Review of the paper Hypertractions and hyperstresses convery the same mechanical information. Authors: Paolo Podio-Guidugli and Maurizio Vianello.

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Hypertractions and hyperstresses convery the same mechanical information

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Reviewed by Francesco dell'Isola

This article contains a description of the theory of second gradient continua, a theory which is attracting increasing interest. This is accounted for by the large number of researchers who are using it for modelling a variety of phenomena ranging from capillarity or phase transition, to plasticity or flow in porous media. The final recognition of the importance of P. Germain's paper [J. Mécanique 12 (1973), 235–274] which concludes and systematizes the works by R. A. Toupin [Arch. Rational Mech. Anal. 17 (1964), 85–112], R. D. Mindlin [Internat. J. Solids Structures 1 (1965), no. 4, 417–438] and P. Casal [C. R. Acad. Sci. Paris Sér. A 274 (1972), 1571–1574; Zbl 0286.76004]—fully justifies its revival. However, such a revival should add some new results to the theory, either clarifying the underlying physical phenomenology or somehow developing the mathematics used. This is not the case in the paper under review: indeed in it, (i) while Paul Germain paid careful attention to introducing a physically meaningful nomenclature for designating the different types of contact actions, here they are all mixed up in a less meaningful "hyper-"designation; (ii) the classification of second gradient materials which do not exert contact edge forces and boundary conditions to be assigned in the framework of second gradient continuum models is claimed to be novel, while instead it has been well known for a long time [see, e.g., F. dell'Isola and P. Seppecher, Meccanica 32 (1997), no. 1, 33–52]; and (iii) instead of using the most suitable, complete and original (multi-index) version of Levi-Civita absolute calculus, the authors try to use a less suitable index-free notation which is well adapted only in contexts where only second-order tensors suffice. In addition, the validity of the Principle of Virtual Powers, not only for the whole continuous body considered but also for all its subbodies, is already explicitly postulated in the work of E. Cosserat and F. Cosserat [Théorie des corps déformables, Hermann, Paris, 1909]. There is no reason for the authors to refer to a strengthened or generalized version of the Principle of Virtual Powers (or Virtual Works) since a word-for-word statement of it can be found, for instance, in the textbook [Mecanique des milieux continus. II, Ellipses, Paris, 1988] of J. Salençon at the École Polytechnique (Paris Tech). On the other hand, the claim that, in the literature, there is no available treatment of second gradient continuum mechanics "à la Cauchy" (i.e., an approach which is starting from the concept of contact actions and is able to deduce the general form of internal powers with a "tetrahedron" argument) is not accurate: such a result has been obtained by dell'Isola and Seppecher [op. cit.], as recognized for instance by G. A. Maugin in his review of that paper, or by S. Forest et al. [Mécanique des milieux continus, Ecole des mines, Paris, 2005–2006; per revr.].