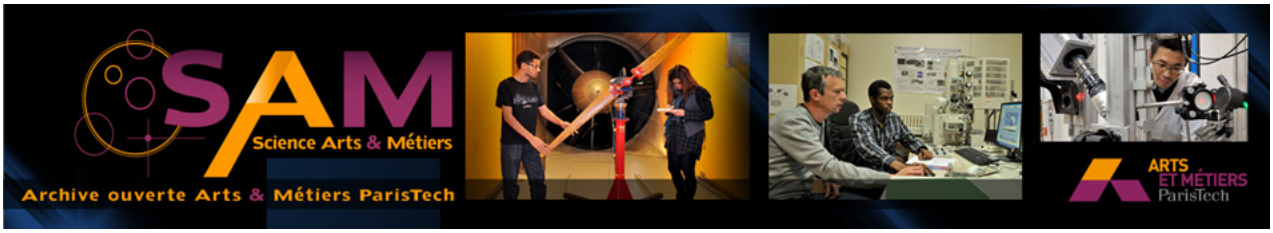

STOCHASTIC POST-PROCESSING CALCULATION OF IRON LOSSES - APPLICATION TO A PMSM

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***Abstract* – To take account of the uncertainties introduced on the magnetic properties during the manufacturing process, the present work focuses on the stochastic modelling of iron losses in electrical machine stators. The investigated samples are composed of 28 slinky stators, coming from the same production chain. The stochastic modelling approach is first described. Thereafter, the Monte-Carlo sampling method is used to calculate, in post-processing, the iron loss density in a PMSM that is modelled by the finite element method. The interest of such approach is emphasized by calculating the main statistical characteristics associated to the losses variability**.

I. INTRODUCTION

Proper modelling and dimensioning of electrical machines require the knowledge of magnetic steel properties, such as the magnetic behaviour law and iron losses. Usually, the dimensioning is reliable when the input data, related to the geometry and physical parameters, are assumed to be well known. In that context, many works have been concerned with the post-processing of iron losses calculation for the numerical modelling of electrical machines [2] in deterministic cases, meaning that uncertainties related to magnetic properties are neglected. Such assumption proves to be insufficient as the manufacturing of electrical machine magnetic parts, from the cutting of laminations till the final magnetic core shape, may introduce uncertainties on the magnetic properties, especially on the iron losses [1]. Therefore, the development of models that take account of these uncertainties is relevant for electrical machine designers. In [3], an experimental approach involving a sample of twenty eight slinky stators, manufactured from the same assembly line, is investigated to take account of the iron loss variability. Studying this kind of stator is of interest as the manufacturing process, consisting in a long strip of steel lamination that is progressively punched and rolled up in a spiral way, may have noticeable impact and variability on the magnetic properties of the material. Following this work, a stochastic model of iron losses parameters is developed in this paper. Note that the modelling approach is similar to those presented in the field of fatigue crack growth in mechanical probabilistic modelling. These models are mainly interested in describing the stochastic aspect of the fatigue of material, using the Paris Erdogan model [10].

In our case, the developed model is used for stochastic Finite Element (FE) post-processing calculation of the iron losses in the stator of a synchronous electrical machine, using the Monte Carlo Simulation Method.

II. IRON LOSSES VARIABILITY

A. Experimental approach

Twenty eight samples of slinky stators made from the same standard lamination grade M800-50A and the same geometry are investigated. In a first step, the variability of the iron losses is quantified from the experiment. To this end, the magnetic characterization is carried out using an excitation winding that creates a magnetic flux in the yoke along its perimeter and a secondary winding that measures the magnetic flux density (Fig.1). Preliminary analysis has been realised, concerning the repeatability of measurements and the influence of manual windings. Results showed that these aspects do not present a significant variability. In the following, all samples have been characterized for 5 levels of the magnetic flux density B_{max} (from 0.75T to 1.5T) and 6 levels of frequency (5Hz to 200Hz).

Fig.1.Stator samples

Statistical analysis were performed on the measured iron losses, from one sample to another, and the variability was quantified in term of the coefficient of variation (Cv), which is the ratio of the standard deviation (Std) to the mean.

At 50Hz, the calculated Cv is approximately 6% for all B_{max} level. Iron losses for the 28 samples measured at 1.5T and 50Hz are illustrated in figure 2.

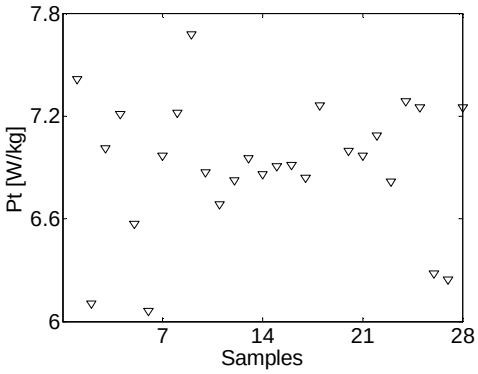

Fig.2.Iron losses at 1.5T and 50Hz of the 28 samples of stator

The variability of the iron losses appears clearly and reflects the non repeatability of the residual constraints introduced by the manufacturing process. More detail on the experimentation may be consulted in [3].

B. Stochastic modelling approach

In order to account for this variability, the following steps have been considered:

- *Deterministic model-parameters identification:* the first step consists in identifying the parameters of the deterministic model for each experimental trajectory. Therefore, classical least square fitting method may be implemented.
- *Probabilistic modelling of parameters*: this step deals with the identification of the probability distribution functions (PDF) of the input parameters. This can be achieved in the context of a parametric approach for which classical probability distribution functions (uniform, Gaussian, lognormal) can be tested with Kolmogorov-Smirnov (KS) test, to check the goodness of fit statistics. More precisely, KS test defines null hypothesis H_0 that consists in assuming that the experimental data are distributed according to the proposed probability density function, at a risk of α%. Then, by computing the maximum distance between empirical Cumulative Distribution Function (CDF) of the experimental data and the CDF of the candidate distribution, one can reject or not the null hypothesis at a risk of α%. In practice, and in many softwares, the distance between these CDF corresponds to a p-value, and the null hypothesis H_0 is rejected if the p-value computed is less than the risk α%. It is also necessary to analyze the inter-dependence of the parameters of the model.
- *Validation of the model:* the validation of the model consists in assessing numerical experimentation, related to the probability distribution of the parameters. Monte Carlo simulations may be used for instance. Moreover, the correlation structure between the parameters has also to be implemented. The KS test is then applied to validate the model in order to check if experimental and simulated CDF are the same. In our case, it is realized for each B_{max} level.
- *Cross Validation (CV) techniques:* for the selected model, one can apply a CV technique [10],[12] which consists in splitting the experimental data in two groups: the model is developed with the first group and then validated with the second group. This approach allows one to check the validity of the probabilistic model for the studied group of samples.

B.1. Deterministic model-Parameters identification

For a sinusoidal supply, the so-called loss separation approach, according to the phenomenological principle proposed by Bertotti [4], is implemented for modelling the iron losses of the samples:

$$P_{tot} = k_h\left(\frac{\Delta B}{2}\right)^{\alpha} f + \frac{1}{T}\int_0^T \left[k_e\left(\frac{dB}{dt}\right)^2 + k_{exc}\left(\frac{dB}{dt}\right)^{3/2}\right] dt \quad (1)$$

where the first term corresponds to the quasi-static hysteresis losses, the second term take account for the classical losses (macroscopic eddy currents) and the third term is related to the excess losses (dynamic behaviour of the magnetic domains). In relation (1), the coefficients (k_h, α, k_e, k_{exc}) are identified for each stator sample, for the whole frequencies and B_{max} levels, with a least square fitting method. The variability of the identified parameters are given in table 1.

TABLE I
VARIABILITY OF IDENTIFIED PARAMETERS OF IRON LOSSES

Parameters	k_h	α	k_e	k_{exc}
Mean	0.067	1.506	6.9×10^{-5}	52×10^{-5}
Sd	47×10^{-4}	23×10^{-3}	9.19×10^{-6}	4.90×10^{-5}
Cv (%)	6.94%	1.57%	13.31%	9.41%

B.2.Probabilistic modelling of the iron loss model parameters

The parametric approach is used to model the variability of the iron losses parameters, and three candidate probability distributions are proposed (uniform, Gaussian, lognormal distribution). These candidate probability distributions are then tested using the Kolmogorov Smirnov (KS) test, at a risk of 5%. For the three candidate probability distribution functions, the p-values of the KS test, at a risk of 5% are summarized in table 2.

TABLE II
P-VALUES OF KOLMOGOROV SMIRNOV TEST FOR THE LOSS MODEL PARAMETERS

	k_h	α	k_e	k_{exc}
Gaussian p-value	0.63	0.73	0.6	0.89
Lognormal p-value	0.53	0.75	0.82	0.75
Uniform p-value	0.0	0.6	0.0	0.0

For the first two lines in this table, the p-values are greater than the risk of 5%, meaning that, for all parameters of the iron loss model, the Gaussian and lognormal distributions are not rejected, at a risk of 5%. At the opposite, the uniform distribution is rejected, except for the parameter α. Moreover, this table suggests also that higher p-value corresponds to a better adequacy between the data and the candidate probability distribution. Therefore, the following conclusion is made:

- (k_h, k_{exc}) are more likely to be Gaussian distributed
- (k_e,α) are more likely to be lognormal distributed

Finally, the Pearson linear correlation between the four parameters is summarized in table 3. One can notice that the correlation is significant for the parameters (k_e,k_{exc}).

TABLE III
LINEAR CORRELATION OF IDENTIFIED PARAMETERS OF IRON LOSSES

	k_h	α	k_e	k_{exc}
k_h	1	0.35	-0.17	-0.04
α	0.35	1	-0.1	0.26
k_e	-0.17	-0.1	1	-0.64
k_{exc}	-0.04	0.26	-0.64	1

B.3.Stochastic Model validation

The validation of the model consists in assessing numerical experimentation, regarding the PDF of the input parameters. It can be achieved by Monte Carlo simulation that involves two main steps: sampling of the probabilistic input variables and performing the deterministic computations to provide the statistic of the output of the model. If a Monte Carlo experiment is to deliver correct results, it has been repeatedly appreciated that correlation among input variables must be taken into account [9]. Indeed, such technique assumes that input variables are independent, meaning that their joint PDF is provided by the product of the marginal distribution. If all the input variables are Gaussian distributed, and only in this case, one can generate Multivariate Gaussian distribution (MGD) random vectors, and the correlation structure is defined by the Pearson linear coefficients [11]. If it is not the case, all the joint distributions among the input variables have to be specified. One can then use the Iman and Conover method [8].

By definition, this method consists in inducing a desired Spearman rank correlation matrix while preserving the marginal distribution for each input variables. It must be noticed that the Pearson and Spearman correlations differ on their theoretical basis. The assumptions underlying the Pearson correlation are that the relationship between two variables is linear and both populations are Gaussian distributed. At the opposite, the Spearman correlation, measuring monotone association, is more flexible in handling nonlinear relationships and is independent of the distribution shape [9].

Therefore, the method takes in its inputs the marginal distributions of individual variables (this can be obtained by using Monte Carlo simulation) and the Spearman rank correlation matrix (identified from experimental data). The correlated vectors are then obtained by rearranging previously generated uncorrelated vectors, according to the rank index of a multivariate reference. The output of the method is a good approximation of the Spearman rank correlation matrix, and the same marginal distribution for each individual parameter [8]~~[8~~-[9]~~9]~~.

The Spearman method has been applied for the iron losses parameters, regarding their marginal PDF. In the first step, 100 000 realizations of the parameter vectors have been simulated independently with Monte Carlo simulation. The rank correlation matrix (table 4), identified from experimental data, and the simulated vectors have been then used as input for the Iman and Conover method.

TABLE IV
SPEARMAN RANK CORRELATION OF IRON LOSSES PARAMETERS

	k_h	α	k_e	k_{exc}
k_h	1	0.27	-0.25	0.028
α	0.269	1	-0.33	0.42
k_e	-0.25	-0.33	1	-0.52
k_{exc}	0.028	0.42	-0.52	1

In the output of the method, the results show that the marginal distribution of each parameter is maintained, and the obtained rank correlation matrix is very close to the desired correlation matrix in table 4.

The parameter realizations from the Monte Carlo method have been then used to simulate the variability of iron losses, at 50Hz and for all B_{max} level. Simulated and experimental CDF, for B_{max}=1.5 T are illustrated in figure 3.

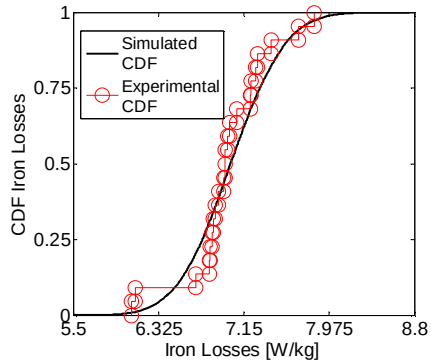

Fig.3. Experimental and simulated CDF for B_{max}=1.5T, at 50Hz

This figure illustrates the good approximation of the experimental data by the simulated CDF. Then, the KS test is applied for all B_{max} levels, at a risk of 5%. For a given B_{max} level, the null hypothesis H_0 assumes that the simulated and experimental CDF are the same. The results show that, for all B_{max} levels, the null hypothesis was not rejected at a risk of 5%, as the p-value of the test were between 0.24 and 0.5.

B.4.Cross Validation Techniques

For the CVT, the iron loss data are split in two subsets: a modelling subset (MS), including 23 trajectories, and a test subsets (TS), constituted of 5 trajectories. The probabilistic model is identified from the MS, and the approach described in the previous section is applied (Independent Monte Carlo simulation and implementation of the Iman and Conover method). From the results, a 95% confidence interval (CI) can be identified and compared to the MS trajectories, as shown in figure 4.

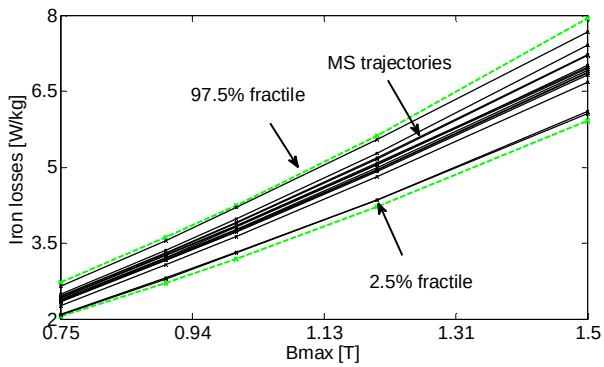

Fig.4. Modelling subsets and 95% Confidence Interval

In this figure, all the MS trajectories lie within the identified CI. This one is compared with the TS trajectories, as shown in figure 5. It can be observed that the entire TS trajectories lie within the 95% CI identified from the MS.

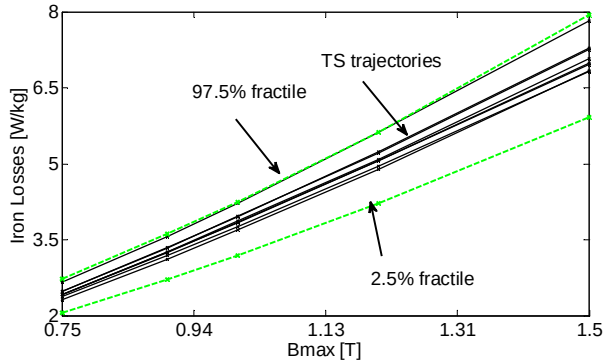

Fig.5. Tests subsets and 95% Confidence Interval

According to all these criteria, it can be considered that the developed stochastic model is representative of the iron losses variability among the studied samples.

In stochastic context, the iron loss model can be written under the form given by (2), where the parameter θ denotes the outcome belonging to the random space.

$$P_{tot} = k_h(\theta)\left(\frac{\Delta B}{2}\right)^{\alpha(\theta)} f + \frac{1}{T}\int_0^T \left[k_e(\theta)\left(\frac{dB}{dt}\right)^2 + k_{exc}(\theta)\left(\frac{dB}{dt}\right)^{3/2}\right] dt \quad (2)$$

One can use the random vector $(k_h(\theta), \alpha(\theta), k_e(\theta), k_{exc}(\theta))$ of the iron loss model obtained in the previous step to deal with the losses calculation in a post-processing step of a FE procedure

III. DETERMINISTIC FINITE ELEMENT MODEL AND UNCERTAINTY PROPAGATION

To investigate the stochastic calculation of the iron losses in a synchronous electrical machine, a 2D FE model is used, for both scalar potential Ω and vector potential **A** formulations (Eq. 3 and 4).

$$\mathrm{div}\left(\mu\left(\mathbf{H}_S - \mathbf{grad}\Omega\right)\right) = 0 \text{ with } \mathbf{curlH}_S = \mathbf{J} \quad (3)$$

$$\mathbf{curl}\left(1/\mu\,\mathbf{curlA}\right) = \mathbf{J} \quad (4)$$

with **J** the current density and μ the magnetic permeability described by the non-linear, and single-valued, magnetic behaviour law model proposed in [5]. Note that the variability on the behaviour law exists but is of second order versus the variability of the iron losses when considering the average B(H) curve from a stator to another. Therefore, in a first step, the variability of the B(H) curve can be neglected and assumed to be deterministic.

The studied electrical machine is a 6 poles, 50Hz three-phase permanent magnet synchronous machine with 36 stator slots (Fig.6) and 6 non-conductive ferrite magnets on the rotor. Considering the symmetry of the system, only a third of the synchronous machine is modelled using 10576 first order elements and 8202 nodes.

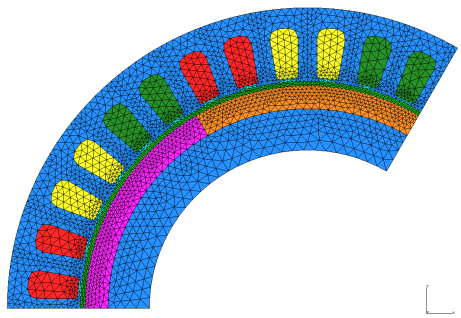

Fig.6. Synchronous machine mesh

IV. RESULTS AND DISCUSSIONS

No load simulations were performed for a remanent magnetic flux density B_r=0.38T in the permanent magnets. A Monte-Carlo procedure, combined with the Iman and Conover method, is then performed to simulate sample of 100,000 realizations of the statistical model according to the distribution of the parameters (k_h, α, k_e, k_{exc}). Thereafter, and for each realization, the expression of iron losses (2) is applied in each element of the stator mesh. On each element, a sample of length 100000 of local losses is then calculated. In this way, a sample of the total losses is obtained by summing all the local losses on each element..

The random distribution of the stator iron losses $P_T(\theta)$ are estimated, and found to be Gaussian distributed (Fig.6), according to the Kolmogorov-Smirnov statistical test. The mean and the standard deviation are given in table 5, showing close results between both formulations. The difference is due to the intrinsic numerical error associated to the discretized formulations but remains acceptable. The variability of the losses is quite large and, for the vector potential **A** formulation, it can be calculated that for a 95% confidence interval, the iron losses are included in the interval [3.43, 4.37] W.

TABLE V
VARIABILITY OF IRON LOSSES FOR A AND Ω FORMULATIONS

	A formulation	Ω formulation
Mean	3.90 W	3.96 W
Std	0.24	0.24
Cv%	6.12%	6.11%

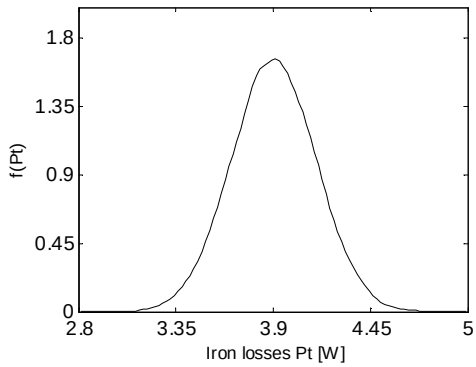

Fig.7. Probability distribution of iron losses density for potential vector **A** formulation

V. CONCLUSION

Stochastic post processing of iron losses is presented in this paper. Iron losses variability of twenty eight SS samples have been first investigated, and stochastic model taking account of this variability is developed. The developed model was validated using statistical test and cross validation techniques in stochastic context. The stochastic model is then used to assess post-processing calculations of iron losses of a pmsm, which can be used to identify confidence interval. Nevertheless, although this approach is well suited for post-processing calculations it can not be used to take account of, for example, the magnetic behaviour law or the geometry uncertainties. In fact, these parameters are intrinsic to the finite element resolution and the FE model must be calculated for each Monte-Carlo realization, which will lead to large computational times. In this context, further simulations will be achieved to reduce the computational time by using the polynomial chaos decomposition approach [6]~~[6~~-[7]~~7]~~.

REFERENCES

[1] W. M. Arshad, T. Ryckebush, F. Magnussen, H. Lendenmann, B.Eriksson, J. Soulard, and B. Malmross, “Incorporating lamination processing and component manufacturing in electrical machine design tools”, in Proc. of Industry Applications Conf., 42nd IAS Annual Meeting, pp. 94–102, 2007

[2] E. Dlala, A. Belahcen, A. Arkkio, “On the Importance of Incorporating Iron Losses in the Magnetic Field Solution of Electrical Machines”, IEEE Trans on Mag. Vol. 46, N°8, pp 3101-3104, 2010.

[3] R. Ramarotafika, A. Benabou, S. Clénet, “Experimental characterization of the iron loss variability in stators of electrical machines”, IEEE Trans on Mag. Vol. 48, N°4 , pp 1629 – 1632, 2012.

[4] G. Bertotti, “General properties of power losses in soft ferromagnetic materials”, IEEE Trans. Mag. Vol. 24, N°1, pp. 621-630, 1998.

[5] Z. Wlodarski, “Analytical description of magnetization curves”, Physica B 373, pp. 323-327, 2006.

[6] R. Gaignaire, S. Clenet, B. Sudret, O. Moreau, “3D Spectral Stochastic Finite Element Method in Electromagnetism”, IEEE Trans on Mag. Vol. 43, N°4 , pp 1209 – 1212, 2007.

[7] H. Mac, S. Clénet, J.C. Mipo, “Transformation method for static field problem with Random Domains”, IEEE Trans. on Mag. Vol. 47, N°5, pp 1446-1449, 2011.

[8] R. L. Iman and W. J. Conover. "distribution-free approach to inducing rank correlation among input variables", in *Communications in Statistics* B11(3), pp 311-334, 1982.

[9] I-T.Yang, "Distribution free Monte Carlo Simulation: Premise and Refinement",Journal of Construction engineering and management, pp 352-360, 2008.

[10] Kotulski, Z. A., “On efficiency of identification of a stochastic crack propagation model based on Virkler experimental data”, Archives on Mechanics, pp 829 – 847, 1998.

[11] R. Ramarotafika, A. Benabou, S. Clénet, “Stochastic modeling of soft magnetic properties of electrical steels: Application to stator of electrical machines”, IEEE Trans. on Mag, *to be published* 2012.

[12] L. Mehrez1, A. Doostan, D. Moens1, D. Vandepitte, “A Validation Study of a Stochastic Representation of Composite Material Properties from Limited Experimental Data”, Proceedings of ISMA 2010-USD 2010, K.U.Leuven, Department Mechanical Engineering, 2010.