



HAL
open science

Energy, decay rate, and effective masses for a moving polaron in a Fermi sea: Explicit results in the weakly attractive limit

Christian Trefzger, Yvan Castin

► **To cite this version:**

Christian Trefzger, Yvan Castin. Energy, decay rate, and effective masses for a moving polaron in a Fermi sea: Explicit results in the weakly attractive limit. 2013. hal-00847562v1

HAL Id: hal-00847562

<https://hal.science/hal-00847562v1>

Preprint submitted on 23 Jul 2013 (v1), last revised 20 Dec 2013 (v3)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Energy, decay rate, and effective masses for a moving polaron in a Fermi sea: Explicit results in the weakly attractive limit

Christian Trefzger and Yvan Castin

Laboratoire Kastler Brossel, École Normale Supérieure and CNRS, UPMC, 24 rue Lhomond, 75231 Paris, France

We study the properties of an impurity of mass M moving through a spatially homogeneous three-dimensional Fermi gas of particles of same spin state and mass m . In the weakly attractive limit, where the effective coupling constant $g \rightarrow 0^-$ and perturbation theory can be used, we analytically calculate the complex energy $\Delta E(\mathbf{K})$ of the moving impurity up to order two included in g . This also gives access to its longitudinal and transverse effective masses $m_{\parallel}^*(\mathbf{K})$, $m_{\perp}^*(\mathbf{K})$, as functions of the impurity wave vector \mathbf{K} . Depending on the modulus of \mathbf{K} and on the impurity-to-fermion mass ratio M/m we identify four regions separated by singularities in derivatives with respect to \mathbf{K} of the second-order term of $\Delta E(\mathbf{K})$, and we discuss the physical origin of these regions. Remarkably, the second-order term of $m_{\parallel}^*(\mathbf{K})$ presents non-differentiable points, as well as a logarithmic divergence for $M = m$, when \mathbf{K} is on the Fermi surface of the fermions.

PACS numbers: 03.75.Ss, 34.50.Cx, 05.30.Fk, 71.10.Ca

Recent cold atom experiments have reached an unprecedented accuracy in measuring the equation of state of an interacting Fermi gas [1–3]. It is now accepted that in strongly spin-polarized configurations the minority atoms dressed by the Fermi sea of the majority atoms form a normal gas of quasi-particles called Fermi polarons [4–6]. Of particular interest is the weakly attractive or BCS limit, where deviations of the polaron equation of state from the grand canonical ideal gas [7] were observed [2].

In general, for an arbitrary interaction strength, a Fermi liquid theory can be used to describe polarons at low momenta [8], with effective parameters that need to be determined with a microscopic theory [4–7, 9–14] or from experiments [1, 2, 15–18]. In this work, we focus on the weakly attractive regime $k_F a \rightarrow 0^-$, where a is the s -wave scattering length of a minority atom and a fermion and k_F is the Fermi wave number of the majority atoms. Using a systematic expansion in powers of $k_F a$, we then go beyond the Fermi liquid description: We determine not only the decay rate, but also the real part of the polaronic complex energy, and being not restricted to low momenta, we have access to momentum-dependent effective masses of the polaron. Within our microscopic approach we have a complete description of the system: We determine a phase diagram where singularities in derivatives of the second-order term of the polaronic complex energy mark the boundaries of different regions as shown in Fig.1. The physical origin of the singularities is also discussed.

At zero temperature, we consider in three dimensions an ideal Fermi gas of particles of same spin state and mass m perturbed by the presence of a moving impurity of mass M and momentum $\hbar\mathbf{K}$. While we assume no interactions among the fermions, the impurity interacts with each fermion through a s -wave interaction of negligible range, e.g. on a broad Feshbach resonance. The system is enclosed in a quantization volume \mathcal{V} with peri-

odic boundary conditions, and is described by the single-channel Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}$:

$$\hat{H}_0 = \sum_{\mathbf{k}} \left(\varepsilon_{\mathbf{k}} \hat{u}_{\mathbf{k}}^{\dagger} \hat{u}_{\mathbf{k}} + E_{\mathbf{k}} \hat{d}_{\mathbf{k}}^{\dagger} \hat{d}_{\mathbf{k}} \right), \quad (1)$$

$$\hat{V} = \frac{g_0}{\mathcal{V}} \sum_{\substack{\mathbf{k}_1, \mathbf{k}_2 \\ \mathbf{k}_3, \mathbf{k}_4}} \delta_{\mathbf{k}_1 + \mathbf{k}_2, \mathbf{k}_3 + \mathbf{k}_4}^{\text{mod}} \hat{d}_{\mathbf{k}_4}^{\dagger} \hat{u}_{\mathbf{k}_3}^{\dagger} \hat{u}_{\mathbf{k}_2} \hat{d}_{\mathbf{k}_1}, \quad (2)$$

where $\hat{u}_{\mathbf{k}}$ ($\hat{d}_{\mathbf{k}}$) annihilates a fermion (impurity) with wave vector \mathbf{k} and kinetic energy $\varepsilon_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m}$ ($E_{\mathbf{k}} = \frac{\hbar^2 k^2}{2M}$). To avoid ultraviolet divergences we use a standard lattice model where the positions of the particles are discretized on a cubic lattice of elementary step $b \ll k_F^{-1}$ and binary interactions between the impurity and a fermion take place at the same lattice site with a bare coupling constant g_0 [19]. The model provides automatically a cutoff in the Fourier space since wave vectors belong to the first Brillouin zone. In Eq. (2), the modified Kronecker delta is to be understood modulo a vector of the reciprocal lattice and ensures conservation of the quasi-momentum. The bare coupling constant g_0 is then related to the scattering length a through the formula

$$\frac{1}{g_0} = \frac{1}{g} - \int_{\text{FBZ}} \frac{d^3 k}{(2\pi)^3} \frac{2\mu}{\hbar^2 k^2} \quad \text{and} \quad g = \frac{2\pi \hbar^2 a}{\mu}, \quad (3)$$

where the integral is taken in the first Brillouin zone, $\text{FBZ} = [-\pi/b, \pi/b]^3$, and $\mu = mM/(m+M)$ is the reduced mass. In practice, at the end of the perturbative calculations to come, we will take the thermodynamic limit, where $\mathcal{V} \rightarrow +\infty$, and also the continuous space limit $b \rightarrow 0^+$, so that the first Brillouin zone will converge to the whole Fourier space.

The goal is to calculate the complex energy of an impurity of wave vector \mathbf{K} moving in the zero temperature Fermi gas of N particles. The ground state of the N fermions is the usual Fermi sea [FS] of energy $E_{\text{FS}}(N)$. In the absence of interactions the ground state of the system is given by the free impurity immersed in the Fermi

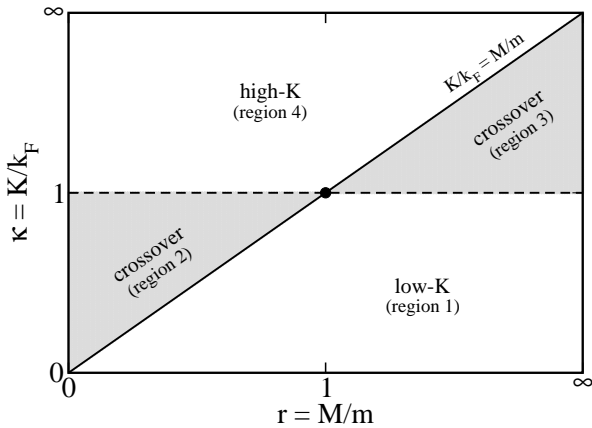


FIG. 1: Phase diagram of the moving polaron in the $M/m - K/k_F$ plane. On the dashed line, $K = k_F$, and on the solid line, $K = k_F M/m$, a logarithmic divergence (discontinuity) appears in the n th order derivative with respect to K of the real part (imaginary part) of the second-order term of the polaronic complex energy (8): $n = 3$ for the dashed line, $n = 4$ for the solid line, and $n = 2$ at the point ($M = m, K = k_F$).

sea

$$|\psi^{(0)}(\mathbf{K})\rangle = \hat{d}_{\mathbf{K}}^\dagger |\text{FS}\rangle, \quad (4)$$

with an energy, measured with respect to the energy of the Fermi sea, given by

$$\Delta E^{(0)}(\mathbf{K}) = \langle \psi^{(0)}(\mathbf{K}) | \hat{H}_0 | \psi^{(0)}(\mathbf{K}) \rangle - E_{\text{FS}}(N) = E_{\mathbf{K}}. \quad (5)$$

We now consider the weakly interacting case $a \rightarrow 0^-$, where a quasi-particle polaronic ground state is known to exist [4–7, 12]. For a fixed lattice spacing b , in the limit where $|a| \ll b$ we can Taylor expand the bare coupling constant (3):

$$g_0 \underset{g \rightarrow 0^-}{=} g + g^2 \int_{\text{FBZ}} \frac{d^3 k}{(2\pi)^3} \frac{2\mu}{\hbar^2 k^2} + O(g^3), \quad (6)$$

and in this weakly attractive limit one can treat \hat{V} in the Hamiltonian with usual perturbation theory. Up to second order in \hat{V} , we find in the thermodynamic limit that the energy of the moving polaron is complex:

$$\Delta E(\mathbf{K}) \underset{a \rightarrow 0^-}{=} E_{\mathbf{K}} + \rho g_0 - \int'_{\text{FBZ}^2} \frac{d^3 k d^3 q}{(2\pi)^6} \left(\frac{g_0^2}{E_{\mathbf{K}+\mathbf{q}-\mathbf{k}} + \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}} - E_{\mathbf{K}} - i\eta} \right) + O(g_0^3), \quad (7)$$

where the prime on the integral means that it is restricted to $0 < q < k_F$ and to $k > k_F$, and we have introduced the mean density of the Fermi sea $\rho = N/\mathcal{V}$ related to the Fermi wave number k_F by the usual relation $k_F = (6\pi^2 \rho)^{1/3}$, $E_F = \frac{\hbar^2 k_F^2}{2m}$, and $\eta \rightarrow 0^+$. Substituting Eq. (6) into Eq. (7) and keeping only terms up to

order g^2 , as announced we get a finite expression for a vanishing lattice spacing

$$\Delta E(\mathbf{K}) \underset{g \rightarrow 0^-}{=} E_{\mathbf{K}} + \rho g + \frac{(\rho g)^2}{E_F} f(K/k_F) + O(g^3). \quad (8)$$

The dimensionless complex function $f(K/k_F)$ is defined as follows:

$$f(K/k_F) = \frac{3r}{r+1} + \frac{E_F}{\rho^2} \int_{\mathbb{R}^6} \frac{d^3 k d^3 q}{(2\pi)^6} \left(\frac{2\mu}{\hbar^2 k^2} - \frac{1}{E_{\mathbf{K}+\mathbf{q}-\mathbf{k}} + \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{q}} - E_{\mathbf{K}} - i\eta} \right), \quad (9)$$

where $r = M/m$ is the impurity-to-fermion mass ratio.

The integral in Eq. (9) can be calculated exactly. Using the Dirac relation $\lim_{\eta \rightarrow 0^+} (x - i\eta)^{-1} = P \frac{1}{x} + i\pi \delta(x)$, where P is the principal value and $\delta(x)$ the Dirac distribution, we can separate the integral into its real and imaginary parts. A first step then consists in averaging over the direction of \mathbf{K} : The expansion of $E_{\mathbf{K}+\mathbf{q}-\mathbf{k}}$ in powers of K reveals that it is convenient to use spherical coordinates of polar axis given by the direction of $\mathbf{k} - \mathbf{q}$. Second, using again spherical coordinates, the integral over \mathbf{k} and \mathbf{q} reduces to a triple integral over k , q and the angle θ between \mathbf{k} and \mathbf{q} . Then taking $\lambda = |\mathbf{k} - \mathbf{q}|$ rather than θ as the integration variable [20] one obtains for the real part

$$\Re f(K/k_F) = \frac{3r}{r+1} + \frac{9r}{2k_F^4} \int_{k_F}^{+\infty} k dk \int_0^{k_F} q dq \int_{k-q}^{k+q} \lambda d\lambda \left[\frac{1}{(r+1)k^2} - \frac{1}{2y} \ln \left| \frac{x+y}{x-y} \right| \right] \quad (10)$$

where the modulus in the argument of the \ln originates from the principal value. For the imaginary part one has

$$\Im f(K/k_F) = -\frac{9\pi r}{8Kk_F^4} \int_{k_F}^{+\infty} k dk \int_0^{k_F} q dq \int_{k-q}^{k+q} d\lambda Y(1-x/y) \quad (11)$$

where Y is the Heaviside function and we have introduced the positive quantities $x = (k^2 - q^2)r + \lambda^2$, and $y = 2\lambda K$. Remarkably, the triple integrals in Eqs. (10,11) can be evaluated explicitly leading to the forthcoming expressions.

The imaginary part of Eq. (8) reveals that due to scattering of the impurity with the fermions, a moving polaron radiates particle-hole pairs [8] and thus decays out of its initial momentum channel, at a rate Γ_0 given by

$$\frac{\hbar \Gamma_0}{2} \underset{g \rightarrow 0^-}{=} -\frac{(\rho g)^2}{E_F} \Im f(K/k_F) + O(g^3). \quad (12)$$

The polaron decay rate is an isotropic function of the impurity wave vector \mathbf{K} . Depending on the impurity-to-fermion mass ratio $r = M/m$ and on the modulus

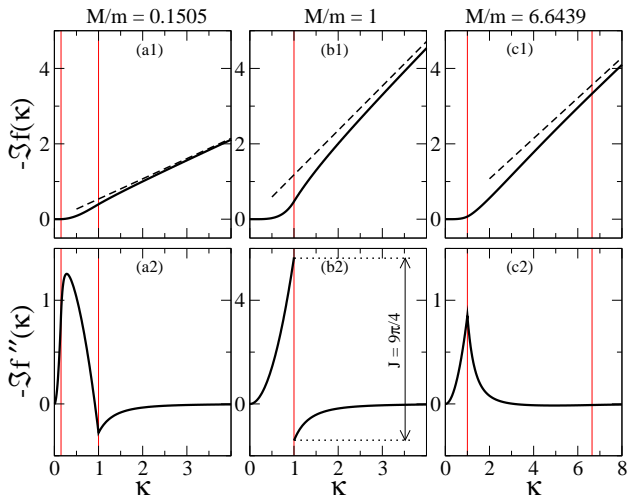


FIG. 2: (Color online) Function $-\Im f(\kappa)$ entering the imaginary part of Eq. (8) (row 1), and its second derivative entering the imaginary part of the inverse effective mass Eq. (22) (row 2), for various mass ratios. Dashed lines correspond to the asymptote (17) of Eq. (16). Red vertical lines correspond to the phase boundaries at $\kappa = 1$ and at $\kappa = M/m$.

of \mathbf{K} , $\Im f(K/k_F)$ assumes different piecewise smooth (indefinitely derivable) functional forms on the four different regions represented in Fig. 1. (i) The low- K region, $0 < K/k_F < \min(1, r)$, where

$$-\Im f(\kappa) \underset{\text{region 1}}{=} \frac{3\pi}{20r} \kappa^4. \quad (13)$$

For an ideal Fermi gas containing a vanishing density of impurities, a similar κ^4 power law was obtained in Ref. [8], but *only* in a small κ expansion of $\Im f(\kappa)$. Eq. (13) is not implied by the result of [8] because it is valid in an entire region of parameters. Notice that for a polaron at rest, i.e. $\mathbf{K} = \mathbf{0}$, one has $\Im f(0) = 0$ as expected. (ii) The $r < 1$ (light impurity) crossover region, $\min(1, r) < K/k_F < \max(1, r)$, where

$$-\Im f(\kappa) \underset{\text{region 2}}{=} \frac{3\pi r}{20(r^2 - 1)^2} \times [(r^2 - 2)\kappa^4 + 10\kappa^2 - 20r\kappa + 15r^2 - 4r^3/\kappa]. \quad (14)$$

(iii) The $r > 1$ (heavy impurity) crossover region, $\min(1, r) < K/k_F < \max(1, r)$, where

$$-\Im f(\kappa) \underset{\text{region 3}}{=} \frac{3\pi r}{20(r^2 - 1)^2} \times \left[\frac{\kappa^4}{r^2} - 10\kappa^2 + 10(r^2 + 1)\kappa - 15r^2 + \frac{6r^2 - 2}{\kappa} \right]. \quad (15)$$

Notice that there is no crossover region for a unit impurity-to-fermion mass ratio, $r = 1$. (iv) Finally the

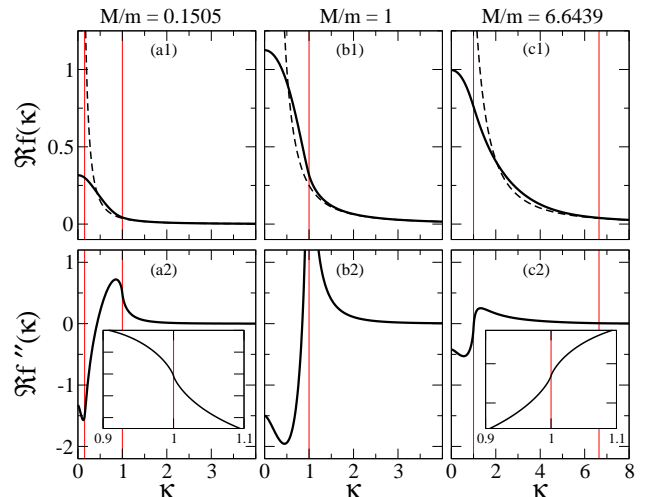


FIG. 3: (Color online) Function $\Re f(\kappa)$ entering the real part of Eq. (8) (row 1), and its second derivative entering the real part of the inverse effective mass Eq. (22) (row 2), for various mass ratios. Dashed lines: Asymptotic behavior (20). Red vertical lines: Phase boundaries at $\kappa = 1$ and at $\kappa = M/m$. Insets: Enlargements showing the non-differentiability of the second-order term of $M/m_{\parallel}^*(\mathbf{K})$ at $K = k_F$.

high- K region, $\max(1, r) < K/k_F$, where

$$-\Im f(\kappa) \underset{\text{region 4}}{=} \frac{3\pi r}{10(r+1)^2} \frac{5\kappa^2 - 2r - 1}{\kappa}. \quad (16)$$

Note that our Eqs. (13, 16) generalize to $r \neq 1$ the results already obtained in [11].

For a large impurity wave vector, $K/k_F \gg \max(1, r)$, the asymptotic value of $-\Im f(\kappa)$ is given by the asymptotic expansion of Eq. (16), and is linear in K :

$$-\Im f(K/k_F) \underset{K/k_F \gg \max(1, r)}{\sim} \frac{3\pi r K}{2(r+1)^2 k_F}. \quad (17)$$

Interestingly, for $r = 1$, Eq. (17) is consistent with the imaginary part of the complex energy (21.20) of Ref. [21], obtained for an unpolarized spin 1/2 Fermi gas. This is due to the fact that, at large K/k_F , the Pauli blocking effect induced by the Fermi sea of impurities (present in Ref. [21] and absent in our case) becomes negligible. For an arbitrary r , one recovers Eq. (17) with a simple kinetic argument which also neglects the Pauli blocking effect provided $K \gg (r+1)k_F$: In a collision of the impurity with a fermion of typical wave vector k_F , their relative velocity is $v \simeq \hbar K/M$, and $\sigma = 4\pi a^2$ is the impurity-to-fermion scattering cross section; the collision rate of the impurity with the atoms in the Fermi sea of density ρ is then given by $\gamma = \rho \sigma v$, and $\hbar \gamma$ reproduces Eq. (17). In Fig. 2 we plot the function $-\Im f(\kappa)$ for different values of the mass ratio between the impurity and the fermions.

Let us now discuss the real part of the polaronic complex energy (8) for which much less was known. We find

that $\Re f(K/k_F)$ is an isotropic function of \mathbf{K} with the following analytic expression

$$\begin{aligned} \Re f(\kappa) = & \frac{3r}{r+1} - \frac{3r}{20(r^2-1)^2} \left\{ \frac{(r^2-1)}{2} (\kappa^2 + 20r - 9) \right. \\ & + \frac{(\kappa-1)^3}{\kappa} [\kappa(\kappa+3)(r^2-2) + 6r^2 - 2] \ln \left| \frac{\kappa-1}{\kappa} \right| \\ & \left. + \frac{(\kappa-r)^4}{\kappa} \left(\frac{\kappa+4r}{r^2} \right) \ln \left| \frac{\kappa-r}{\kappa} \right| + (\kappa \rightarrow -\kappa) \right\}. \quad (18) \end{aligned}$$

The locus of points where the moduli in the argument of the \ln functions vanish forms the boundaries of the same physical regions discussed for $\Im f(K/k_F)$, and sketched in Fig. 1. We have numerically observed that Eq. (18) is a monotonically decreasing function of κ . It has a vanishing derivative at the origin, $\Re f'(0) = 0$, where it reaches its maximum value

$$\Re f(0) = \frac{9r(1-r^2+2r^2 \ln r)}{4(r^2-1)^2}. \quad (19)$$

An expression equivalent to (19) was obtained in [9, 10] in the context of a Λ -particle in a Fermi sea of nucleons with $\nu = 4$ spin-isospin states, hence an extra factor ν in those references. Eq. (18) vanishes at large K/k_F :

$$\Re f(\kappa) \underset{\kappa \gg \max(1,r)}{=} \frac{r}{4\kappa^2} + O(1/\kappa)^4. \quad (20)$$

This indicates that, for large K/k_F and up to second-order in g , the dispersive effect of the Fermi sea on the energy of the impurity reduces to the mean-field term ρg . Contrary to a first impression, Eq. (18) has a finite limit for a unit mass ratio

$$\begin{aligned} \lim_{r \rightarrow 1} \Re f(\kappa) = & \frac{3}{2} - \frac{3}{40} \left[\frac{11}{2} + \kappa^2 + (2\kappa^3 + 4\kappa^2 + 6\kappa + 3) \right. \\ & \left. \times \frac{(\kappa-1)^2}{\kappa} \ln |\kappa-1| - 2\kappa^4 \ln |\kappa| + (\kappa \rightarrow -\kappa) \right], \quad (21) \end{aligned}$$

and as well as for $\Im f(K/k_F)$ the crossover region is absent in this special case $M = m$. In the weakly attractive limit, $g \rightarrow 0^-$, the second-order effect of the Fermi sea on the energy of the impurity moving through it can then be summarized as follows for an arbitrary mass ratio: On the real (imaginary) part of the resulting complex energy the effect of the medium is maximal (minimal) when the impurity is at rest, $\mathbf{K} = \mathbf{0}$, while the effect of the medium is minimal (maximal) when the impurity is moving fast.

A related observable of interest is the effective mass of the polaron. Since the impurity is moving with momentum $\hbar\mathbf{K}$, one can define the effective mass $m_{\parallel}^*(\mathbf{K})$ along the direction of \mathbf{K} , and the effective mass $m_{\perp}^*(\mathbf{K})$ along the direction perpendicular to \mathbf{K} [22]. $m_{\parallel}^*(\mathbf{K})$ is related to the second derivative of the complex energy Eq. (8) with respect to the impurity wave vector K as follows:

$$\frac{M}{m_{\parallel}^*(\mathbf{K})} \underset{g \rightarrow 0^-}{=} 1 + \frac{r}{2} \left(\frac{\rho g}{E_F} \right)^2 f''(\kappa) + O(g^3), \quad (22)$$

and $m_{\perp}^*(\mathbf{K})$ is related to the first derivative of the complex energy with respect to K :

$$\frac{M}{m_{\perp}^*(\mathbf{K})} \underset{g \rightarrow 0^-}{=} 1 + \frac{r}{2} \left(\frac{\rho g}{E_F} \right)^2 \frac{f'(\kappa)}{\kappa} + O(g^3). \quad (23)$$

When $K \rightarrow 0$, Eqs. (22, 23) have the same limit, which coincides with the result of [11], and one recovers the usual rotational symmetry of the effective mass tensor. At non-zero K , Eq. (23) is a differentiable function of K , whereas Eq. (22) presents interesting singularities and is non-differentiable in $\kappa = 1$ as shown in Figs. 2, 3.

Furthermore, for $r = 1$, $\Re f''(\kappa)$ presents a logarithmic divergence $\propto \ln |\kappa - 1|$ in $\kappa = 1$, as can be immediately seen in Eq. (21). This suggests that for $r = 1$, $M/m_{\parallel}^*(\mathbf{K})$ cannot be Taylor expanded in powers of g at the point $\kappa = 1$, as e.g. the function $g^2 \ln |g|$, and a non-perturbative approach must be used, which is beyond the scope of this work. Instead, $\Im f''(\kappa)$ has a jump $J = \Im f''(1^+) - \Im f''(1^-) = 9\pi/4$ in $\kappa = 1$ for $r = 1$.

We now physically interpret the boundaries between regions sketched in Fig. 1. Intuitively, singularities in the polaron complex energy may originate from the fact that (i) the energy denominator in Eq.(7) can vanish, and (ii) the domains of variation of \mathbf{k} and \mathbf{q} in Eq.(7) have sharp boundaries $k = q = k_F$. One thus investigates the form of the Fourier space domain where the energy denominator vanishes for \mathbf{k} and \mathbf{q} *both* at the Fermi surface. For $K < k_F$, it is found that this domain supports all values of $\theta \in [0, \pi/2]$, where θ is the angle between \mathbf{K} and $\mathbf{k} - \mathbf{q}$. For $K > k_F$, on the contrary, the allowed values of θ range from $\arccos(k_F/K)$ to $\pi/2$. This designates $K = k_F$ as a first peculiar line in the plane $M/m - K/k_F$. Note that $\pi - 2\theta$ is the scattering angle of an incoming impurity of wave vector \mathbf{K} on the Fermi sea when *both* the resulting scattered fermion and hole are at the Fermi surface. To obtain the second peculiar line in Fig. 1, we introduce the additional idea that the energy denominator as well as its first order derivatives with respect to \mathbf{k} and to \mathbf{q} vanish at $k = q = k_F$, which indeed leads to $K = r k_F$.

In conclusion, in the weakly attractive limit, we have calculated analytically the energy, the decay rate, and the effective masses of an impurity moving with momentum $\hbar\mathbf{K}$ through a fully polarized Fermi sea. In what concerns the second-order contributions in the coupling constant g , our results show the existence of four physical regions separated by singularities of system's observables, as e.g. the effective mass. We have characterized the order and the physical origin of these singularities, which lays in the impurity-to-fermion scattering at the Fermi surface.

We acknowledge financial support from the ERC Project FERLODIM N.228177. C. Trefzger acknowledges support from a Marie Curie Intra European grant INTERPOL N.298449 within the 7th European Community Framework Programme.

-
- [1] N. Navon, S. Nascimbène, F. Chevy, C. Salomon, *Science* **328**, 729 (2010).
- [2] S. Nascimbène, N. Navon, K. Jiang, F. Chevy, C. Salomon, *Nature (London)* **463**, 1057 (2010).
- [3] K. Van Houcke, F. Werner, E. Kozik, N. Prokof'ev, B. Svistunov, M.J.H. Ku, A.T. Sommer, L.W. Cheuk, A. Schirotzek, M.W. Zwierlein, *Nature Phys.* **8**, 366 (2012).
- [4] C. Lobo, A. Recati, S. Giorgini, S. Stringari, *Phys. Rev. Lett.* **97**, 200403 (2006).
- [5] F. Chevy, *Phys. Rev. A* **74**, 063628 (2006); R. Combescot, A. Recati, C. Lobo, F. Chevy, *Phys. Rev. Lett.* **98**, 180402 (2007).
- [6] N. Prokof'ev, B. Svistunov, *Phys. Rev. B* **77**, 020408(R) (2008); *ibid.*, *Phys. Rev. B* **77**, 125101 (2008).
- [7] C. Mora, F. Chevy, *Phys. Rev. Lett.* **104**, 230402 (2010).
- [8] G.M. Bruun, A. Recati, C.J. Pethick, H. Smith, S. Stringari, *Phys. Rev. Lett.* **100**, 240406 (2008).
- [9] J.D. Walecka, *Nuovo Cimento* **16**, 342 (1960).
- [10] R.F. Bishop, *Nucl. Phys. B* **17**, 573 (1970).
- [11] R.F. Bishop, *Ann. Phys.* **78**, 391 (1973).
- [12] R. Combescot, S. Giraud, *Phys. Rev. Lett.* **101**, 050404 (2008).
- [13] P. Massignan, G.M. Bruun, *Eur. Phys. J. D* **65**, 83 (2011).
- [14] J. Vlietinck, J. Ryckebusch, K. Van Houcke, *Phys. Rev. B* **87**, 115133 (2013).
- [15] A. Schirotzek, Cheng-Hsun Wu, A. Sommer, M. W. Zwierlein, *Phys. Rev. Lett.* **102**, 230402 (2009).
- [16] S. Nascimbène, N. Navon, K.J. Jiang, L. Tarruell, M. Teichmann, J. McKeever, F. Chevy, C. Salomon, *Phys. Rev. Lett.* **103**, 170402 (2009).
- [17] S. Nascimbène, N. Navon, S. Pilati, F. Chevy, S. Giorgini, A. Georges, C. Salomon, *Phys. Rev. Lett.* **106**, 215303 (2011).
- [18] C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, G. M. Bruun, F. Schreck, R. Grimm, *Nature* **485**, 615 (2012).
- [19] C. Mora, Y. Castin, *Phys. Rev. A* **67**, 053615 (2003); L. Pricoupenko, Y. Castin, *J. Phys. A* **40**, 12863 (2007); E. Burovski, N. Prokof'ev, B. Svistunov, M. Troyer, *New J. Phys.* **8**, 153 (2006); O. Juillet, *New J. Phys.* **9**, 163 (2007).
- [20] C. Trefzger, Y. Castin, *Europhys. Lett.* **101**, 30006 (2012).
- [21] E. M. Lifshitz and L.P. Pitaevski, *Statistical Physics Part 2*, Pergamon Press 1980.
- [22] Neil W. Ashcroft, N. David Mermin, *Solid State Physics*, HRW International Editions (1976).