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Disturbance Decoupling of Timed Event Graphs by output feedback controller

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Abstract This paper deals with control of Timed Event Graphs (TEG). In a first part the disturbance decoupling problem for TEG is defined. In a second part we propose the synthesis of an optimal output feedback controller ensuring the disturbance decoupling and preserving the state of the system.

Keywords : Dioid, Residuation, Timed Event Graphs, Control of Discrete Events Systems.

1 Introduction

About 20 years ago a linear theory is introduced for a particular class of Discrete Event Dynamic Systems (DEDS) called Timed Event Graphs (TEG). Timed Event Graphs (TEG) constitute a subclass of timed Petri nets of which each place has exactly one upstream and one downstream transition. It is well known that the timed/event behavior of a TEG, under the earliest functioning rule¹, can be expressed by linear relations over some dioids [1]. Strong analogies then appear between the classical linear system theory and the $(\max, +)$ -linear system theory. In particular, the concept of *control* is well defined in the context of TEG study. In the literature, an *optimal control* for TEG exists and is proposed in [9], [16]. It is an open-loop control that requires the knowledge of the whole reference input trajectory to compute the control law. For a given reference input, this open-loop control yields the *latest* input firing date in order to obtain the output before the desired date. Moreover, recent works deal with the problem of *closed-loop* control [11], [12], which consists in synthesizing a controller in a *model matching* objective. Furthermore the proposed controllers allow delaying, as much as possible, the tokens input inside the TEG. Several recent studies lead to extend the range of systems admitting a linear representation in these algebraic structures (see [15] for unstationary TEG, [17] for nonlinear TEG and [5] for continuous TEG).

Moreover, these algebraic structures are apprehended since 1996 under a geometric approach [6], [7], [8]. In the classical linear systems theory, the interest of the geometric point of view has been shown [19], [2]. The notions of *controllability* and *observability* amount to surjectivity, resp. injectivity, of certain linear operators. Hence images and kernels as geometric objects are central. In this paper the disturbance decoupling problem for TEG is introduced.

In section 2, we summarize some theoretical results from the $(\max, +)$ literature. In the next parts, modeling and properties of TEG in these algebraic structures are presented [1]. The fifth part introduces the problem of disturbance decoupling into dioids. In particular, by taking into account the nature of these systems, the direction to give to this problem is discussed. Section 6 presents synthesis of an output feedback controller allowing the disturbance decoupling for TEG. Finally, an illustration of these results is given in section 7.

¹*i.e.* a transition is fired as soon as it is enabled

2 Algebraic Preliminaries

We first recall in this section some notions from the dioid theory. The reader is invited to consult [1] for a complete presentation².

2.1 Dioid

Definition 1 (Dioid) A dioid \mathcal{D} is a set endowed with two inner operations denoted by \oplus (addition) and \otimes (multiplication), both associative and both having neutral elements denoted by ε and e respectively, such that \oplus is also commutative and idempotent (i.e. $a \oplus a = a$). The \otimes operation is distributive with respect to \oplus , and ε is absorbing for the product (i.e. $\varepsilon \otimes a = a \otimes \varepsilon = \varepsilon, \forall a$). When \otimes is commutative, the dioid is said to be commutative. The symbol \otimes is often omitted.

Definition 2 (Order relation) An order relation can be associated with a dioid \mathcal{D} by the following equivalence : $\forall a, b \in \mathcal{D}, a \succeq b \Leftrightarrow a = a \oplus b$.

Definition 3 (Complete dioid) A dioid \mathcal{D} is complete if it is closed for infinite sums and if the product distributes over infinite sums too.

Example 1 The set $\overline{\mathbb{Z}} = \mathbb{Z} \cup \{-\infty, +\infty\}$ endowed with the max operator as sum and the classical sum + as product is a complete dioid, usually denoted by $\overline{\mathbb{Z}}_{\max}$, with $\varepsilon = -\infty$ and $e = 0$.

Remark 1 If \mathcal{D} is a dioid, the set $\mathcal{D}^{n \times n}$ of $n \times n$ matrices with coefficients in \mathcal{D} is also a dioid. Sum and product are defined in the following way:

$$(A \oplus B)_{ij} = A_{ij} \oplus B_{ij} \quad , \quad (A \otimes B)_{ij} = \bigoplus_{k=1}^n A_{ik} \otimes B_{kj}.$$

Theorem 1 Over a complete dioid \mathcal{D} , the implicit equation $x = ax \oplus b$ admits $x = a^*b$ as least solution, where $a^* = \bigoplus_{i \in \mathbb{N}} a^i$ (Kleene star operator) with $a^0 = e$.

²An electronic version is available on <http://www-rocq.inria.fr/scilab/cohen/SED/book-online.html>.

Notation 1 The Kleene star operator, defined on a complete dioid \mathcal{D} , will be represented by the application

$$\mathcal{K} : \mathcal{D} \rightarrow \mathcal{D}$$

$$x \mapsto \bigoplus_{i \in \mathbb{N}} x^i.$$

Property 1 ([14]) Let \mathcal{D} be a complete dioid.

$\forall a, b \in \mathcal{D}$

$$(a^*)^* = a^* \quad (1)$$

$$a^* a^* = a^* \quad (2)$$

$$a(ba)^* = (ab)^* a \quad (3)$$

$$(a \oplus b)^* = (a^* b)^* a^* \quad (4)$$

Definition 4 (Kernel [6],[7]) Let $C : \mathcal{X} \rightarrow \mathcal{Y}$ be a mapping. We call kernel of C (denoted by $\ker C$), the equivalence relation over \mathcal{X} :

$$x \overset{\ker C}{\sim} y \Leftrightarrow C(x) = C(y). \quad (5)$$

Remark 2 The usual kernel definition $\{x \in \mathcal{X} \mid C(x) = \varepsilon\}$ becomes meaningless in dioid algebra. The definition (5) corresponds to the kernel definition of a mapping defined on lattices [13].

Definition 5 (Isotone mapping) A mapping f defined over ordered sets is isotone (resp. antitone) if $a \preceq b \Rightarrow f(a) \preceq f(b)$ (resp. $f(a) \succeq f(b)$).

Definition 6 (Closure mapping) An isotone mapping $f : \mathcal{E} \rightarrow \mathcal{E}$ defined on an ordered set \mathcal{E} is a closure mapping if $f \succeq \text{Id}_{\mathcal{E}}$ and $f \circ f = f$.

Example 2 The Kleene Star mapping (see Theorem 1) is a closure mapping since $a^* = \bigoplus_{i \in \mathbb{N}} a^i \succeq a$ and $(a^*)^* = a^*$ (see Property 1).

Definition 7 (Restricted mapping) Let $f : \mathcal{E} \rightarrow \mathcal{F}$ be a mapping and $\mathcal{A} \subseteq \mathcal{E}$. We will denote $f|_{\mathcal{A}} : \mathcal{A} \rightarrow \mathcal{F}$ the mapping defined by $f|_{\mathcal{A}} = f \circ \text{Id}|_{\mathcal{A}}$ where $\text{Id}|_{\mathcal{A}} : \mathcal{A} \rightarrow \mathcal{E}$, $x \mapsto x$ is the canonical injection. Identically, let $\mathcal{B} \subseteq \mathcal{F}$ with $\text{Im} f \subseteq \mathcal{B}$. Mapping ${}_{\mathcal{B}}f : \mathcal{E} \rightarrow \mathcal{B}$ is defined by ${}_{\mathcal{B}}f = \text{Id}|_{\mathcal{B}} \circ_{\mathcal{B}} f$, where $\text{Id}|_{\mathcal{B}} : \mathcal{B} \rightarrow \mathcal{F}$, $x \mapsto x$ is the canonical injection.

2.2 Residuation theory

The residuation theory provides, under some assumptions, *optimal* solutions to inequalities such as $f(x) \preceq b$, where f is an isotone mapping defined over ordered sets. One can note that a complete presentation of this theory is given in [3], and see [1] for a specialization to dioids.

Definition 8 (Residuated mapping) *An isotone mapping $f : \mathcal{E} \rightarrow \mathcal{F}$, where $(\mathcal{E}, \preceq_{\mathcal{E}})$ and $(\mathcal{F}, \preceq_{\mathcal{F}})$ are ordered sets, is a residuated mapping if for all $y \in \mathcal{F}$ the least upper bound of the subset $\{x \in \mathcal{E} \mid f(x) \preceq_{\mathcal{F}} y\}$ exists and belongs to this subset.*

Theorem 2 *Let f be an isotone mapping from the complete dioid $(\mathcal{E}, \preceq_{\mathcal{E}})$ into the complete dioid $(\mathcal{F}, \preceq_{\mathcal{F}})$. The following two statements are equivalent :*

(i) *f is residuated.*

(ii) *There exists an isotone mapping $f^{\sharp} : \mathcal{F} \rightarrow \mathcal{E}$ such that $f \circ f^{\sharp} \preceq_{\mathcal{F}} \text{Id}_{\mathcal{F}}$ and $f^{\sharp} \circ f \succeq_{\mathcal{E}} \text{Id}_{\mathcal{E}}$.*

Consequently, f^{\sharp} is unique. When f satisfies these properties, it is said to be residuated and f^{\sharp} is called its residual.

Proposition 1 ([10], [11]) *Let $f : \mathcal{E} \rightarrow \mathcal{E}$ be a closure mapping. The mapping $\text{Im}f|f$ is a residuated mapping of which residual is the canonical injection $\text{Id}|_{\text{Im}f} : \text{Im}f \rightarrow \mathcal{E}$, $x \mapsto x$.*

It means that a closure mapping restricted to its image is a residuated mapping, with the canonical injection as residual.

Proof: According to Theorem 2, $\text{Im}f|f$ is residuated if there exists a mapping g such that $\text{Im}f|f \circ g \preceq \text{Id}$ and $g \circ \text{Im}f|f \succeq \text{Id}$, where identity mappings are respectively identity on $\text{Im}f$ and on \mathcal{E} . By setting $g = \text{Id}|_{\text{Im}f}$, we both verify $\text{Im}f|f \circ \text{Id}|_{\text{Im}f} = \text{Im}f|f|_{\text{Im}f} = \text{Id}$ (identity on $\text{Im}f$) since $f \circ f = f$, and $\text{Id}|_{\text{Im}f} \circ \text{Im}f|f = f \succeq \text{Id}$ (by Definition 6). ■

2.3 Residuation theory and dioid

Theorem 3 ([1]) *Let $f : \mathcal{D} \rightarrow \mathcal{E}$ be a mapping where \mathcal{D} and \mathcal{E} are complete dioids of which bottom elements are respectively denoted by $\varepsilon_{\mathcal{D}}$ and $\varepsilon_{\mathcal{E}}$. Then, f is residuated iff $f(\varepsilon_{\mathcal{D}}) = \varepsilon_{\mathcal{E}}$ and $\forall A \subseteq \mathcal{D}$,*

$$f\left(\bigoplus_{x \in \mathcal{A}} x\right) = \bigoplus_{x \in \mathcal{A}} f(x).$$

Example 3 ([1]) The mappings $L_a : x \mapsto ax$ and $R_a : x \mapsto xa$ defined over a complete dioid \mathcal{D} are both residuated. Their residuals are usually denoted by respectively $L_a^\sharp(x) = a \backslash x = \frac{x}{a}$ and $R_a^\sharp(x) = x / a = \frac{x}{a}$.

Theorem 4 ([4]) Let $A \in \mathcal{D}^{n \times n}$, are equivalent :

(i) $A = A^*$

(ii) $A = A \phi A$

We recall that A^* belongs to the image of \mathcal{K} (denoted by $\text{Im}\mathcal{K}$).

Proposition 2 ([10]) The mapping $\text{Im}\mathcal{K}|_{\mathcal{K}}$ (Kleene star operator) is a residuated mapping of which residual is $(\text{Im}\mathcal{K}|_{\mathcal{K}})^\sharp = \text{Id}|_{\text{Im}\mathcal{K}}$.

Proof: The proof is a direct application of Proposition 1, since \mathcal{K} is a closure mapping. ■

3 TEG representation

3.1 Transfer function

Timed Event Graphs (TEG) are well adapted to model synchronization phenomena; moreover, they can be seen as linear dynamic systems in dioid algebra [1]. TEG behavior can be expressed over many dioids, for instance in the dioid of formal power series in one variable γ and coefficients in $\overline{\mathbb{Z}}_{\max}$. This dioid is usually denoted by $\overline{\mathbb{Z}}_{\max}[[\gamma]]$ in literature.

Consequently, for a given TEG we can obtain the following representation over the dioid $\overline{\mathbb{Z}}_{\max}[[\gamma]]$

$$\begin{cases} x &= Ax \oplus Bu \\ y &= Cx \oplus Du \end{cases} \quad (6)$$

where,

- $x(\cdot) \in \mathcal{X}$ represents the state vector ; $u(\cdot) \in \mathcal{U}$ represents the control vector ; $y(\cdot) \in \mathcal{Y}$ represents the output vector

- The mappings $A_{n \times n} : \mathcal{X} \rightarrow \mathcal{X}$, $B_{n \times r} : \mathcal{U} \rightarrow \mathcal{X}$, $C_{m \times n} : \mathcal{X} \rightarrow \mathcal{Y}$ et $D_{m \times r} : \mathcal{U} \rightarrow \mathcal{Y}$ are represented by constants matrices of which terms are in $\overline{\mathbb{Z}}_{\max}[\gamma]$.

We propose to describe the behavior of the TEG given in Fig 1. It models an elementary production workshop composed of three machines (M_1 to M_3). The machine M_1 can process 2 parts simultaneously, each processing lasts 6 times units. The machine M_3 processes the parts released by machines M_1 and M_2 . For this TEG (Fig 1.), a state representation is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6\gamma^2 & \varepsilon & \varepsilon \\ \varepsilon & 6\gamma^2 & \varepsilon \\ 7 & 8 & 6\gamma^2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \oplus \begin{pmatrix} 11 & \varepsilon \\ \varepsilon & 9 \\ \varepsilon & \varepsilon \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (7)$$

$$\begin{pmatrix} y \end{pmatrix} = \begin{pmatrix} \varepsilon & \varepsilon & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (8)$$

Figure 1: TEG model of an assembly workshop

4 Periodicity, causality and realizability

The transfer function of a TEG is characterized by some periodic and causal properties which we recall below. Let us consider a series $s = \bigoplus_{k \in \mathbb{Z}} s(k)\gamma^k$ in $\overline{\mathbb{Z}}_{\max}[\gamma]$. The support of s is defined by $\text{Supp}(s) = \{k \in \mathbb{Z} \mid s(k) \neq \varepsilon\}$, and its valuation corresponds to the lower bound of $\text{Supp}(s)$, *i.e.* $\text{val}(s) = \min\{k \in \mathbb{Z} \mid s(k) \neq \varepsilon\}$. A series $s \in \overline{\mathbb{Z}}_{\max}[\gamma]$ such that $\text{Supp}(s)$ is finite is said to be polynomial.

Definition 9 (Causality) A series $s \in \overline{\mathbb{Z}}_{\max}[\gamma]$ is causal if $s = \varepsilon$ or if $\{\text{val}(s) \geq 0 \text{ and } s \succeq \gamma^{\text{val}(s)}\}$. The set of causal elements of $\overline{\mathbb{Z}}_{\max}[\gamma]$ has a complete dioid structure denoted by $\overline{\mathbb{Z}}_{\max}^+[\gamma]$.

Definition 10 (Periodicity) A series $s \in \overline{\mathbb{Z}}_{\max}[\gamma]$ is said to be periodic if it can be written as $s = p \oplus q(\tau\gamma^\nu)^*$ with p and q two polynomials and $\nu, \tau \in \mathbb{N}$. A matrix is said to be periodic if all its entries are periodic.

Definition 11 (Realizability) A series $s \in \overline{\mathbb{Z}}_{\max}[\gamma]$ is said to be realizable if there exist three matrices A, B and C with entries in $\mathbb{N} \cup \{-\infty, +\infty\}$ such that $s = C(\gamma A)^* B$. A matrix is said to be realizable if all its entries are realizable³.

Theorem 5 ([1]) The following statements are equivalent :

- (i) A series s is realizable.
- (ii) A series s is periodic and causal.

The set of periodic series of $\overline{\mathbb{Z}}_{\max}[\gamma]$ has a dioid structure which is not complete. Nevertheless, we have the following property.

Theorem 6 ([4]) Let s_1 and s_2 be two periodic series of $\overline{\mathbb{Z}}_{\max}[\gamma]$, then $s_1 \setminus s_2$ is also a periodic series.

Theorem 7 ([11]) The canonical injection $\text{Id}_{|_+} : \overline{\mathbb{Z}}_{\max}^+[\gamma] \rightarrow \overline{\mathbb{Z}}_{\max}[\gamma]$ is residuated. We denote by $\text{Pr}_+ : \overline{\mathbb{Z}}_{\max}[\gamma] \rightarrow \overline{\mathbb{Z}}_{\max}^+[\gamma]$ its residual.

The series $\text{Pr}_+(s)$ is the greatest causal series less than or equal to s . From a practical point of view, for all $s \in \overline{\mathbb{Z}}_{\max}[\gamma]$, the computation of $\text{Pr}_+(s)$ is obtained by :

$$\text{Pr}_+ \left(\bigoplus_{k \in \mathbb{Z}} s(k)\gamma^k \right) = \bigoplus_{k \in \mathbb{Z}} s_+(k)\gamma^k \quad \text{where}$$

$$s_+(k) = \begin{cases} s(k) & \text{if } (k, s(k)) \geq (0, 0) \\ \varepsilon & \text{otherwise.} \end{cases}$$

Theorem 8 ([11]) Let s be a periodic (not necessarily causal) series of $\overline{\mathbb{Z}}_{\max}[\gamma]$. Then $\text{Pr}_+(s)$ is the greatest realizable series less than or equal to s .

³In other words, a series s is realizable if there exists a TEG of which s is the transfer relation

5 Disturbance decoupling in dioid

In this chapter we first discuss the sense of the disturbance decoupling problem for TEG. Examination of this problem leads naturally to give a practical sense to the kernel definition of a mapping in dioid. Let us consider the system :

$$x = Ax \oplus Bu \oplus Sq \quad (9)$$

$$y = Cx \quad (10)$$

The term q in (9) represents a disturbance which is assumed to be not directly measurable by the controller.

Let us assume that q belongs to a set \mathcal{Q} and that the mapping $S : \mathcal{Q} \rightarrow \mathcal{X}$ is a time-invariant mapping.

In the conventional linear system theory [18], the disturbance decoupling problem consists in finding a control u such that disturbance q has no influence on the controlled output y . A particular problem is to find (if possible) an output feedback F , *i.e.* $u = Fy$, which allows reaching this objective. The disturbance decoupling problem can also be solved by the mean of a state feedback, $y = Fx$. From an algebraic point of view, it amounts to find F such that the state trajectory remains in a subspace of the kernel of mapping C , *i.e.* a state which leads to a null output $\forall q$.

Our problem must be stated in a different way since trajectories u, x, y and q are monotonous and no decreasing (date $x_i(k+1)$ is later than date $x_i(k)$). The output cancellation is consequently meaningless in this context. Nevertheless find a control u which keeps the state x in the kernel of C for all disturbance q is relevant, provided that the notion of kernel be redefined. In dioid (see Definition 4), the kernel of C is an equivalence relation⁴, *i.e.* the space splits up in equivalence classes (each class contains all the elements which map to the same image, in [6] and [20], the term "fibration" is used). Here, the disturbance decoupling control amounts to find a control which keeps the state x in the same equivalence class as the one due to q disturbance, *i.e.* in the class of states that yields the same output. Obviously the set of these controls may contains many elements, hence we are interested in computing the greatest one, since it is the one which satisfies the just-in-time criterion. Formally the disturbance decoupling problem can be established in the

⁴*i.e.* the kernel of C is not a 'subspace' of X

following way. The explicit solution of (9) is :

$$x = A^*Bu \oplus A^*Sq,$$

which leads to the output

$$y = CA^*Bu \oplus CA^*Sq.$$

This equation allows establishing that all controls u such as $CABu \preceq CA^*Sq$ keep unchanged the output generated by q . In agreement with the objective stated previously, the disturbance decoupling for TEG consists in establishing the greatest control u satisfying this inequation. This greatest control u allows delaying as much as possible the tokens input inside the TEG. Actually, it is useless that tokens be inserted too soon in the TEG since the uncontrollable disturbance q delay the output firing. The residuation of mapping L_{CA^*B} (Theorem 3) yields

$$u = \frac{CA^*Sq}{CA^*B}. \quad (11)$$

Practically this control computation requires the disturbance⁵ knowledge. Our problem is then to find a feedback F which allows avoiding this assumption.

6 Output feedback for disturbance decoupling

In this part we discuss the existence and the computation of two output feedbacks controllers which lead to a closed-loop system making the disturbance decoupling. The objective of the first controller (denoted by F_1) is to keep unchanged output y whatever be the disturbance q . The second controller (denoted by F_2) kept unchanged state x whatever be the disturbance q .

⁵In a manufacturing system, q may represent the supply of raw material which is a priori known. The problem is then very similar to the problem introduced in [16] which establishes an optimal open-loop control in presence of known uncontrollable inputs.

Figure 2: The output feedback control

A system provided with a controller F is represented Figure 2. Its behaviour is described by the following equations:

$$x = Ax \oplus BFy \oplus Sq \quad (12)$$

$$y = Cx \quad (13)$$

6.1 Output Feedback which kept the state

In this section we are looking for a controller which achieves disturbance decoupling by keeping the state x .

The Equation (12) yields :

$$x = Ax \oplus BFCx \oplus Sq = (A \oplus BFC)x \oplus Sq, \quad (14)$$

thanks to Theorem 1, we establish :

$$x = (A \oplus BFC)^*Sq.$$

This equation is to be compared with the expression of the transfer between the state and the disturbance in the absence of the controller : $x = A^*Sq$.

The problem of disturbance decoupling by the mean of a feedback controller which kept the state x , can then be formally expressed as the controller synthesis F_1 such that :

$$(A \oplus BF_1C)^*S = A^*S. \quad (15)$$

This controller is such that the output remains unchanged with respect to q , *i.e.* x is kept in the kernel equivalence classes of C generated by q , formally :

$$C((A \oplus BF_1C)^*S) = CA^*S \Leftrightarrow (A \oplus BF_1C)^*S \stackrel{ker C}{\sim} A^*S \quad (16)$$

Among the controllers satisfying the objective (Eq. ??) we seek the greatest one, *i.e.* the one which will generate the greatest control : $u = F_1y$. By Property (4) equation (17) becomes:

$$(A^*BF_1C)^*A^*S = A^*S \quad (17)$$

Equation (18) can be also written :

$$(A^*BF_1C)^* = \frac{A^*S}{A^*S} \quad (18)$$

Term $A^*S \not\leq A^*S \in \text{Im}\mathcal{K}$ (Theorem 4), according to Proposition 2, we know that mapping \mathcal{K} is a residuated mapping whose residual $(\text{Im}\mathcal{K}|\mathcal{K})^\sharp$ is $\text{Id}_{\text{Im}\mathcal{K}}$, then equation (19) is equivalent to :

$$A^*BF_1C = \frac{A^*S}{A^*S},$$

from where we can extract the controller F_1 :

$$F_1 = \frac{\frac{A^*S \not\leq A^*S}{A^*B}}{C} \quad (19)$$

This controller is the greatest which preserves state x generated by q unchanged what implies an unchanged exit.

From a practical point of view it is the controller which will generate the greatest control $u = F_1y$, *i.e.* the controller F_1 is then the feedback controller leading to the control trajectory which delays as much as possible the input of tokens. For the disturbances having effect to delay the fire of these internal transitions, delay input u avoids an useless accumulation of tokens in places of TEG upstream to the disturbed transitions.

6.2 Output feedback which maintains the output

Let us resolve (Theorem 1) directly implicit Equation (12) :

$$x = A^*BF_2y \oplus A^*Sq,$$

hence output y is expressed by :

$$y = CA^*BF_2y \oplus CA^*Sq. \quad (20)$$

Equation (21) is also an implicit equation using Theorem 1 again, we obtain

$$y = (CA^*BF_2)^*CA^*Sq.$$

The objective of this controller is to leave the output unchanged whatever be disturbance q , *i.e* formally :

$$(CA^*BF_2)^*CA^*S = CA^*S$$

As in the previous part, we can write the above equation as:

$$(CA^*BF_2)^* = \frac{CA^*S}{CA^*S}$$

Again, the term $\frac{CA^*S}{CA^*S} \in \text{Im}\mathcal{K}$, thanks to Proposition 2, we know that the residual $(\text{Im}\mathcal{K}|\mathcal{K})^\sharp$ is $\text{Id}_{\text{Im}\mathcal{K}}$, then

$$CA^*BF_2 = \frac{CA^*S}{CA^*S}$$

Finally the controller F_2 is equal to :

$$F_2 = \frac{CA^*S \cancel{CA^*S}}{CA^*B} \quad (21)$$

This new controller F_2 is the one which keep unchanged the output.

7 Application

Let the TEG of Figure 1, for which the various transfer matrices (7),(8) are calculated. Matrice S linking the disturbance to the state is missing :

$$S = \begin{pmatrix} e & \varepsilon & \varepsilon \\ \varepsilon & e & \varepsilon \\ \varepsilon & \varepsilon & e \end{pmatrix},$$

Trajectories q_1, q_2 and q_3 represent the input delaying the pieces output of machines M_1, M_2 and M_3 . They can represent the history of machines stops due to external causes in the model. Our objective is thus to

synthesis the greatest state feedback controller $F = \begin{pmatrix} F_{11} \\ F_{21} \end{pmatrix}$ such as the control $u = Fy$ is the greatest leaving the unchanged state. Relation (20) provides the expression of this controller :

$$F = \begin{pmatrix} -19(6\gamma^2)^* \\ -18(6\gamma^2)^* \end{pmatrix}$$

All the coefficients of this matrice are periodicals, but they all are not causal (Definition 9). This corrector is thus not realizable (Theorem 5), it is necessary for that to consider the Theorem 7 which makes it possible to project the noncausal elements in the set of causal elements $\overline{\mathbb{Z}}_{\max}^+[[\gamma]]$. For controller F it leads to matrice F_+ equal to :

$$F_+ = \text{Pr}_+(F) = \begin{pmatrix} 5\gamma^8(6\gamma^2)^* \\ \gamma^6(6\gamma^2)^* \end{pmatrix} \quad (22)$$

A realization in the form of TEG of this controller is given on the Figure 3. It leads to the following control laws in $\overline{\mathbb{Z}}_{\max}$:

$$\begin{cases} u_1(k) = 6u_1(k-2) \oplus 5y(k-8) \\ u_2(k) = 6u_1(k-2) \oplus y(k-6) \end{cases}$$

Figure 3: In solid black lines system H , in dotted arcs a realization of the output feedback controller F_+ .

7.1 Numerical application

Let the following trajectories for inputs :

$$\begin{aligned} v_1 &= e \oplus 2\gamma \oplus 5\gamma^3 \oplus 8\gamma^5 \oplus +\infty\gamma^6, \\ v_2 &= 2\gamma \oplus 5\gamma^2 \oplus 8\gamma^4 \oplus 10\gamma^5 \oplus +\infty\gamma^6. \end{aligned}$$

The states are delayed by disturbances whose trajectories are as follows:

$$\begin{aligned} q_1 &= 20 \oplus 22\gamma \oplus 30\gamma^2 \oplus 32\gamma^3 \oplus +\infty\gamma^6, \\ q_2 &= 22 \oplus 23\gamma \oplus 31\gamma^2 \oplus 32\gamma^3 \oplus +\infty\gamma^6, \\ q_3 &= 21 \oplus 25\gamma \oplus 33\gamma^2 \oplus 34\gamma^4 \oplus +\infty\gamma^6. \end{aligned}$$

The state X of the TEG (Figure 1, with $V = U$) in open loop is equal to :

$$\begin{aligned} X &= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = A^*B \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \oplus A^*S \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \\ &= \begin{pmatrix} 20 \oplus 22\gamma \oplus 30\gamma^2 \oplus 32\gamma^3 \oplus 36\gamma^4 \oplus 38\gamma^5 \oplus +\infty\gamma^6 \\ 22 \oplus 23\gamma \oplus 31\gamma^2 \oplus 32\gamma^3 \oplus 37\gamma^4 \oplus 38\gamma^5 \oplus +\infty\gamma^6 \\ 30 \oplus 31\gamma \oplus 39\gamma^2 \oplus 40\gamma^3 \oplus 45\gamma^4 \oplus 46\gamma^5 \oplus +\infty\gamma^6 \end{pmatrix} \end{aligned}$$

By considering the GET in closed loop (Figure 3), now state vector of system is

$$X = A^*BFY \oplus A^*BV \oplus A^*SQ,$$

with Y equal to :

$$Y = (CA^*BF)^*CA^*BV \oplus (CA^*BF)CA^*SQ.$$

The controller objective (18) calculated previously was : taking into account the disturbances acting on the TEG, to find a controller allow delaying, as much as possible, the token input inside the TEG (decreasing work-in-process) without slowing down more than the disturbances on the state of TEG. The controller F calculated in the previous part (20) allows to achieve this goal. Considering trajectories (u_1, u_2, q_1, q_2 and q_3), it is possible to determine the greatest closed loop control U_{BF} giving the same state which the open loop control :

8 Conclusion

In this paper is introduced the problem of disturbance decoupling in dioids. The objective is to synthesize a control law maintaining state x in the kernel of C , which presents a strong analogy with the disturbance decoupling of the traditional automatic. It is however necessary to note that the reached objective does not lead to a output cancellation, indeed the specific kernel definition of a mapping on a lattice and the nature of the considered systems leads to obtain the greatest control such as the output remains unchanged whatever is the disturbance.

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