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A stability property of Weibull tail-distributions

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Abstract

We exhibit a new stability property of Weibull tail-distributions. It is shown that the power of a random variable with a Weibull tail-distribution still has a Weibull tail-distribution and the associated Weibull tail-coefficient is exhibited.

Weibull tail-distributions are defined through their distribution function by:

$$F(x) = 1 - \exp \left\{ -x^{1/\theta} \ell(x) \right\} \quad (1)$$

where ℓ is a slowly-varying function, see Table 1 for examples, [22] for mixture properties and [20] for a review. A sufficient condition for subexponentiality has been established in [19], Theorem 1. A new diagnostic tool has been proposed in [21].

There exist numerous estimators of $\theta > 0$. Berred [5] uses record values while most of estimators are based on the largest observations of the sample: Asimit *et al.* [1], Beirlant *et al.* [2, 3, 4], Broniatowski [6], Diebolt *et al.* [7, 8], Dierckx *et al.* [9], Gardes *et al.* [10, 11, 12, 13, 18], Girard [14], Goegebeur *et al.* [15, 16] and Mercadier *et al.* [17]. Our main result is a general result on Weibull tail-distributions which has not been shown yet in the extreme-value literature.

Theorem. *Let X be a random variable from the Weibull-tail model (1). Let $Y = X^\alpha$ with $\alpha > 0$. Then, Y follows a Weibull tail-distribution with associated Weibull tail-coefficient $\alpha\theta$.*

Proof. Let us consider

$$P(Y \leq y) = P(X^\alpha \leq y) = P(X \leq y^{1/\alpha}) = F(y^{1/\alpha}) = 1 - \exp \left\{ -y^{1/(\alpha\theta)} \ell(y^{1/\alpha}) \right\}.$$

Remarking that $y \rightarrow \ell(y^{1/\alpha})$ is a slowly-varying function, the conclusion follows.

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Distribution	θ
Gaussian $\mathcal{N}(\mu, \sigma^2)$	1/2
Gamma $\Gamma(\alpha, \lambda)$	1
Weibull $\mathcal{W}(\alpha, \lambda)$	1/ α

Table 1: Weibull tail-distributions