



HAL
open science

A stability property of Weibull tail-distributions

Kim Yu

► **To cite this version:**

| Kim Yu. A stability property of Weibull tail-distributions. 2013. ⟨hal-00851253⟩

HAL Id: hal-00851253

<https://hal.science/hal-00851253v1>

Preprint submitted on 13 Aug 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

A stability property of Weibull tail-distributions

Kim Yu

Shanghai Jiao Tong University, 800 Dong Chuan Road, Shanghai 200240, P.R.China

Abstract

We exhibit a new stability property of Weibull tail-distributions. It is shown that the power of a random variable with a Weibull tail-distribution still has a Weibull tail-distribution and the associated Weibull tail-coefficient is exhibited.

Weibull tail-distributions are defined through their distribution function by:

$$F(x) = 1 - \exp \left\{ -x^{1/\theta} \ell(x) \right\} \quad (1)$$

where ℓ is a slowly-varying function, see Table 1 for examples, [22] for mixture properties and [20] for a review. A sufficient condition for subexponentiality has been established in [19], Theorem 1. A new diagnostic tool has been proposed in [21].

There exist numerous estimators of $\theta > 0$. Berred [5] uses record values while most of estimators are based on the largest observations of the sample: Asimit *et al.* [1], Beirlant *et al.* [2, 3, 4], Broniatowski [6], Diebolt *et al.* [7, 8], Dierckx *et al.* [9], Gardes *et al.* [10, 11, 12, 13, 18], Girard [14], Goegebeur *et al.* [15, 16] and Mercadier *et al.* [17]. Our main result is a general result on Weibull tail-distributions which has not been shown yet in the extreme-value literature.

Theorem. *Let X be a random variable from the Weibull-tail model (1). Let $Y = X^\alpha$ with $\alpha > 0$. Then, Y follows a Weibull tail-distribution with associated Weibull tail-coefficient $\alpha\theta$.*

Proof. Let us consider

$$P(Y \leq y) = P(X^\alpha \leq y) = P(X \leq y^{1/\alpha}) = F(y^{1/\alpha}) = 1 - \exp \left\{ -y^{1/(\alpha\theta)} \ell(y^{1/\alpha}) \right\}.$$

Remarking that $y \rightarrow \ell(y^{1/\alpha})$ is a slowly-varying function, the conclusion follows.

References

- [1] V. Asimit, D. Li, and L. Peng. Pitfalls in using Weibull tailed distributions. *Journal of Statistical Planning and Inference*, 140:2018–2024, 2010.

- [2] J. Beirlant, C. Bouquiaux, and B. Werker. Semiparametric lower bounds for tail index estimation. *Journal of Statistical Planning and Inference*, 136:705–729, 2006.
- [3] J. Beirlant, M. Broniatowski, J.L. Teugels, and P. Vynckier. The mean residual life function at great age: applications to tail estimation. *Journal of Statistical Planning and Inference*, 45:21–48, 1995.
- [4] J. Beirlant, J.L. Teugels, and P. Vynckier. *Practical analysis of extreme values*. Leuven University Press, Leuven, Belgium, 1996.
- [5] M. Berred. Record values and the estimation of the Weibull tail-coefficient. *Comptes-Rendus de l'Académie des Sciences*, T. 312, Série I:943–946, 1991.
- [6] M. Broniatowski. On the estimation of the Weibull tail coefficient. *Journal of Statistical Planning and Inference*, 35:349–366, 1993.
- [7] J. Diebolt, L. Gardes, S. Girard, and A. Guillo. Bias-reduced estimators of the Weibull tail-coefficient. *Test*, 17:311–331, 2008.
- [8] J. Diebolt, L. Gardes, S. Girard, and A. Guillo. Bias-reduced extreme quantiles estimators of Weibull tail-distributions. *Journal of Statistical Planning and Inference*, 138:1389–1401, 2008.
- [9] G. Dierckx, J. Beirlant, D. De Waal, and A. Guillo. A new estimation method for Weibull-type tails based on the mean excess function. *Journal of Statistical Planning and Inference*, 139(6):1905–1920, 2009.
- [10] L. Gardes and S. Girard. Estimating extreme quantiles of Weibull tail-distributions. *Communication in Statistics - Theory and Methods*, 34:1065–1080, 2005.
- [11] L. Gardes and S. Girard. Comparison of Weibull tail-coefficient estimators. *REVSTAT - Statistical Journal*, 4(2):163–188, 2006.
- [12] L. Gardes and S. Girard. Estimation of the Weibull tail-coefficient with linear combination of upper order statistics. *Journal of Statistical Planning and Inference*, 138:1416–1427, 2008.
- [13] L. Gardes, S. Girard, and A. Guillo. Weibull tail-distributions revisited: a new look at some tail estimators. *Journal of Statistical Planning and Inference*, 141(1):429–444, 2011.
- [14] S. Girard. A Hill type estimate of the Weibull tail-coefficient. *Communication in Statistics - Theory and Methods*, 33(2):205–234, 2004.
- [15] Y. Goegebeur, J. Beirlant, and T. de Wet. Generalized kernel estimators for the Weibull-tail coefficient. *Communications in Statistics - Theory and Methods*, 39(20):3695–3716, 2010.

- [16] Y. Goegebeur and A. Guillou. Goodness-of-fit testing for Weibull-type behavior. *Journal of Statistical Planning and Inference*, 140(6):1417–1436, 2010.
- [17] C. Mercadier and P. Soulier. Optimal rates of convergence in the Weibull model based on kernel-type estimators. *Statistics and Probability Letters*, 82:548–556, 2011.
- [18] J. El Methni, L. Gardes, S. Girard, and A. Guillou. Estimation of extreme quantiles from heavy and light tailed distributions. *Journal of Statistical Planning and Inference*, 142(10):2735–2747, 2012.
- [19] K. Yu. On the subexponentiality of Weibull tail-distributions. <http://hal.archives-ouvertes.fr/hal-00766277>, 2012.
- [20] K. Yu. Weibull tail-distributions: A bibliography. <http://hal.archives-ouvertes.fr/hal-00764041>, 2012.
- [21] K. Yu and C. Zhu. A diagnostic tool for Weibull tail-distributions. <http://hal.archives-ouvertes.fr/hal-00770524>, 2013.
- [22] K. Yu and C. Zhu. Mixtures of Weibull tail-distributions. <http://hal.archives-ouvertes.fr/hal-00769531>, 2013.

Distribution	θ
Gaussian $\mathcal{N}(\mu, \sigma^2)$	1/2
Gamma $\Gamma(\alpha, \lambda)$	1
Weibull $\mathcal{W}(\alpha, \lambda)$	1/ α

Table 1: Weibull tail-distributions