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Comparing Eugene P. Wigner, Raymond Duval and Richard P. Feynman

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Abstract. Inspired by some basic formal aspects of *standard model* of fundamental physics, in this paper, putting into comparison the Eugene Wigner's theoretical definition of elementary particle with some aspects of the semiotic theory according to Raymond Duval, it will be possible to lay out the former into the latter theory which will get a further epistemological foundation also on the basis of the experimental validations of the standard model itself. Moreover, a final comparison with some epistemological considerations due to Richard Feynman will be also explained in the light of the previous ones. Finally, where possible, the discussion will be also compared with the general ontological perspective.

Sunto. Sulla base di alcuni precipui aspetti formali del *modello standard* della fisica delle interazioni fondamentali, mettendo, in particolare, a confronto la definizione formale di particella elementare secondo Eugene Wigner con alcuni aspetti della teoria semiotica di Raymond Duval, sarà possibile collocare la prima definizione nella seconda teoria che riceverà ulteriore fondamento epistemologico pure sulla scorta delle validazioni sperimentali dello stesso modello standard. Inoltre, un confronto finale con alcune considerazioni epistemologiche dovute a Richard Feynman sarà pure condotto alla luce di quanto precedentemente stabilito. Infine, laddove possibile, la discussione sarà pure vagliata dal generale punto di vista ontologico.

1. A preliminary philosophy of mathematics contextualization

It well-known what basic and primary role has played the formal description of that entity called *elementary particle* in fundamental physics above all *d'après* the pioneering 1930s work of Eugene Paul Wigner (1902-1995). In the period in which he wrote this paper, an epistemological revolution was beginning in physics. Namely, the group theoretical methods started to pervade most of theoretical physics, event which was designed with the (in a certain sense pejorative) appellative of *gruppenpest* by E. Schrödinger, W. Pauli and J.C. Slater (see Wybourne (1973)). Indeed, in that period, on the wake of special relativity and its formulation, group theory bit by bit revealed to be of extreme utility from a mathematical viewpoint both in explaining and foresighting many aspects of theoretical physics. The group theory textbooks of Hermann Weyl (see (Weyl 1928)) and Eugene P. Wigner (see¹ (Wigner 1931)), with their remarkable insights, were rightly celebrated and still unsurpassed models. Thereafter, the group theory was a vital part of theoretical and mathematical framework of physics (and not only) until nowadays. In particular, the group representation theory showed its highest theoretical power, and the 1939 Wigner paper is only one of the main historical cases of this event. This Wigner work has marked a milestone in theoretical physics of elementary particle theory, with great theoretical and experimental consequences for the whole framework of physics until recent time. Likewise, such a work has also notable contributed to give a great impulse towards the epistemological reflection on the foundations of physics if one take into consideration the principles of ontology. The modern philosophy of knowledge (or gnoseology) has put attention on the methodologies of knowledge, or else on the procedures, tools and methods used to achieve it. In this regards, one of the main knowledge's approach is given by the analysis of the various and

¹ Although the Wigner's book has been published in 1931, its content is the material of lecture notes of various courses held by Wigner since 1925. Thereafter, both Weyl (1928) and Wigner (1931) textbooks were enlarged and revised above all in their American translations.

possible *representations* of a given object of knowledge, and their *reality*, that is to say their relations with the external world. All that will be then the basis for a definition of the *entity* defining this object of knowledge, so that we are naturally led to the realm of ontology.

D'après Martin Heidegger work on existentialistic ontology, a crucial role is played by his notion of *ontological difference*, i.e., very roughly, the difference between the Being and the Entity. As early as in the Greek philosophy, the adjective *ontologic* refers to something concerning the essential aspects of being, while the adjective *ontic* refers to the empiric or concrete entity. Nevertheless, it was Heidegger, since 1927, to give them a great importance and relevance from a wider philosophical stance. Following Brezzi (1994), Nicholson (1996) and Abbagnano (1998), there is an irreducible and unavoidable difference between the *entity* (roughly, all that exists) and the *being* (roughly, the very existence of what exists), which refers to the distinction between *ontic truth*, which concerns the entity in its empiricity and factual determination (like in physics, in which, for example, an empirical property of an object is an ontic property), and *ontological truth*, which is a consideration of the entity's being. Heidegger masterfully states that the being is not definable because the definition always remains at the ontic level (or that of the entity). Therefore, the ontological difference establishes "the *not* between being and entity". Its place of revelation is the *Dasein* which, transcending itself, relate with the other entities, so comprising or including their being. Therefore, the being, in itself devoid of foundation, supports secretly all the various entities thanks to this *Dasein*. Thus, what is intelligible and definable only concerns the ontic truth of the entity. On the other hand, from an empirical perspective, the ontology is meant to be as the ordinate exposition of the fundamental characters of being (inherent its ontic nature) that the experience detect in a repeated and constant manner. This last is just one of the main assumptions of Galileian *scientific method*, that is to say, the systematicity, regularity and repeatability of the features of a physical phenomenon. A *representation*, from the general philosophical viewpoint, may mean either the reproduction's act of an object² and the content itself of this act. It can be expressed either as image or idea. The central problem of the modern philosophy regards the *reality* properties of the representation, that is to say, the *concernancy*'s manners with the external world. In this sense, this problematic cannot be eluded by epistemology of physics, in which the notion of representation has assumed a primary (epistemological) *statute* since the occurrence of group theory³. Now, the comparison of the various possible representations is the main way to ascertain these last requirements of relationships with external world, and the group theory provide the main formal tools to get this at least when such representations may have suitable formal dresses, like in theoretical physics.

Following Font et al. (2007)

«To speak about representation is equivalent speaking about, for example, knowledge, meaning, comprehension and modelling. Without doubt, these notions make up one of the central nuclei, not only of our discipline, but also of epistemology, psychology and other sciences and technologies that study human cognition, its nature, origin and development».

According to these authors, who develop an interesting ontological perspective in mathematics called *onto-semiotic approach* (OSA) to representations in mathematics education and foundations,

«The onto-semiotic approach to mathematics cognition tackles the problem of meaning and knowledge representation by elaborating an explicit mathematical ontology based on anthropological [...], semiotic and socio-cultural theoretical frameworks [...]. It assumes socio-epistemic relativity for mathematical knowledge since knowledge is considered to be

² This last being understood, from the general philosophical viewpoint, as that thing to which is oriented an arbitrary mental human act and is passibile of judgment. In this terms, therefore, we are forced to attain to the general notion of representation from the psychological standpoint.

³ In this, a certain preliminary role has been played by Felix Klein Erlangen Program and its philosophy.

indissolubly linked to the activity in which the subject is involved and is dependent on the cultural institution and the social context of which it forms part [...]».

Their starting point is the notion of *semiotic function* due to L. Hjelmslev in linguistic, which will be suitably extended. Precisely, they affirm as follows

«We use 1940s Hjelmslev's notion of function of sign (named by U. Eco as semiotic function), the dependence between a text and its components and between these components themselves. In the onto-semiotic approach, a semiotic function is conceived, interpreting this idea, as the correspondences (relations of dependence or function) between an antecedent (expression, signifier) and a consequent (content, signified or meaning), established by a subject (person or institution) according to a certain criteria or corresponding code. These codes can be rules (habits, agreements) that inform the subjects about the terms that should be put in correspondence in the fixed circumstances. For us, the relations of dependence between expression and content can be representational (one object which is put in place of another for a certain purpose), and instrumental (an object uses another or others' objects as an instrument). In this way, semiotic functions and the associated mathematics ontology take into account the essentially relational nature of mathematics and generalize the notion of representation: the role of representation is not totally undertaken by language (oral, written, graphical, gestures, ...). In accordance with Peirce's semiotics, it is proposed that the different types of objects (problem situations, procedures, concepts, properties and arguments), can also be expressions or content of the semiotic functions».

Clearly, the discussions and the subsequent arguments treated by these authors are more articulated and profound in relation to the context in which lay out their work, and that, amongst other things, mainly starts from the previous work made by G. Goldin. Nevertheless, we stress on the main fact that their debating is centred around the essential *relational nature* of knowledge through which it will be then possible to descry a certain ontological stance by comparing all the possible representation of a given object of knowledge, and go beyond. In the general theory of knowledge, the notion of representation is therefore proposed as a *key element* of a primary unified theoretical framework for research into mathematics education, and not only. According to Goldin

«We should draw the natural conclusion - that it is time to set aside dismissive epistemologies, in order to proceed with concepts that can unify the understandings reached from disparate perspectives».

Again, according to Godino & Font (2010),

«The theory of representations set out by Goldin illustrates how the notion of representation is a potentially key element of a unifying theoretical framework for research into mathematics education. Systems of representation and the development of representational structures during mathematics learning and problem solving are considered as essential components of this framework. [...] The onto-semiotic approach (OSA) to mathematical knowledge [...] show that the notion of semiotic function and the categories of mathematical objects and processes used in the OSA complement the notion of representation, underlining, therefore, the importance of these notions as instruments for analysing mathematical thought and activity».

Herein, we only point out on some possible useful comparisons between this view and perspective, taking into account the Raymond Duval pioneering work, and some of the main aspects of the epistemology of physics after the pioneering work of Eugene P. Wigner in theoretical particle

physics. The resulting discussion will be also compared with some little known but extremely useful elementary epistemological considerations made by Richard P. Feynman on physics, which will receive more evidence in the light of what previously said.

2. The Wigner classification of irreducible representations of Poincaré group

In this section, we mainly will follow Sternberg (1994, Chapter 3, Section 3.9). Minkowski's space-time is a physical-mathematics model of space and time without gravity. The transformations on this model are the Lorentz transformations, known to form the group $O(1,3)$, named *Lorentz group*. The identity component of $O(1,3)$ is $SO^+(1,3)$, named the *proper Lorentz group*. The identity connected component of $SO^+(1,3)$, taken together the translations $\mathbb{R}^{(1,3)}$ of Minkowskian space-time, is the *Poincaré group*, which is its symmetry group. The Poincaré group is a very special group because it is deeply involved in formal description of elementary particles, *d'après* Wigner. Simply, it is the set of all transformations that preserve the Minkowski's metric of Minkowski's space-time. $SL(2, C)$ is the connected component of the identity of $SO^+(1,3)$, and the Poincaré group is therefore defined to be $SL(2, C) \ltimes \mathbb{R}^{(1,3)}$. It is also said to be the *inhomogeneous Lorentz group*.

In 1939, Eugene P. Wigner, in the seminal paper (Wigner 1939), classified the fundamental particles using the irreducible representations of Poincaré group. Wigner was motivated by the idea that symmetry underlies all the physical laws. In particular, a physical experiment should come up with the same results regardless of where, when, or what orientation the experiment is done. An experiment's result should also be invariant whether the experiment is at rest or moving at a constant velocity. It is common to combine several systems together such almost like when protons and electrons combine to form atoms. When this is done, the overall symmetry of the system should be related to the individual symmetries of its components. These components are simply the irreducible representations of the related symmetry group.

The double cover of the Poincaré group acts to classify the fundamental particles in physics and explain patterns in their behaviours. In particular, the particles are most easily classified by the irreducible representations of the double cover of Poincaré group. The representations are determined by the different orbits in the group. These orbits serve to classify types of particles. The first two types of orbits correspond to the value $m^2 > 0$. The value m^2 is equivalent to the idea of mass in physics. Thus elements in the first two orbits correspond to massive particles which travel slower than the speed of light. The particles that correspond to the light cones are those with $m^2 = 0$, which are particles that travel at the speed of light. Examples of these particles are *photons*, from which the light cone gains its name, and *gravitons*, which at present only exist in theory. The particles on the single-sheet hyperboloid with values $m^2 < 0$ are called *tachyons*. They travel faster than the speed of light and have imaginary mass. As tachyons have never been observed, we will not find the representations corresponding this orbit. Finally, the orbit in which $m^2 = 0$ and the particles are not moving through time corresponds to vacuum. Vacuum is devoid of all particles.

It is somewhat surprising that the correspondences between the orbits of the double cover of the Poincaré group and types of particles line up as well as they do. The spin values of particles predicted by physicists are exactly the values that come as an overcome of finding the irreducible representations of the double cover of the Poincaré group. This fact is further explained by the fact that the Poincaré group's covering space is connected and has good properties that emulate those observed in quantum spin. Therefore, summing up, all the recent theories of elementary particles have been shaped by the paper (Wigner 1939), containing the classification of the irreducible representations of $SL(2, C) \ltimes \mathbb{R}^{(1,3)}$. According to Sternberg (1994, Chap. 3, Section 3.9), it is difficult to overestimate the importance of this paper which will certainly stand as one of the great intellectual achievements of 20th century. It has not only provided a framework for the physical

search for elementary particles but has also had a deep influence on the further development of modern mathematics, in particular the theory of group representations.

Summarizing the Wigner's main points, since the logic of physics is quantum mechanics, it follows that the symmetry group of a physical system manifests itself as a unitary (or anti-unitary) representation. Ignoring the anti-unitaries (for example, considering connected groups), one posits that: (i) an elementary particle is an irreducible⁴ unitary representation of the symmetry group G of the given physical theory, where the representations are required to satisfy certain physically reasonable restrictions; (ii) the group G is the group $SL(2, \mathbb{C}) \ltimes \mathbb{R}^{(1,3)}$, the semidirect product of $SL(2, \mathbb{C})$ and the translation group in Minkowski space, i.e. the group G is the double universal covering of the Poincaré group P of those transformations of special relativity which can be continuously deformed to the identity; (iii) all the irreducible representations of G satisfying the physical conditions, can be parametrized by two parameters m and s , where m is a non-negative real number and where s is constrained by $s = 0, 1/2, 1, 3/2$, etc if m is positive, and $s = 0, \pm 1/2, \pm 1, \pm 3/2$, etc if $m = 0$. Here, the parameter m can be identified with the *rest mass* of the particle, and the parameter s can be identified with the *spin*, which, as said above, is a notion related to the intrinsic angular momentum of the particle. The crucial point of the Wigner's notion of elementary particle is that a physical object – in this case, an elementary particle – is cognizable as a G -invariant of the theory having G as symmetry group. And this, as we will see later, is of a fundamental importance from an ontological viewpoint as regards that entity called *elementary particle*.

3. Some elementary notions of group theory

Following Sternberg (1994, Chap. 2), a *representation* of a group G in a vector space V is a homomorphism $\rho_V: G \rightarrow GL(V)$, where $GL(V)$ is the group of invertible linear transformations $h: V \rightarrow V$ with functional composition as group law. If one wants to specify as well the occurrence of the group G , then it is possible to write more specifically ρ_V^G . For a given group G , there may be many different choices for V and ρ_V . The *regular* representation for finite groups assigns to each element of G its own basis element \vec{e}_h in V , so $\rho_V: G \rightarrow \mathbb{R}^n$, where $n = \text{card } G$. Define the representation by $\rho_V(g)(\vec{e}_h) = \vec{e}_{gh}$. A linear subspace W is said to be *invariant* under ρ_V if $\rho_V(G)(W) \subseteq W$. Any non-trivial vector spaces V has $\{0\}$ and V as improper invariant subspaces. A representation $\rho: G \rightarrow GL(V)$ is said to be *irreducible* if the only invariant subspaces of V are $\{0\}$ and V . Often, rather than looking at the representation itself, we instead look at the trace of the representation, which is the basis of further metric properties. Being the trace of a matrix the sum of the elements on the main diagonal, then we define the *character* of ρ to be $\chi_\rho: G \rightarrow \mathbb{C}$ defined by $\chi_\rho(g) = \text{Tr}(\rho(g))$.

What has been said above might be largely generalized considering an arbitrary non-void set A (instead of a vector space V) together its group of automorphisms $\text{Aut}(A)$ (instead of $GL(V)$), hence consider the theory of group action on an arbitrary set (see Bourbaki (1970, Chapitre I, §§ 3, 4, 5)). On the other hand, following De Bartolomeis (1993, Chap. 3, Section 3.3), if V_K is an arbitrary n -dimensional linear space on the scalar field K and \mathcal{B} is a basis of it, then the known map $\mathcal{L}_{\mathcal{B}}^{\mathcal{B}}: GL(n, K) \rightarrow \text{Aut}(V_K)$ which assigns to each automorphism of V_K the corresponding matrix respect to the basis \mathcal{B} , defines a general structure which has a fundamental importance in mathematics, namely the action of a group on a *geometrical object* (in this case V_K), or else the representation of a group (in this case $GL(n, K)$) as a group of transformations of a geometrical object. Any group representation determines a group action. The recalls to the Felix Klein Erlangen Program are also evident.

⁴ The irreducibility just corresponds to the required elementarity's condition of the given particle. If such a condition does not hold, then the particle is composite.

4. The first elements of Duval theory: a brief outline

4.1 *A very brief introduction to R. Duval work.* According to Font et al. (2007), the philosophy of mathematics education, in agreement with the related experimental context, has pointed out what primary and fundamental role play *representations* in mathematics as well as in the general gnoseological context, for their capability to give an ontological foundation to knowledge, as said in section 1. As regards then the fundamental work made by Raymond Duval⁵ in philosophy of mathematics education, we quote what said Gérard Kuntz in reviewing (Duval 1995), namely

«La particularité de l'apprentissage des mathématiques tient à ce que ces activités cognitives requièrent l'utilisation de systèmes d'expression et de représentation autres que le langage naturel ou que les images: systèmes variés d'écriture pour les nombres, notations symboliques pour les objets, écritures algébrique et logique (qui prennent le statut de langues parallèles au langage naturel pour exprimer les relations et les opérations), figures géométriques, représentations en perspective, graphes cartésiens, réseaux, diagrammes, schémas, etc. Pour Raymond Duval, l'utilisation de multiples systèmes sémiotiques de représentation et d'expression est indispensable au développement de l'activité mathématique. Le progrès des connaissances s'accompagne toujours de la création et du développement de systèmes sémiotiques nouveaux et spécifiques qui coexistent plus ou moins avec le premier d'entre eux, la langue naturelle. L'élève qui sait coordonner les représentations relevant de systèmes sémiotiques variés, qui passe harmonieusement des unes aux autres se distingue (l'expérimentation le prouve) par une modification profonde de ses initiatives, de ses démarches et par un saut qualitatif de ses productions. La sémosis (acte d'appréhender des représentations sémiotiques imaginées par d'autres et d'en produire soi-même, mentalement ou matériellement) est intimement liée à la qualité de la néosis, l'acte de concevoir par la pensée (appréhension conceptuelle d'un objet, discrimination d'une différence ou compréhension d'une inférence). Pour Duval, il n'y a pas de néosis sans sémosis. La capacité de coordonner des registres sémiotiques n'est pas une conséquence de la conceptualisation. Au contraire, le degré de maîtrise de la sémosis (impliquant pluralité et coordination de registres sémiotiques) détermine l'étendue et la profondeur du champ de conceptualisation d'un individu. La sémosis est intrinsèque au fonctionnement de la pensée humaine».

All this, we suppose, may be enough to understand the primary principles underlying Duval work on semiotic theory, which will be further discussed in the next section 4.3.

4.2 *Some physical cases study.* In Physics, there are many remarkable cases in which the group representation theory plays a fundamental role to achieve such an ontological knowledge. For instance, in mathematical and theoretical physics, it is well-known the distinction between a *physical* (or *non-eliminable*) *singularity* from a *coordinate* (or *eliminable*) *singularity*: the first one has a real ontological existence differently from the second one which is eliminable with a suitable change of coordinates (hence, with a group transformation). A meaningful case of this last fact is given by formal expressions of electric field some of which seem to show a singularity in a certain coordinate representation (like the Cartesian one) that disappear when one passes to another (like the polar one). This falls into the general framework of symmetries of electric and magnetic field, thanks to which it is possible to deduce fundamental physical properties of these entities as well as to correctly and deeply solve problems related to them: from the educational viewpoint, the only two-volumes general physics textbook to my knowledge which extensively use group theory in

⁵ Who is a psychologist also with interests in mathematics education.

basic university physics is (Picasso 1998, 1999). His treatise, as well as all other books written by him, are very original and profound; they constitute the main textbooks used at Pisa.

An analogous but more complex case is the one concerning the *Schwarzschild (static) solution* of Einstein equations of general relativity, the first vacuum neutral spherical symmetry solution of them, which shows to have singularities at $r = 0$ and at the *Schwarzschild ray* $r = 2M$ (if M is the mass of the gravitational field source) which identifies the *event horizon*. Nevertheless, if one considers another coordinate system different from the *Schwarzschild coordinates*, like, for example, the *Lemaître* or *Kruskal-Szekeres* or *Eddington-Finkelstein coordinates*, then the singularity $r = 0$ still holds, whereas the singularity $r = 2M$ no longer holds. Hence, the former is a non-eliminable or *essential* singularity, whereas the latter is an eliminable one which will lead to the definition of the notion of *event horizon* and to the first notion of *black hole*. The distinction between essential and eliminable singularities is besides linked to certain curvature group invariants related to the given space-time metric under examination, which are finite when the singularity is non-eliminable, so that we again come back to group representation theory in establishing whether one has to do with a real physical entity or not. In any case, from all that, it is clear what primary role plays group representation theory to achieve an ontological knowledge. As regards, then, the mathematical context, it is enough just to recall the group theoretical foundation of the celebrated Felix Klein *Erlanger programm* and what historical impact and epistemological legacy have had such a Klein's work together the new philosophy of space and time to which it gave rise. But a deeper view of this gnoseological perspective comes from recent education mathematics research (see next subsection).

4.3 First elements of R. Duval semiotic theory. Following D'Amore (2001, pp. 321-358) and references therein⁶, Raymond Duval work on semiotic representation registers starts from the originary Cartesian emphasis on the primary role played by representation's activity into any knowledge process, which are inseparable each of other. Descartes and Kant have stressed on content or object of the representation, around which revolves any knowledge process. In mathematics, *d'après* Duval, the conceptual acquisition of an object necessarily goes through the acquisition of one of more semiotic representations, so that there exists no *noetic* (meant as the conceptual acquisition of an object) without *semiotics* (understood as the acquisition of a representation realized through signs⁷).

Let r^m $m = 1, 2, 3 \dots$ be a semiotic register and let $R_i^{r^m}(A)$ $i = 1, 2, 3, \dots$ be the *i*th *semiotic representation* having content or object A respect to the *semiotic register* r^m (i.e. common language, algebraic, iconographic, arithmetic language, etc), which give the possible distinctive features of A . In $R_i^{r^m}(A)$, if the semiotic register r^m changes, then the semiotic representation changes as well, whereas not necessarily takes place the contrary, that is to say, the latter may changes notwithstanding the former be the same. In this last case, two different semiotic representations on the same semiotic register r^m are denoted by $R_i^{r^m}(A)$ and $R_j^{r^m}(A)$ with $i \neq j$. The representation transformation $R_i^{r^m}(A) \rightarrow R_j^{r^m}(A)$ is said to be a *treatment's transformation*, whereas a semiotic register transformation $R_i^{r^n}(A) \rightarrow R_i^{r^m}(A)$ is said to be a *conversion* (within the same *i*th representation). Semiotic representation, treatment's transformation and conversion are the three basic cognitive features of semiotics which give rise to noetic.

Therefore, the knowledge acquisition of a mathematical concept or object A corresponds to the acquisition of an its *i*th semiotic representation $R_i^{r^m}(A)$ respect to a given semiotic register r^m , through which A manifests itself to our own consciousness. It speaks of a concept or object A represented into r^m , which gives its aspect related to the semiotic register r^m , so that it is a partial

⁶ For a complete view of the remarkable Duval's work on semiotic theory, see Duval (1995), whereas, for very brief outlines, see also Dalla Chiara et al. (2004, pp. 34-35).

⁷ In this context, the symbols are considered to be particular signs.

construction of A . To attain the full comprehension of A , it is necessary to know all the conversions of the type $R_i^{r^n}(A) \rightarrow R_j^{r^m}(A)$ for each i, j and n, m . This is the pivotal semiotic point around which is centred the *noetic apprehension* of a concept or object A . A crucial point to take into account is then the *relativity of the registers* (B. D'Amore) in the sense that, from $R_i^{r^m}(A)$, it is no possible to infer, with absolute certainty, to what semiotic register it belongs or else this last (i.e. r^m) is not deducible by a single representation of it (i.e. $R_i^{r^m}(A)$).

5. Comparing Wigner with Duval

Taking into account what said in the above Section 2 about group representations, a semiotic register r^m may be put into correspondence with the linear space V , while the i th representation will correspond to the generic group element g of the group G ; whence $R_i^{r^m}(A)$, equivalently rewritten as $R_{r,m}^i(A)$, will correspond to ρ_V^G . Therefore, with these prescription rules, a concept or object A will be cognizable only when, a priori, it is known the set $\{R_i^{r^m}(A)\}$ obtained varying, in all the possible ways, the i th semiotic representation and the semiotic register r^m . As said at the end of the previous Section 2, an elementary particle *is* an irreducible unitary representation of the symmetry group G of the physical theory under consideration, that is to say, it is given by the map $\rho_V^G: G \rightarrow GL(V)$ with $\rho_V^G(g): V \rightarrow V$ for each $g \in G$. Now, considering r^m as an element of V and the i th semiotic representation as corresponding to g , then the map $\rho_V^G(g) = \psi_g$ is of the type $\psi_g = \psi_i: r^m \rightarrow r^n$, that is to say, it gives a *conversion* (of semiotic registers) respect to the i th representation and, therefore, $\rho_V^G(g) = \psi_g$ gives a generic conversion respect to the i th representation g . So, ρ_V^G give all the possible conversions at varying of the i th representation. Therefore, the Wigner's definition of elementary particle is placeable within the general semiotic scheme of Raymond Duval, which, in turn, receives, as say, a sort of epistemological confirmation or corroboration just from the celebrated Wigner's formal model that has played a very fundamental rule in building up the theoretical framework of the *standard model* of fundamental physics; this last then have already received numerous experimental validations. In conclusion, with this comparison, the semiotic Duval theory gets indirect validation from an epistemological viewpoint. Vice versa, it would be possible, in principle, that such a Wigner's model might receive further extensions in view of the above comparison with the wider Duval's theoretical framework.

6. Comparing Feynman and Bion with Duval

Amongst the wide and deep *œuvres* of Richard P. Feynman, mainly devoted to fundamental physics, there has been too a certain interest toward the educational aspects as testified by the notable, original and clear Feynman works on General Physics and its epistemological foundations. In this regards, it is enough to recall the celebrated *The Feynman Lectures on Physics*, as concerns the most properly educational aspects. Instead, as regards the epistemological ones, we wish to stress on a series of conferences freely held by Feynman in 1964 at Cornell University and collected in the book (Feynman 1965). Precisely, we outline only those points strictly correlated to the arguments here considered, without any claim of completeness. The preface to the Italian translation due to the theoretical physicist Luigi A. Radicati di Bròzolo, expose the main lines of thought that have driven Feynman in drawing up these lectures. This book had the chief intention to outline as proceed the physics, its methodological essence in seeking for the laws of nature, together its motivations. In this, the book has clearly an epistemological setting, even if the author's intention wasn't originally this. He indirectly or unknowingly has nevertheless greatly contributed to originally clarify the philosophical foundations of physics, as no other could do. Besides, the style of exposition is fully original and typical of Feynman manner to do and to teach science: it is masterfully impregnated by that vivid immediacy of the New York *slang*, of its picturesque

metaphors as well as by that sublime scorn toward that rigid and usual syntax and stylistic elegance typical of tradition. At a first sight, all this could horrify the mainstream philosophical methodology, giving the sensation to be sacrilegious against it. Instead, after a deep mediation on its contents, it will be possible to identify valuable hints just from the epistemological side, as we will try to do as regards those arguments pertinent to what said above.

The Feynman epistemological intuitions that we want to stress, may be laid out into the general philosophical discussion of the *simplicity property of theoretical constructs* as made, for example, by Baženov (1977) in which, inter alia, the related literature on this argument is discussed and compared. This last has had a great epistemological importance: it is often named *principle of descriptive simplicity* like for instance in (Reichenbach 1958), in which the author makes wide use of this argument. In (Baženov 1977), the author, just in discussing this principle, inserts the Feynman work. Exactly, Baženov discusses on the so-called *principle of the truth of possible equivalent interpretative schemes* or *principle of the multiplicity and variety of the equivalent formulations*, that he simply calls *Feynman epistemological principle*. Feynman appeal many times to this principle in identifying the presence, in the general science (above all, in physics), of many formulations theoretically equivalent. He recall some known equivalent formal descriptions of quantum theory, like, for example, those related to matrix and wave mechanics (as Schrödinger himself for first proved – see his original papers in Ludwig (1968)). Due to his non-common critical insight, Feynman wonders whether there is a some deeper philosophical truth in this sort of formal equivalence. In seeking this, Feynman retrace some of the main stages of history of physics. In particular, in Chapter 2 of (Feynman 1965) devoted to the relationships between mathematics and physics, he compares the *Babylonian period*, in which the mathematics gave a certain flexibility to the axiomatic basis from which to start in such a manner to can have different methods with which build up various theories often in relationships amongst them, with the *Greek period*, in which instead there was the usual procedure to uniquely choose a given axiomatic basis that gave rise to an as many unique theory built up on it. According to Feynman, in physics is preferable to opt for the first one because it is difficult to have a unique omnicomprehensive axiomatic system for the whole physics framework. Therefore, there will exist different axiomatic systems (for mechanics, thermodynamics, electromagnetism, quantum mechanics and field theory, statistical mechanics, and so on) each of which allow to give a formal description of certain phenomenological aspects of nature. Such descriptions may change if one changes the related axiomatic setting as well as may exist common aspects described by different theories⁸.

Now, in case there are different descriptions which are formally equivalent amongst them, then Feynman claims that this fact has a deep ontological content because it means that them identify a same aspect of the nature seen by different representations, which will give rise to those fundamental laws of physics more or less extensively invariant. Feynman was fascinated and astonished by this last fact: it seems that the fundamental laws of physics be expressible in different ways, according to a great variety of formal manners or distinct representations. In this, the recalls to group representation theory are evident, as well as manifest are the analogies with what said by Duval in regards to the mathematical ontology. In short, the recurrent theme of these discussions seem to be a certain ontological feature inherent an a-priori invariance respect to many different (formal) representations, just that that will characterize the ontic truth of the object or entity under examination. Indeed, this last is attainable only by means of all the possible its representations only whose invariants will have ontological content. According to Feynman, it is just this different descriptive modalities to say us that we have to do with a same reality's aspect. The Feynman conclusions are surely supported by his wide experience of great physicist, so that it is quite

⁸ In this regards, the Husserlian phenomenology, which was at the basis of Heideggerian framework, speak of certain *regional ontologies* as ideal sciences of entities which are empirically study's object of many sciences that therefore describe them from different perspectives just in order to determine their ontological foundations. Besides, E. Husserl devoted a great part of his work to mathematical ontology (see Ferraris (Chapter 4)).

unlikely to put into discussion his ideas. Himself, in the 1965 *Nobel Lectures*⁹, stated that he was led to formulate the quantum electrodynamics just making repeatedly use of this his *principle of the variety of the possible interpretative schemes* (see Feynman (1971, Chapter 2, p. 60)) or, as Baženov names it, the *principle of the truth of possible equivalent interpretative schemes*, where truth here is meant to be related to ontic truth which is always yearns to the ontological one. Precisely, Feynman, at the end of his Nobel Lectures, states as follows

«[...] This completes the story of the development of the space-time view of quantum electrodynamics. I wonder if anything can be learned from it. I doubt it. It is most striking that most of the ideas developed in the course of this research were not ultimately used in the final result. For example, the half-advanced and half-retarded potential was not finally used, the action expression [...] was not used, the idea that charges do not act on themselves was abandoned. The path-integral formulation of quantum mechanics was useful for guessing at final expressions and at formulating the general theory of electrodynamics in new ways - although, strictly it was not absolutely necessary. The same goes for the idea of the positron being a backward moving electron, it was very convenient, but not strictly necessary for the theory because it is exactly equivalent to the negative energy sea point of view.

We are struck by the very large number of different physical viewpoints and widely different mathematical formulations that are all equivalent to one another. The method used here, of reasoning in physical terms, therefore, appears to be extremely inefficient. On looking back over the work, I can only feel a kind of regret for the enormous amount of physical reasoning and mathematically re-expression which ends by merely re-expressing what was previously known, although in a form which is much more efficient for the calculation of specific problems. Would it not have been much easier to simply work entirely in the mathematical framework to elaborate a more efficient expression? This would certainly seem to be the case, but it must be remarked that although the problem actually solved was only such a reformulation, the problem originally tackled was the (possibly still unsolved) problem of avoidance of the infinities of the usual theory. Therefore, a new theory was sought, not just a modification of the old. Although the quest was unsuccessful, we should look at the question of the value of physical ideas in developing a new theory.

Many different physical ideas can describe the same physical reality. Thus, classical electrodynamics can be described by a field view, or an action at a distance view, etc. Originally, Maxwell filled space with idler wheels, and Faraday with fields lines, but somehow the Maxwell equations themselves are pristine and independent of the elaboration of words attempting a physical description. The only true physical description is that describing the experimental meaning of the quantities in the equation - or better, the way the equations are to be used in describing experimental observations. This being the case perhaps the best way to proceed is to try to guess equations, and disregard physical models or descriptions. For example, McCullough guessed the correct equations for light propagation in a crystal long before his colleagues using elastic models could make head or tail of the phenomena, or again, Dirac obtained his equation for the description of the electron by an almost purely mathematical proposition. A simple physical view by which all the contents of this equation can be seen is still lacking.

Therefore, I think equation guessing might be the best method to proceed to obtain the laws for the part of physics which is presently unknown. Yet, when I was much younger, I tried this

⁹ Exactly, see the Richard P. Feynman Nobel Lecture: *The Development of the Space-Time View of Quantum Electrodynamics*, December 11, 1965, in *Nobel Lectures in Physics (1963-1970)*, Amsterdam, London and New York: Elsevier Publishing Company, 1972; see also Lundqvist (1998).

equation guessing and I have seen many students try this, but it is very easy to go off in wildly incorrect and impossible directions. I think the problem is not to find the best or most efficient method to proceed to a discovery, but to find any method at all. Physical reasoning does help some people to generate suggestions as to how the unknown may be related to the known. Theories of the known, which are described by different physical ideas may be equivalent in all their predictions and are hence scientifically indistinguishable. However, they are not psychologically identical when trying to move from that base into the unknown. For different views suggest different kinds of modifications which might be made and hence are not equivalent in the hypotheses one generates from them in ones attempt to understand what is not yet understood. I, therefore, think that a good theoretical physicist today might find it useful to have a wide range of physical viewpoints and mathematical expressions of the same theory (for example, of quantum electrodynamics) available to him. This may be asking too much of one man. Then new students should as a class have this. If every individual student follows the same current fashion in expressing and thinking about electrodynamics or field theory, then the variety of hypotheses being generated to understand strong interactions, say, is limited. Perhaps rightly so, for possibly the chance is high that the truth lies in the fashionable direction. But, on the off-chance that it is in another direction - a direction obvious from an unfashionable view of field theory - who will find it? Only someone who has sacrificed himself by teaching himself quantum electrodynamics from a peculiar and unusual point of view; one that he may have to invent for himself. I say sacrificed himself because he most likely will get nothing from it, because the truth may lie in another direction, perhaps even the fashionable one.

But, if my own experience is any guide, the sacrifice is really not great because if the peculiar viewpoint taken is truly experimentally equivalent to the usual in the realm of the known there is always a range of applications and problems in this realm for which the special viewpoint gives one a special power and clarity of thought, which is valuable in itself. Furthermore, in the search for new laws, you always have the psychological excitement of feeling that possible nobody has yet thought of the crazy possibility you are looking at right now.

So what happened to the old theory that I fell in love with as a youth? Well, I would say it's become an old lady, that has very little attractive left in her and the young today will not have their hearts pound anymore when they look at her. But, we can say the best we can for any old woman, that she has been a very good mother and she has given birth to some very good children. And, I thank the Swedish Academy of Sciences for complimenting one of them. Thank you».

Maybe, this Feynman's claimed variety of representations or determinations of the entity is an attempt toward the transcendental manifestation of the Dasein. Indeed, since we may have awareness (of something) only by means of a (Hegelian) dialectic opposition, the being is partially recognizable only through its opposition with the various entities with which it is in relation, that is to say, through the Dasein (the not between the Being and the Entity) which, in turn, is recognizable through its intrinsic transcendental nature. And the comparison of various and multiple representations is just one of main manner to try to approach such a *transcendence* which refers to the properties that the things or objects have in common amongst them and that therefore exceed (that is, *transcend*) the diversity. Only in this way, it shall be possible to reach to some an ontological foundation. In a wider sense, *ontological questions* concern too natural sciences insofar we turn our attention toward the existence of those entities which are their object's study and that

exhibit detectable special features. The central problem of such a type of studies just regard these last detectable features whose *existence* has to be ascertained.

Finally, we conclude this comparison recalling, in this regards, some aspects of the complex Wilfred Ruprecht Bion (1897-1979) psychoanalytic thought. Precisely, in (Bion 1962, Chapter I), the author speak of *functions* as of those complex mental activities as turn out from the concomitant action of different *factors* meant as elementary mental activities. These functions then underwent to *realizations* which are meant as representations of the formal system given by all the above functions. Bion appeals just to the corresponding mathematical notions in formulating these, taking into account the algebraic projective geometry paradigm, hence the Felix Klein viewpoint. Afterwards, Bion apply all this to the psychoanalytic context that, in this place, we cannot treat.

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