



HAL
open science

PML Implementation for TLM Propagation Model

Pierre Chobeau, Judicaël Picaut, David Ecotiere, Guillaume Dutilleux

► **To cite this version:**

Pierre Chobeau, Judicaël Picaut, David Ecotiere, Guillaume Dutilleux. PML Implementation for TLM Propagation Model. AIA-DAGA 2013 Conference on Acoustics, Mar 2013, Merano, Italy. 4p, fig., graphiques, schéma. hal-00845709

HAL Id: hal-00845709

<https://hal.science/hal-00845709>

Submitted on 17 Jul 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

PML Implementation for the TLM Propagation Model in acoustics

Pierre Chobeau^{1,2}, D. Ecotière², G. Dutilleux², J. Picaut¹

¹ LUNAM Université, IFSTTAR, AME, LAE, Email: pierre.chobeau@ifsttar.fr

² LR Strasbourg/ERA32, pierre.chobeau@developpement-durable.gouv.fr

Introduction

Computational resources are often a limitation for the simulation of long range sound propagation. Especially, time domain methods such as FDTD or TLM methods for acoustics still require consequent computational power. Simulations of free-field wave propagation should be equivalent to wave propagation in an infinite domain but in practice computational domain is always finite. Furthermore, the truncation of the propagation domain can be seen as an approach to minimize computational burden. Therefore, the truncation, i.e. the artificial numerical boundary, should not impact the propagation domain of interest. The methods that prevent from phenomena such as unwanted reflections can be divided into two categories: absorbing boundaries and absorbing layers. Each approach aims at simulating free-space propagation in order to allow a truncation of the computational domain. Considering the transmission-line matrix (TLM) method, absorbing boundaries have shown limited interest, i.e. some of them induce instabilities and/or are only efficient for normal incidence. Therefore, absorbing layer methods appear to be more promising for the TLM method. Two absorbing layer methods have already been tested with the TLM method: the dissipative medium [1] and the matched connexion law [2, 3]. The perfectly matched layer (PML) proposed by Bérenger in electromagnetism has been used in several methods for acoustic simulations. This approach shows convincing performances in most calculation methods used for acoustics but no rigorous implementation of Bérenger's perfectly matched layers has been proposed yet for TLM applications in acoustics.

The first part of this document presents the basics of the TLM principle. The second part briefly reminds the approaches used to implement absorbing conditions to the TLM method. The method to adapt the PML to the TLM method is presented in the third part. A condition on the solutions and the validity of every approximate PML formulation is discussed in part four and five. Finally, the numerical results are presented and discussed in the conclusion.

The TLM method for acoustics

Incident pulses ${}_tI_{(i,j)}^n$ and scattered pulses ${}_tS_{(i,j)}^m$ for a given node position (i, j) are depicted by figure 1. Indexes n and m correspond to the number of the branch where pulses are traveling. Incident and scattered pulses



Figure 1: Incident ${}_tI_{(i,j)}^n$ and scattered ${}_tS_{(i,j)}^m$ pulses for a node (i, j) at the time t for an homogeneous and non dissipative medium (2D).

can be put into vectors as follow:

$${}_t\mathbf{I}_{(i,j)} = [{}_tI^1; {}_tI^2; {}_tI^3; {}_tI^4]_{(i,j)}^T, \quad (1)$$

$${}_t\mathbf{S}_{(i,j)} = [{}_tS^1; {}_tS^2; {}_tS^3; {}_tS^4]_{(i,j)}^T \quad (2)$$

where (i, j) corresponds to the node position in the discretized TLM network and the exponent T denotes the transposed of the original vectors ${}_t\mathbf{I}_{(i,j)}$ and ${}_t\mathbf{S}_{(i,j)}$. The scattering process for each node of the transmission-line network is written as:

$${}_t\mathbf{S}_{(i,j)} = {}_t\mathbf{D}_{(i,j)} {}_t\mathbf{I}_{(i,j)}, \quad (3)$$

where ${}_t\mathbf{D}_{(i,j)}$ is the scattering matrix, which define the proportion of reflected and transmitted pulses at every node of the TLM network.

In the case of outdoor sound propagation, physical phenomena such as atmospheric turbulence induce heterogeneities of the propagation medium. Atmospheric absorption induces additional dissipation which should be taken into account in the model. Both phenomena are implemented in the TLM method through the introduction of specific additional branches.

Heterogeneities

It has been shown [6] that an additional branch at each node can be used to set the local sound celerity. This approach introduces the parameter η that allows to modify the local impedance through an additional fifth branch of length $\delta l/2$. This branch presents a characteristic impedance Z_0/η (see Fig. 2), where Z_0 corresponds to the characteristic impedance of the main branches.

Dissipation

Dissipation phenomena additional to the geometrical divergence can be simulated for a given frequency by adding a branch. This branch presents an anechoic termination and a specific impedance Z_0/ζ . As the termination of the dissipation's branch is anechoic, the wave that travels in such a branch is neither transmitted nor reflected.

Therefore, to simulate acoustic propagation in an heterogeneous and dissipative medium the TLM network has to be modified as shown in figure 2, which give rise to five incident and scattered pulses.

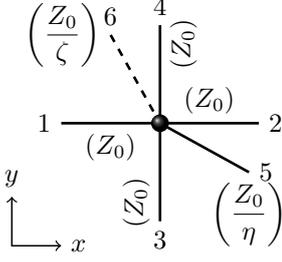


Figure 2: Additional branches for acoustic propagation in an heterogeneous and dissipative medium (2D).

$${}_t\mathbf{I}_{(i,j)} = [{}_tI^1; {}_tI^2; {}_tI^3; {}_tI^4; {}_tI^5]_{(i,j)}^T, \quad (4)$$

$${}_t\mathbf{S}_{(i,j)} = [{}_tS^1; {}_tS^2; {}_tS^3; {}_tS^4; {}_tS^5]_{(i,j)}^T, \quad (5)$$

where ${}_tI^5$ and ${}_tS^5$ corresponds to incident and scattered pulses from the additional fifth branch which aims at introducing heterogeneities in the transmission-line network.

The superposition principle allows to write the pressure at a given node position as follows:

$${}_tP_{(i,j)} = \frac{2}{\eta_{(i,j)} + \zeta_{(i,j)} + 4} \left[\sum_{n=1}^4 {}_tI_{(i,j)}^n + \eta_{(i,j)} {}_tI_{(i,j)}^5 \right]. \quad (6)$$

At a given node position (i, j) , each scattered pulse ${}_tS^m$ travel along the discretized distance δl during the time δt and becomes an incident pulse ${}_{t+\delta t}I^n$ at an adjacent node. For the node position (i, j) , this is written as:

$${}_{t+\delta t}I_{(i,j)}^1 = {}_tS_{(i-1,j)}^2, \quad (7a)$$

$${}_{t+\delta t}I_{(i,j)}^2 = {}_tS_{(i+1,j)}^1, \quad (7b)$$

$${}_{t+\delta t}I_{(i,j)}^3 = {}_tS_{(i,j-1)}^4, \quad (7c)$$

$${}_{t+\delta t}I_{(i,j)}^4 = {}_tS_{(i,j+1)}^3. \quad (7d)$$

$${}_{t+\delta t}I_{(i,j)}^5 = {}_tS_{(i,j)}^5. \quad (7e)$$

From the combination of the matrix relation, the connexion laws and the nodal pressure definition, the iterative schemes for heterogeneous and dissipative TLM network is written as:

$$\begin{aligned} {}_{t+\delta t}P_{(i,j)} = & \frac{2}{\eta_{(i,j)} + \zeta_{(i,j)} + 4} [{}_tP_{(i+1,j)} + {}_tP_{(i-1,j)} \\ & + {}_tP_{(i,j+1)} + {}_tP_{(i,j-1)} + \eta_{(i,j)} {}_tP_{(i,j)}] \\ & - \frac{\eta_{(i,j)} - \zeta_{(i,j)} + 4}{\eta_{(i,j)} + \zeta_{(i,j)} + 4} {}_{t-\delta t}P_{(i,j)}, \end{aligned} \quad (8)$$

where time and spatial derivatives can be identified to give the wave following wave equation:

$$\left[\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) - \frac{\eta + 4}{2} \frac{\delta t^2}{\delta l^2} \frac{\partial^2}{\partial t^2} - \zeta \frac{\delta t}{\delta l^2} \frac{\partial}{\partial t} \right] {}_tP_{(i,j)} = 0. \quad (9)$$

Absorbing conditions for the TLM method

The review of the absorbing conditions for TLM method shows that absorbing layer approach presents the most convincing results for the truncation of the propagation domain of interest. For the use of TLM method in acoustics, the proposed absorbing layers [2, 3] are mostly based on empirical approaches. Guillaume [3] proposed an optimized formulation of the matched connexion laws, which is named Guillaume's matched connexion law (GMCL). The author reminded that the principle of a PML is to attenuate acoustic wave following the direction normal to the layer boundary. Therefore, instead of applying the attenuation factor on the four connexion laws as it was initially proposed by de Cogan [2], the GMCL treats only connexion law which corresponds to the main propagation axis.

Adaptation of the PML to the TLM method

The only TLM calculation in acoustics truncated by a PML [5] has been carried out through the connexion of an FDTD PML calculation which surrounded the TLM domain of interest. Although PML is seen as convincing method to truncate the propagation domain, still no rigorous formulation is available for acoustic simulation using the TLM method.

Based on an analogy between electromagnetic and acoustic fields, for a 2D acoustic wave propagation case, the acoustic pressure p is split into two additive components: $p = g(x) + h(y)$. The functions $g(x)$ and $h(y)$ are the pressure components that respectively only depend on the spatial variable x and y . The particle velocity v is composed of two vectorial components v_x and v_y following respectively x and y directions. The split-field PML formulation involves the four acoustic components: g, h, v_x, v_y , through the mass continuity and momentum equations. In 2D Cartesian coordinates the PML can be written as:

$$\frac{\partial g(x)}{\partial t} + c_0^2 \frac{\partial v_x}{\partial x} + \sigma_x g(x) = 0, \quad (10a)$$

$$\frac{\partial h(y)}{\partial t} + c_0^2 \frac{\partial v_y}{\partial y} + \sigma_y h(y) = 0, \quad (10b)$$

$$\frac{\partial v_x}{\partial t} + \frac{\partial p}{\partial x} + \sigma_x v_x = 0, \quad (10c)$$

$$\frac{\partial v_y}{\partial t} + \frac{\partial p}{\partial y} + \sigma_y v_y = 0, \quad (10d)$$

where c_0 is the sound celerity, v_x and v_y are the particle velocity components, p the acoustic pressure which can be split into two components: $g(x)$ and $h(y)$. After rearrangement, equations (10) give rise to the following propagation system:

$$\frac{\partial^2 g(x)}{\partial t^2} - c_0^2 \frac{\partial^2 p}{\partial x^2} + 2\sigma_x \frac{\partial g(x)}{\partial t} + \sigma_x^2 g(x) = 0, \quad (11a)$$

$$\frac{\partial^2 h(y)}{\partial t^2} - c_0^2 \frac{\partial^2 p}{\partial y^2} + 2\sigma_y \frac{\partial h(y)}{\partial t} + \sigma_y^2 h(y) = 0. \quad (11b)$$

Assuming that $\sigma = \sigma_x + \sigma_y$, equations (11) can be

rewritten into an unsplit form as follows:

$$\frac{\partial^2 p(x, y, t)}{\partial t^2} - c_0^2 \left(\frac{\partial^2 p(x, y, t)}{\partial x^2} + \frac{\partial^2 p(x, y, t)}{\partial y^2} \right) + 2\sigma \frac{\partial p(x, y, t)}{\partial t} + \sigma^2 p(x, y, t) = 0, \quad (12)$$

where the pressure p and the acoustic velocity v are unsplit. Equation (12) is named as the unsplit PML wave equation.

Using finite centered differences and the TLM notations, equation (12) can be discretized and written as an iterative propagation scheme:

$$\begin{aligned} {}_{t+\delta t}p_{(i,j)} &= \frac{1}{2} \frac{1}{(1 + \sigma\delta t)} [{}_t p_{(i+1,j)} + {}_t p_{(i-1,j)} \\ &+ {}_t p_{(i,j+1)} + {}_t p_{(i,j-1)} - 2\sigma^2 \delta t^2 {}_t p_{(i,j)}] \\ &- \frac{1 - \sigma\delta t}{1 + \sigma\delta t} {}_{t-\delta t} p_{(i,j)}. \end{aligned} \quad (13)$$

The propagation schemes (13) and (8) are compared and each factor are identified from one equation to the other. From this identification a system of three linear equations can be written:

$$\frac{2}{\eta + \zeta + 4} = \frac{1}{2} \frac{1}{1 + \sigma\delta t}, \quad (14a)$$

$$\frac{2\eta}{\eta + \zeta + 4} = -\frac{\sigma^2 \delta t^2}{1 + \sigma\delta t}, \quad (14b)$$

$$\frac{\eta - \zeta + 4}{\eta + \zeta + 4} = \frac{1 - \sigma\delta t}{1 + \sigma\delta t}, \quad (14c)$$

where η is the heterogeneity term and ζ is the dissipative term inherent to the TLM simulations in heterogeneous and dissipative media. The term σ is the PML attenuation factor. To not overload the notation in this section the indexes (i, j) are not written, keeping in mind that each variable η , ζ and σ may depend on the node location (i, j) in the TLM model.

The system (14) is made up of three linear equations and two unknown variables: η and ζ . As (14) is an inconsistent system there no unique solution for η and ζ . However, three independent combinations of equations can be studied to solve a part of the system. The three combinations of equations give rise to three independent sets of solutions for η and ζ . Each set of solution can be seen as an approximate PML formulation for the TLM method.

- **The first set of solutions** for η and ζ is calculated from equations (14a) and (14b). After rearrangement, equation (14a) gives directly:

$$\eta = -2\sigma^2 \delta t^2. \quad (15)$$

The expression of η (15) can be injected in (14b). By substitution, this gives:

$$\zeta = 4\sigma\delta t - 2\sigma^2 \delta t^2, \quad (16)$$

this corresponds to the first approximate PML formulation in this document.

- **The second set of solutions** is calculated from equations (14b) and (14c). This system gives:

$$\eta = -\frac{4\sigma^2 \delta t^2}{2 + \sigma^2 \delta t^2}, \quad (17)$$

and

$$\zeta = \frac{8\sigma\delta t}{2 + \sigma^2 \delta t^2}, \quad (18)$$

this corresponds to the second approximate PML formulation.

- **The third set of solutions** is calculated from equations (14a) and (14c). This system gives:

$$\eta = 0, \quad (19)$$

and

$$\zeta = 4\sigma\delta t, \quad (20)$$

this corresponds to the third approximate PML formulation.

Condition on the first set of solutions

A condition on σ is introduced which enables to verify every equation of the system (14) with the solutions (15) and (16). From equation (14a) the following equality can be written:

$$\eta + \zeta + 4 = 4(1 + \sigma\delta t), \quad (21)$$

and using (15), (16) and (21) in equation (14c) enable to write:

$$\frac{\eta - \zeta + 4}{\eta + \zeta + 4} = \frac{1 - \sigma\delta t - \sigma^2 \delta t^2}{1 + \sigma\delta t}. \quad (22)$$

If relation (22) is related to equation (14c), then the following condition should be satisfied: $\sigma^2 \delta t^2 \ll 1 - \sigma\delta t$. This induces the following inequality:

$$\frac{\sigma^2 \delta t^2}{1 - \sigma\delta t} < \varepsilon, \quad (23)$$

where ε should be minimized. This condition introduces a limitation on the value of the PML attenuation factor σ which is written as:

$$\sigma < \frac{1}{2\delta t} (\sqrt{\varepsilon^2 + 4\varepsilon} - \varepsilon). \quad (24)$$

This condition (24) on σ allow to verify every equation of the system (14). From equation (24) it can be seen that the maximum value for the PML attenuation factor σ_{max} is determined as a function of the variable ε . This can be written as:

$$\sigma_{max} = \frac{1}{2\delta t} (\sqrt{\varepsilon^2 + 4\varepsilon} - \varepsilon). \quad (25)$$

For the following simulations the parameter ε is set equal to: $\varepsilon = 10^{-2}$. Similarly, the condition on the second set of solutions can be derived in order to verify every equation of the system (14).

Discussion on the validity of every set of solutions

Whereas the first and second sets of solutions can be optimized to satisfy every equation of the system (14) by setting a condition on the parameter σ , the third solution can not be used to solve the whole system of equations as it gives rise to incoherent condition. Thus, the third solution is rejected from this study.

However, it can be noticed that the third set of solutions corresponds to a purely dissipative TLM propagation scheme. This can be the object for further work on the absorbing conditions.

Assessment of the absorbing layers

In order to assess the numerical efficiency of a given absorbing condition, an ideal free-field (p_{ff}) wave propagation is compared to the real pressure (p) in presence of the absorbing layer as depicted by figure 3. Therefore, Sheu *et al.* [4] defined the mean error level to evaluate the impact of absorbing layers on the truncated propagation domain of interest:

$$\text{error}(x, y) = 10 \log_{10} \frac{\sum_{t=0}^T |p_{ff}(x, y, t) - p(x, y, t)|^2}{\sum_{t=0}^T |p_{ff}(x, y, t)|^2}. \quad (26)$$

The free-field pressure is calculated for an infinite propagation domain (D_0) and used as a reference. Whereas the true pressure is calculated inside the test area depicted by figure 3. The width of the absorbing layer is denoted by e_{AL} . It is important to distinguish the propagation domain (D_0) and the absorbing layer domain (D_1) where the tested absorbing formulation is implemented. The

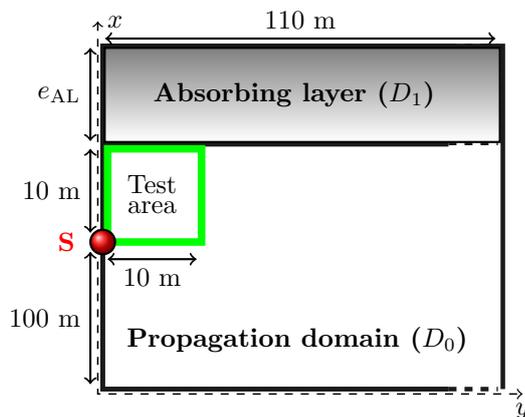


Figure 3: Computational domain implemented to assess the performances of an absorbing layer inside the test area (green square).

contour of the whole computational domain is made up of rigid boundaries. The source position allows to simulate wave propagation which can be considered as semi-infinite following the y -axis in the 2D cartesian plan, for a simulation duration $t_{sim} = 0.5$ s.

Numerical results

Numerical simulations has been carried with the parameter: $\varepsilon = 10^{-2}$, which gives a maximum value for the

attenuation factor: $\sigma_{max} = 190$.

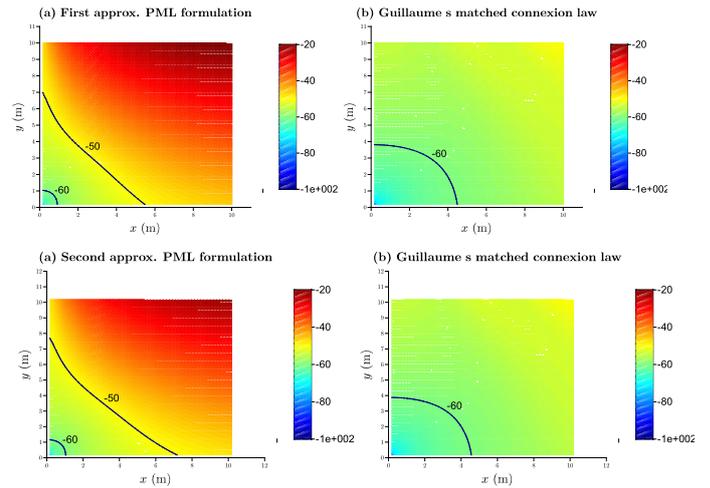


Figure 4: Mean error function level (dB) inside the test area for an absorbing layer thickness $40nodes \Leftrightarrow N_\lambda = 2$, $f_{signal} = 100$ Hz: (a) first and second approximate PML formulations ; (b) GMCL.

Conclusion

The results show that both first and second approximate PML formulations are typical of the introduction of a discontinuity between the propagation domain and the absorbing layer. This phenomenon caused by the negative heterogeneity fluctuation. Although the third set of solutions as been rejected from this study, it is interesting to not that it give rise to purely dissipative TLM propagation scheme. This dissipative propagation scheme would give rise to further investigations.

References

- [1] Hofmann, J., Heutschi, K.: Simulation of outdoor sound propagation with a transmission line matrix method. *Applied Acoustics* **68** (2007), 158-172
- [2] de Cogan, D., O'Connor, W. J., Pulko, S.: *Transmission Line Matrix in Computational Mechanics*. CRC Press Edition, Taylor & Francis, 2006
- [3] Guillaume, G. ; Picaut, J.: A simple absorbing layer implementation for transmission line matrix modeling. Submitted to JSV (2013).
- [4] Sheu, Y. L. and Li, P. C.: Simulations of photoacoustic wave propagation using a finite-difference time-domain method with Brenger's perfectly matched layers. *J. Acoust. Soc. Am.* **124** (2008), 3471-3480
- [5] Porti, J. A. and Morente, J. A.: TLM methods and acoustics. *Int. J. Numer. Model.* **14** (2001), 171-183
- [6] Kagawa, Y. ; Tsuchiya, T. ; Fuji, B.; Fujioka, K.: Discrete Huygen's model approach to sound wave propagation. *J. Sound Vib.* **218** (1998), 419-444