



**HAL**  
open science

## Robust stabilization of the current profile in tokamak plasmas using sliding mode approach in infinite dimension

Oumar Gaye, Emmanuel Moulay, Sylvain Brémond, Laurent Autrique, R. Nouailletas, Jean-Francois Artaud, Y. Orlov

### ► To cite this version:

Oumar Gaye, Emmanuel Moulay, Sylvain Brémond, Laurent Autrique, R. Nouailletas, et al.. Robust stabilization of the current profile in tokamak plasmas using sliding mode approach in infinite dimension. *Control Engineering Practice*, 2013, 21 (10), pp.1350-1358. 10.1016/j.conengprac.2013.05.003 . hal-00845383

**HAL Id: hal-00845383**

**<https://hal.science/hal-00845383>**

Submitted on 17 Jul 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Robust stabilization of the current profile in Tokamak plasmas using sliding mode approach in infinite dimension

O. Gaye<sup>a</sup>, E. Moulay<sup>b</sup>, S. Brémond<sup>c</sup>, L. Autrique<sup>a,\*</sup>, R. Nouailletas<sup>c</sup>,  
J.F. Artaud<sup>c</sup>, Y. Orlov<sup>d</sup>

<sup>a</sup>LISA (EA 4094) – Université d'Angers, 62 avenue Notre Dame du Lac, 49000 Angers – France, Tel: +33 241 226 518, Fax: +33 241 226 561

<sup>b</sup>Xlim-SIC (UMR CNRS 7252), Université de Poitiers - Bât. SP2MI, Bvd Marie et Pierre Curie – BP 30179, 86962 Futuroscope Chasseneuil Cedex – France

<sup>c</sup>CEA/DSM/IRFM CEA-Cadarache, 13108 Saint Paul Lez Durance – France

<sup>d</sup>CICESE Research Center, Km 107. Carretera Tijuana-Ensenada, Ensenada, B.C. 22860 – Mexico

---

## Abstract

This paper deals with the robust stabilization of the spatial distribution of tokamak plasmas current profile using a sliding mode feedback control approach. The control design is based on the 1D resistive diffusion equation of the magnetic flux that governs the plasma current profile evolution. The feedback control law is derived in the infinite dimensional setting without spatial discretisation. Numerical simulations are provided and the tuning of the controller parameters that would reject uncertain perturbations is discussed. Closed loop simulations performed on realistic test cases using a physics based tokamak integrated simulator confirm the relevance of the proposed control algorithm in view of practical implementation.

*Keywords:* Tokamak plasmas control, Parabolic partial differential equations, Lyapunov stabilization, sliding mode control, Robust control

---

## 1. Introduction

Fossil fuels (oil, gas, coal) account for approximately 85% of the worldwide sources of primary energy today. But they should run out with in some tens of years and they are responsible for a climate change via the contribution in the greenhouse effect of the CO<sub>2</sub> generated by their combustion.

---

\*Corresponding author

*Email addresses:* [oumar.gaye@etud.univ-angers.fr](mailto:oumar.gaye@etud.univ-angers.fr) (O. Gaye),  
[emmanuel.moulay@univ-poitiers.fr](mailto:emmanuel.moulay@univ-poitiers.fr) (E. Moulay), [sylvain.bremond@cea.fr](mailto:sylvain.bremond@cea.fr) (S. Brémond),  
[laurent.autrique@univ-angers.fr](mailto:laurent.autrique@univ-angers.fr) (L. Autrique), [remy.nouailletas@cea.fr](mailto:remy.nouailletas@cea.fr)  
(R. Nouailletas), [jean-francois.artaud@cea.fr](mailto:jean-francois.artaud@cea.fr) (J.F. Artaud), [yorlov@cicese.mx](mailto:yorlov@cicese.mx)  
(Y. Orlov)

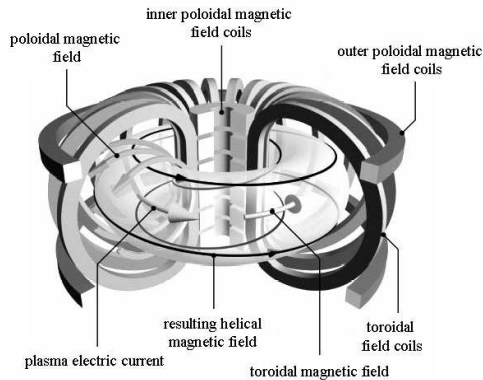


Figure 1: Tokamak magnetic configuration

The controlled thermonuclear fusion is one of the options being studied in order to eventually provide an answer to these issues. Its main assets are to be a potential inexhaustible and safe source of energy because the reserves in nuclear fuel are plenty (Deuterium can be extracted from sea water, and Lithium, that has to be used to breed Tritium, can be found in continental crust) and because there is no risk of runaway reaction nor long lasting radioactive waste.

The key world project in the domain, ITER, is led by seven partners (Europe, United States of America, Japan, China, India, South Korea, Russia) accounting for one half of the world population. The main objective of the ITER project is to demonstrate the scientific feasibility of thermonuclear fusion.

Several conditions have to be met to produce fusion reactions (Wesson (2004)): the fuels have to be heated up to very high temperature (around 100 millions degrees) in order to overcome the electrostatic potential barrier between positively charged nucleus. To reach such a temperature, the ionized gas or plasma must be confined, for example by magnetic confinement which seems to be the most promising way. This magnetic confinement is obtained in a tokamak (Wesson (2004) and Saoutic et al. (2009)) by superimposing different electric currents in a torus-like configuration device, including a high current, of the order of the MA, within the plasma itself (see Fig. 1).

This plasma current can be produced by inductive means, in particular at the beginning of the plasma pulses, and by non-inductive means through the injection of fast particles and /or waves (Wesson (2004) and Fisch (1987)). The 1D radial profile of this plasma current, via the so-called safety factor profile, is known to be a key parameter for tokamak plasma performance. It indeed plays a crucial role in the global magnetohydrodynamic (MHD) (Wesson (2004) and Freidberg (2007)) stability of plasma experiments. Moreover, it has been observed that some specific profiles may generate some enhanced confinement of the plasma energy (see for example the work of Baranov et al. (2004), Challis et al. (2002), Taylor (1997) and Wolf (2003)). It is obvious that such profiles are

very attractive and may at the end reduce the size and cost of future fusion reactors. Several approaches have been developed regarding the control of tokamak plasmas current profile. The control of one single shape parameter, based on Single Input - Single Output (SISO) semi-empirical approaches, has been performed experimentally (e.g plasma internal inductance by Wijnands et al. (1997) or non-inductive current drive profile width by Mazon et al. (2002)). But this is clearly not enough to match the main requirements of MHD stability and/or internal transport barrier issues in advanced tokamak scenarios.

The control of the safety factor profile in a few number of points, based on a Multi Input Multi Output (MIMO) approach in finite dimensional setting was developed using linear models identified from experimental data (Moreau et al. (2008)). It was experimentally tested but showed severe limitations in terms of robustness, given also the lack of power from the actuators as shown by Ariola et al. (2008). Some other recent approaches have been developed on simulations (see the work of Ou & Schuster (2009a), Xu et al. (2010), Argomedo et al. (2010), Ou et al. (2011) and Gaye et al. (2013)).

Some nonlinear control techniques for tokamak reactors are presented in Ou et al. (2007), Ou & Schuster (2009b) and Ou et al. (2010). In Ou et al. (2007), extremum seeking approach is implemented for optimal control of a nonlinear distributed parameter system. In Ou & Schuster (2009b) and Ou et al. (2010), the evolution of the poloidal magnetic flux profile (modeled by a nonlinear partial differential equation), is controlled using a proper orthogonal decomposition (POD) and Galerkin projection.

The approach proposed in this paper is to design the control law in the infinite dimensional setting (without spatial discretisation) and without linearisation. This allows to keep as long as possible the physical phenomena at stake, and in particular the meaning of the terms manipulated, and thus to better integrate the knowledge of model uncertainties and disturbances during the design control process (whereas for linear finite dimensional models obtained for instance from identification, the physics parameters are mixed up in the resulting transfert functions of  $A$ ,  $B$ ,  $C$ ,  $D$  matrices coefficients, thus making the physical uncertainties very difficult to handle). The model reduction is performed at the latest stage, that is to say when the real engineering inputs have to be set using a model based optimisation process (a realistic estimation of the infinite dimensional control inputs is then obtained).

In order to investigate the practical relevance of the proposed control approach, closed loop simulations using one of the internationally recognised physics based tokamak integrated simulator have been performed.

The paper is organized as follows. The overall description of the control problem is given in section 2. In section 3 a mathematical analysis of the distributed control model is proposed. Details are given on the transformation of the PDE that are required in order to prepare the control design. Section 4 is devoted to the construction of a control law. Simulation results, using the METIS (see the work of Artaud (2008)) code dedicated to plasma scenario studies, are provided in section 5. A preliminary version of the proposed results without simulation METIS has been published by Gaye et al. (2011). Finally,

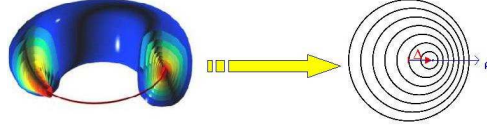


Figure 2: 1D geometry of the simplified formulation of the problem

conclusion remarks are collected in section 6.

## 2. Control problem description

The quantity to be controlled is the so called  $q$  safety factor profile. Indeed, this physics quantity which characterises the twisting of the magnetic field lines on the magnetic equilibrium surfaces ( $q$  measures the number of turns around the tokamak main axis that a magnetic field lines has to make before going back to the same position in a meridian plane) is extremely important in tokamak research. In particular, different energy confinement regimes were observed depending on the  $q$  profile being either monotonous or reversed. Rational values of  $q$  play also a very important role for what concerns MHD stability.

An equivalent output quantity is the magnetic flux profile relative to its edge value (at  $x = 1$ )

$$\psi_r(t, x) = \psi(t, x) - \psi(t, 1) \quad (1)$$

as

$$\psi_r(t, x) = a^2 B_0 \int_x^1 \frac{r}{q(t, r)} dr.$$

In order to calculate  $\psi_r^{target}$ , we have the relation between the new variable  $\psi_r(t, x)$  and the safety factor  $q(t, x)$ . Then, considering a realistic desired distribution of the safety factor  $q$  (issued for example from experimentations or from analytical resolution),  $\psi_r^{target}$  can be defined.

Provided usual assumptions (axisymmetry, MHD equilibrium, averaging over the magnetic surfaces, cylindrical approximation, etc. (see the work of Blum (1989) and Witrant et al. (2007))), the evolution of the plasma safety factor  $q$  can be obtained by solving the following 1D PDE

$$\begin{cases} \frac{\partial \psi}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi}{\partial x} \right) + \eta_{||} R_0 j_{ni}; \\ \frac{\partial \psi}{\partial x} \Big|_{x=0} = 0, \quad \frac{\partial \psi}{\partial t}(t, 1) = -V_0(t). \end{cases} \quad (2)$$

$$q = -\frac{a^2 x B_0}{\frac{\partial \psi}{\partial x}}$$

and

$$j_T = -\frac{1}{\mu_0 R_0 a^2 x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi}{\partial x} \right)$$

where  $x \in (0, 1)$  is the 1D (radial) profile coordinate,  $\psi(t, x)$  the magnetic flux.  $R_0$  the major radius,  $a$  the minor radius of the plasma boundary (assumed to be fixed),  $\mu_0$  the permeability of vacuum,  $B_0$  the toroidal magnetic field at  $R_0$  are known parameters. Plasma loop voltage  $V_0$  is the boundary control and non inductive current density  $j_{ni}$  is the interior control.  $\eta_{||}(t, x)$  the parallel resistivity of the plasma is not accurately known and depends on the plasma temperature. However, considering thermal evolution in tokamak plasma, it is natural to consider for physical reasons that  $\eta_{||}$  is bounded.  $j_T(t, x)$  is the total current density.  $q(t, x)$  is the safety factor to be controlled (plant output).

The second boundary condition in (2)  $\frac{\partial \psi}{\partial t}(t, 1) = -V_0$  (based on a time derivative) can be also found in numerous references Witrant et al. (2007), Argomedo et al. (2010), Ouarit et al. (2011) and Gaye et al. (2011).

The control variables in the infinite dimensional setting are the plasma loop voltage  $V_0(t)$  and the non-inductive current density  $j_{ni}(t, x)$ .  $V_0$  can basically be directly set using the inner poloidal magnetic field coils voltage (see Fig. 1). The non-inductive current density is composed of the bootstrap current density which is self-generated by the plasma (Hirshman (1998)) and of additional source terms provided by different actuators, namely the Lower Hybrid, Electron Cyclotron or Ion Cyclotron wave systems and/or the Neutral Beam Injection system. The most efficient is the Lower Hybrid Current Drive (LHCD) system that is routinely used on the Tore Supra tokamak (Saoutic et al. (2009)), which now has a capability to inject up to around 7 MW in steady state that shall allow to sustain plasma currents in the megaAmpere range on very long pulse (see the work of Ekedahl et al. (2010) and Becoulet et al. (2011)).

The control law design that will be derived in section 4 is generic to all kinds of current drive system. However, in this paper, the numerical simulations will be performed on Tore Supra typical conditions, using mainly Lower Hybrid waves as current drive means, so that in this particular case :

$$\dot{j}_{ni} = \dot{j}_{bs} + \dot{j}_{lh}$$

where  $j_{bs}$  and  $j_{lh}$  are respectively the bootstrap current density and the current density provided by the LHCD system.  $j_{bs}$  can be modelled by a nonlinear function of the flux (Hirshman (1998)).  $j_{lh}$  is modelled by Gaussian functions controlled by two engineering parameters: the LH power  $P_{lh}$  and the wave refractive index  $N_{lh}$ , (see the work of Witrant et al. (2007) for further details).  $V_0$ ,  $P_{lh}$  and  $N_{lh}$  are the considered control variables (plant inputs) for the simulations of section 5.

In the present investigation, the resistivity  $\eta_{||}(t, x)$  is assumed to be lower and upper bounded by some positive constants  $\eta_1$  and  $\eta_2$ . The upper- and lower boundedness of the resistivity is required to validate the applicability of the sliding mode approach from Orlov (2000) to the present case. Moreover, let us consider that  $\eta_{||}(t, x)$  is available for feedback purposes through some on-line estimation, basically from electronic temperature measurements (see the work of Witrant et al. (2007) for more details).

The feedback control designed in this paper is composed of two steps (see

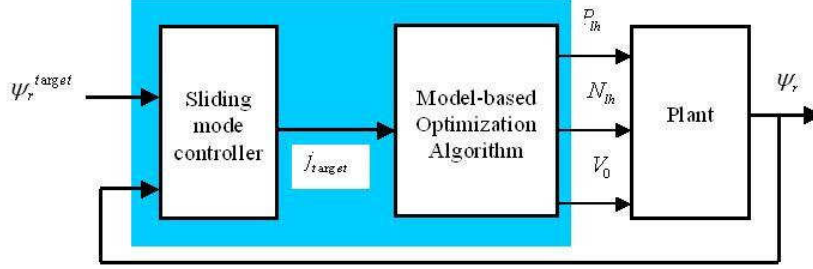


Figure 3: Control scheme of the system

Fig. 3).

The first step provides the current density  $j_{target}$  to be applied to the system. Here, a sliding mode controller has been designed in infinite dimensional setting whose derivation will be detailed in the following. It takes as inputs the target flux profile  $\psi_r^{target}$  and the current flux profile provided by the plant. The output  $j_{target}$  has to be reachable by the actuators. Thus preliminary numerical simulations or experimental observations have to be taken into account. The difference between the target current density  $j_{target}$  and the one that the actuators can provide is handled by the robustness of the controller as an unknown bounded disturbance on the input ( $|j_T - j_{target}| < \Delta_j, \forall x \in [0, 1]$ ).

The second step is an optimization process which finds the best set of parameters  $P_{lh}$ ,  $N_{lh}$  and  $V_0$  to fit the current density target  $j_{target}$  provided by the sliding mode control law in the infinite dimensional setting. The optimisation algorithm minimizes the quadrature (Ouarit et al. (2011))

$$\varepsilon(P_{lh}, N_{lh}, V_0) = \int_0^1 (j_{target} - (j_\Omega + j_{lh} + j_{bs}))^2 dx$$

under the constraints

$$V_{\min} < V_0 < V_{\max}, P_{\min} < P_{lh} < P_{\max}, N_{\min} < N_{lh} < N_{\max}$$

where  $j_\Omega = j_T - j_{ni}$  is the steady state ohmic part of the current density, equal to  $\frac{-V_0}{\eta_{||} R_0}$ .

In the simulations, the minimum / maximum value of the engineering inputs will be taken from the Tore Supra tokamak constraints, i.e.

$$-5V < V_0 < 5V, 0MW < P_{lh} < 7MW, 1.43 < N_{lh} < 2.37$$

At each time step, the optimization algorithm looks for the set of control variables that minimize the quadratic criterion  $\varepsilon(P_{lh}, N_{lh}, V_0)$  considering the previous constraints on  $P_{lh}$ ,  $N_{lh}$  and  $V_0$ . Then  $P_{lh}$ ,  $N_{lh}$  and  $V_0$  are applied at the following time step. In practice, the minimization is performed using a numerical MATLAB optimization toolbox (gradient-based algorithm *fmincon*).

### 3. Control model mathematical analysis

The state equation (2), rewritten in terms of  $\psi_r$ , reduces to

$$\begin{cases} \frac{\partial \psi_r}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r}{\partial x} \right) + \eta_{||} R_0 j_{ni} + V_0(t) \\ \frac{\partial \psi_r}{\partial x} \Big|_{x=0} = 0, \quad \psi_r(t, 1) = 0 \end{cases} \quad (3)$$

In order to obtain the second boundary condition in (3), the definition of  $\psi_r$  given in (1) is considered:

$$\psi_r(t, x) = \psi(t, x) - \psi(t, 1) \text{ implies } \psi_r(t, 1) = \psi(t, 1) - \psi(t, 1) = 0.$$

Then, let us introduce the error variable

$$\phi(t, x) = \psi_r(t, x) - \psi_r^{target}(x) \quad (4)$$

with respect to the steady state target  $\psi_r^{target}(x)$  which has to be reached in the Sobolev space

$$W^{2,2}(0, 1) = \left\{ \psi \in L^2(0, 1) : \frac{\partial \psi}{\partial x} \in L^2(0, 1) \text{ and } \frac{\partial^2 \psi}{\partial x^2} \in L^2(0, 1) \right\}$$

of differentiable functions, whose spatial derivative is square integrable on the interval  $(0, 1)$ . Then the error variable is governed by

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \phi}{\partial x} \right) + \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r^{target}}{\partial x} \right) + \eta_{||} R_0 j_{ni} + V_0(t); \\ \frac{\partial \phi}{\partial x} \Big|_{x=0} = - \frac{\partial \psi_r^{target}}{\partial x} \Big|_{x=0}, \quad \phi(t, 1) = -\psi_r^{target}(1). \end{cases} \quad (5)$$

In order to deal with the regular term  $\frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r^{target}}{\partial x} \right)$  in (5) and homogeneous boundary conditions let us assume that

$$\lim_{x \rightarrow 0} \left| \frac{1}{x} \frac{\partial \psi_r^{target}}{\partial x} \right| < +\infty; \quad (6)$$

$$\frac{\partial \psi_r^{target}}{\partial x} \Big|_{x=0} = 0; \quad \psi_r^{target}(1) = 0. \quad (7)$$

These assumptions lead to the system with homogeneous boundary conditions

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \phi}{\partial x} \right) + \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r^{target}}{\partial x} \right) + \eta_{||} R_0 j_{ni} + V_0(t); \\ \frac{\partial \phi}{\partial x} \Big|_{x=0} = 0, \quad \phi(t, 1) = 0. \end{cases} \quad (8)$$

Since

$$\frac{\eta_{||}}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \phi}{\partial x} \right) = \frac{\partial}{\partial x} \left( \eta_{||} \frac{\partial \phi}{\partial x} \right) + \left( \frac{\eta_{||}}{x} - \frac{\partial \eta_{||}}{\partial x} \right) \frac{\partial \phi}{\partial x} \quad (9)$$



the equation (8) with the singular term  $\frac{1}{x}$  can be brought into the regular form without singularities by applying the following feedback transformation

$$\eta_{||}R_0j_{target} = \eta_{||}R_0j_{ni} + V_0 = \eta_{||}u + v \quad (10)$$

with a virtual control input  $u$  and the relation

$$v = \frac{1}{\mu_0 a^2} \left( -\frac{\eta_{||}}{x} + \frac{\partial \eta_{||}}{\partial x} \right) \frac{\partial \phi}{\partial x} - \frac{\eta_{||}}{\mu_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r^{target}}{\partial x} \right) \quad (11)$$

is deduced from the state transformations (1) and (4). Then substituting (10) subject to (11) into (8) yields

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\eta_{||}}{\mu_0 a^2} \frac{\partial \phi}{\partial x} \right) + \eta_{||}u; \\ \frac{\partial \phi}{\partial x} \Big|_{x=0} = \phi(t, 1) = 0. \end{cases} \quad (12)$$

The feedback transformation method is first used to bring the infinite-dimensional system with a non self-adjoint operator into the standard form captured by the sliding mode approach Orlov (2000).

The resulting equation is a standard parabolic equation in the Sobolev space

$$H = \left\{ \phi \in W^{2,2}(0, 1) : \frac{\partial \phi}{\partial x} \Big|_{x=0} = \phi|_{x=1} = 0 \right\}$$

of the square integrable functions subject to the boundary conditions corresponding to the boundary value problem (12) and equipped with the norm

$$\|\phi\|_H = \sqrt{\int_0^1 \phi^2 dx} + \sqrt{\int_0^1 \left( \frac{\partial \phi}{\partial x} \right)^2 dx} + \sqrt{\int_0^1 \left( \frac{\partial^2 \phi}{\partial x^2} \right)^2 dx}. \text{ The operator}$$

$$A\phi = \frac{\partial}{\partial x} \left( \frac{\eta_{||}}{\mu_0 a^2} \frac{\partial \phi}{\partial x} \right), \quad (13)$$

that appears in the right-hand side of the PDE (12), is defined on the domain  $D(A) = H \subset L^2(0, 1)$ . This operator is recognized as a Sturm-Liouville operator (see Naylor & Sell (1982)) and its spectrum consists of the discrete values

$$\lambda_k = -\frac{1}{\mu_0 a^2} \left( \frac{\left(k - \frac{1}{2}\right) \pi}{\int_0^1 \frac{ds}{\eta_{||}(s)}} \right)^2, \quad k = 1, 2, \dots$$

that correspond to the following eigenfunctions

$$\phi_k^0(x) = \cos \left( \left( k - \frac{1}{2} \right) \pi \frac{\int_0^x \frac{ds}{\eta_{||}(s)}}{\int_0^1 \frac{ds}{\eta_{||}(s)}} \right) \quad (14)$$

for the scalar product  $\langle f|g \rangle_{\frac{1}{\eta_{||}}} = \int_0^1 f(x)g(x) \frac{dx}{\eta_{||}(x)}$  and for all  $k$

$$I_1(k) = \|\phi_k^0(x)\|_{\frac{1}{\eta_{||}}} = \sqrt{\int_0^1 \phi_k^0(x)^2 \frac{dx}{\eta_{||}(x)}}.$$

The Sturm-Liouville operator  $A$  generates an exponentially stable semigroup.

The relation  $\eta_1 \leq \eta_{||}$  implies that operator  $A$  is exponentially stable and  $\eta_{||} \leq \eta_2$  implies that operator  $B$  ( $Bu = \eta_{||}u$ ) is bounded. In order to apply the sliding mode controller in infinite dimensional based on the approach proposed in Orlov (2000), the system must be exponentially stable.

The following theorem, being applied to (12), shows that the system (12) has continuous solutions (see the work of Cannarsa et al. (2004) and Rosier (2007) for more details).

**Theorem 1.** *For given  $z_0 \in \mathbf{H}$  where  $\mathbf{H}$  is a Hilbert space, let us consider the Cauchy problem*

$$\begin{cases} \dot{z} = Az + Bu; \\ z(0) = z_0. \end{cases} \quad (15)$$

where  $A$  is the infinitesimal generator of a strongly continuous semigroup  $(S(t))_{t \geq 0}$  on  $L^2(0, 1)$ . Let  $u$  belongs to  $L^2((0, T) \times (0, 1))$  for all  $T > 0$ . Then for all  $z_0 \in D(A)$  where  $D(A)$  is the domain of the operator, there exists a unique strong solution  $z \in \mathcal{C}([0, +\infty); D(A))$  of (15); and for all  $z_0 \in L^2(0, 1)$  there exists a unique weak solution  $z \in \mathcal{C}([0, +\infty); L^2(0, 1))$  of (15). The strong solution of (15) is given by the Duhamel formula

$$z(t) = S(t)z_0 + \int_0^t S(t-s)Bu(s)ds \quad \forall t \in [0, T].$$

The above formula is still meaningful and defines the weak solution of (15).

The system (12) is null controllable if  $u$  belongs to  $L^2$  (see the work of Cannarsa et al. (2004, 2006) for details). It implies that the system (12) is exponentially stabilizable ((Rosier, 2007, Theorem 4.12, p. 407)). Consequently, the sliding mode strategy developed by Orlov (2000) can be applied.

The controller developed in Orlov (2000) is applicable only for systems which are exponentially stable. The partial differential equation system (2) contains a non self-adjoint infinitesimal operator that has not been addressed in the literature.

In order to control (2), the control law is expressed in order to obtain a new PDE (12) that is exponentially stable. Then, in this new framework, the approach proposed by Y. Orlov in Orlov (2000) is developed in order to build a robust control law.

#### 4. Sliding mode controller

The problem of the control of partial differential equations (PDEs) is an active area of research (see the work of Coron (2007) and Smyshlyaev & Krstic

(2010) for example), but very few constructive methods are available. For robust stabilization, a sliding mode strategy has nevertheless recently been developed by Orlov (2000).

The sliding mode control approach, developed by Orlov (2000) for infinite dimensional systems, is further adapted to be applied to our control problem. The virtual control input  $u$  is designed as follows:

$$u(\phi) = - \left( \mathcal{N} + L \sqrt{\sum_{k=0}^{k=N_d} (P^k(\phi))^2} \right) \text{sign} \left( \sum_{k=0}^{k=N_d} c_k P^k(\phi) \right) \quad (16)$$

where  $C = (c_k)_{k=0}^{N_d}$  is the matrix which defines the sliding surface,  $\mathcal{N}$  is determined to reject the disturbances,  $L$  is determined to ensure the Lyapunov stability and  $N_d$  is the number of projections  $P^k$  on the eigenfunctions of the operator  $A$  defined by  $P^k(\phi) = \frac{1}{\Gamma_1(k)^2} \int_0^1 \phi(t, x) \phi_k^0(x) \frac{dx}{\eta_{||}}$ ,  $k \in \{0, 1, \dots\}$ . The following relation describes the sliding motion along the sliding surface (see the work of Orlov (2000))

$$\dot{x}_1 = [A_1 - B_1 (CB_1)^{-1} CA_1] x_1 \quad (17)$$

where  $x_1 = (P^0 \phi(t, \cdot), \dots, P^{N_d} \phi(t, \cdot))^T$  is the projection of  $\phi$  on the first  $N_d + 1$  eigenfunctions of operator  $A$  and where

$$A_1 = \text{diag} \{ \lambda_k \}_{k=0}^{N_d} \in \mathbb{R}^{N_d+1 \times N_d+1}$$

and

$$B_1 = (P^0 \eta_{||}, P^1 \eta_{||}, \dots, P^{N_d} \eta_{||})^T.$$

Let us consider

$$A_c = A_1 - B_1 (CB_1)^{-1} CA_1 \quad (18)$$

then

$$CA_c = CA_1 - CB_1 (CB_1)^{-1} CA_1 = 0 \Leftrightarrow A_c^T C^T = 0. \quad (19)$$

Consequently  $C^T$  is an eigenvector of  $A_c^T$  associated to the eigenvalue  $\lambda = 0$ . Now let us introduce  $K = (CB_1)^{-1} CA_1$ . From (17),

$$\dot{x}_1 = [A_1 - B_1 K] x_1. \quad (20)$$

The matrix  $K$  is chosen in order to define an appropriate system dynamics on the sliding surface. The choice of  $K$  does not affect the stability but determine the characteristics of sliding mode.  $K$  is chosen such that  $A_c^T$  has a first eigenvalue equal to zero while other eigenvalues are strictly negative. In order to determine  $L$ , the system in  $x_1$  without disturbance ( $\mathcal{N} = 0$ ) is considered

$$\dot{x}_1 = A_1 x_1 + B_1 u \quad (21)$$

and the Lyapunov function

$$V = \frac{1}{2} S^2 > 0, \quad \text{with } S = Cx_1.$$

It follows that

$$\begin{aligned}\frac{dV}{dt} &= S\dot{S} = Cx_1C(A_1x_1 + B_1u) \\ &= Cx_1CA_1x_1 - Cx_1CB_1L \|x_1\| \text{sign}(S).\end{aligned}$$

Then  $\dot{V} < 0$  if and only if

$$Cx_1CA_1x_1 < CB_1L \|x_1\| |Cx_1|. \quad (22)$$

Knowing that

$$Cx_1CA_1x_1 \leq |Cx_1| \|CA_1\| \|x_1\| \quad (23)$$

and

$$CB_1L \|x_1\| |Cx_1| \leq |Cx_1| \|CB_1\| L \|x_1\|. \quad (24)$$

In order to have  $\dot{V} < 0$  it is sufficient to have

$$|Cx_1| \|CA_1\| \|x_1\| < CB_1L \|x_1\| |Cx_1|.$$

Then from (23) and (24),

$$L > \frac{\|CA_1\|}{\|CB_1\|}. \quad (25)$$

The constant  $L$  is lower bounded. Moreover, the proposed control law (16), specified with (12), rejects any additive external disturbances satisfying a matching condition

$$\alpha(t, x) = \eta_{||} h(t, x) \quad (26)$$

$$\begin{cases} \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x} \left( \frac{\eta_{||}}{\mu_0 a^2} \frac{\partial \phi}{\partial x} \right) + \eta_{||} u + \alpha(t, x); \\ \left. \frac{\partial \phi}{\partial x} \right|_{x=0} = \phi(t, 1) = 0. \end{cases} \quad (27)$$

with a priori known upper bounds  $\mathcal{H} > 0$  provided that

$$\|h\| \leq \mathcal{H} < \mathcal{N}. \quad (28)$$

The difference between the target current density  $j_{target}$  and  $j_T$  (which can be provided by real actuators) is denoted by  $\delta(t, x) = j_{target} - j_T$ . It appears in EDP system in the form  $\eta_{||} R_0 \delta = \eta_{||} \tilde{h}$ , which is the matching condition. Then the difference  $\delta(t, x)$  is handled by the robustness of the controller if  $\|\tilde{h}\| < \mathcal{N}$ .

The optimal approximation of the theoretically obtained controller is utilized and reasoning of this approach is then verified numerically.

In equation (16),  $\mathcal{N}$  and  $L$  are constant. The parameter  $L$  is lower bounded (see (25)) and ensures the stability in the nominal case (without perturbations). Parameter  $\mathcal{N}$  allows to reject the disturbances  $h$  if  $\mathcal{N} > \|h\|$  (see (28)).

Summarizing the following result is obtained.

**Theorem 2.** *Consider the error system (12) with the assumption (25). Let it be driven by the sliding mode controller (16) under the constraints (25)-(28). Then the closed-loop system is globally exponentially stable in the state space  $H$ .*

The stability remains in force even if system (27) is additively affected by an external disturbance (26) satisfying (28).

In the controller (16), the matrices  $C$  are given by the formula (18) with

$$A_1 = \text{diag} \{ \lambda_k \}_{k=0}^{N_d} \in \mathbb{R}^{N_d+1 \times N_d+1}, \quad B_1 = (P^0 \eta_{||}, P^1 \eta_{||}, \dots, P^{N_d} \eta_{||})^T.$$

The system (12) is approximated by the finite dimensional system (21) which is indeed finite time stabilized by the proposed controller (16). However, the infinite-dimensional rest of the system possesses the spectrum with negative real parts so that it is globally exponentially stable in the open-loop and it remains so while driven by the sliding mode controller, vanishing in finite time.

## 5. Simulation results

In this section, numerical simulations are performed in order to illustrate the sliding mode approach relevance in the context investigated (robust stabilisation of the current profile in Tokamak plasmas in infinite dimension). Evolution of the current spatial profile  $q$  from an initial state to a desired target profile is considered (see Fig. 4) (the goal is to ensure that the surface  $q = 1.5$  is not in the plasma in order to avoid MHD instability). We then also checked the effect of the disturbance

$$h(t, x) = \begin{cases} 0, & 0s \leq t < 10s \\ -\frac{5}{100\mu_0 R_0 a^2} \frac{1}{x} \frac{\partial}{\partial x} \left( x \frac{\partial \psi_r^{target}}{\partial x} \right), & 10s \leq t \leq 20s \end{cases} \quad (29)$$

added at the time instant  $t = 10s$  and representing 5% of the target total current density  $j_T^{target}$ . First of all, without engineering parameters research optimization process, parameters of the sliding mode controller are tuned. In order to illustrate closed loop system behavior, time evolution of  $\psi_r$  is drawn at point  $x = 0.4$  (behaviors are similar for other points). In Fig. 5a, Fig. 5b and 6 we have the result of simulations without using the METIS code. In Fig. 5a, effect of parameter  $L$  is shown (while other control parameters are fixed). In Fig. 5b, effect of parameter  $\mathcal{N}$  is shown for disturbances rejection purpose. As expected, (i) the closed loop system is stable, (ii) the larger the parameter  $L$ , the smaller the steady state error resulting from a disturbance; moreover time response can be speeded up using large parameter  $L$  at the expense of possibly overshoots, (iii) parameter  $\mathcal{N}$  allows to reject the steady state disturbance ( $\mathcal{N}$  in section 4 aims to reject the disturbances  $\alpha(t, x) = \eta_{||} h(t, x)$  if  $\mathcal{N} > \|h\|$ ), (iv) large values of parameter  $\mathcal{N}$  can speed up the rejection of steady state disturbance (but can induce large overshoots). In Fig. 6 we have the evolution of the variable  $\psi_r$  at some instants.

Then, in order to evaluate if the proposed control approach is realistic from the experimental point of view, simulations using METIS (Minute Embedded Tokamak Integrated Simulator) (Artaud (2008)) tokamak plant simulator are performed. METIS is a simplified version of the CRONOS suite of codes developed by Artaud et al. (2010) adapted to the final simulation test of tokamak

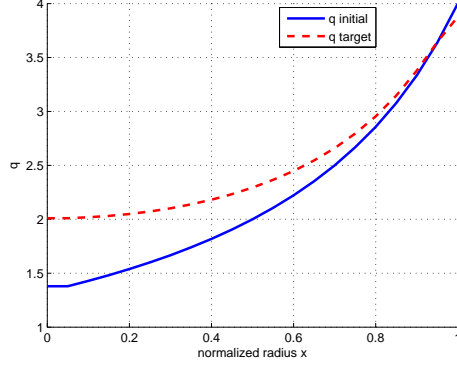
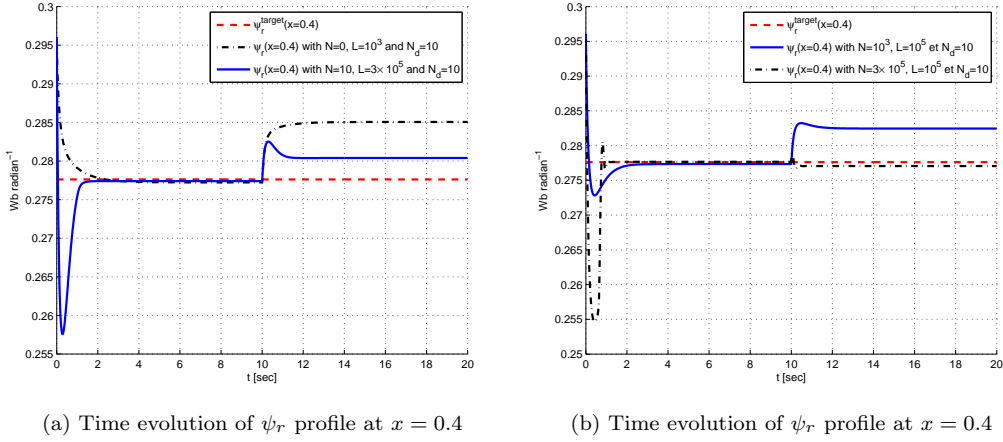


Figure 4: Initial and target profile of safety factor  $q$



(a) Time evolution of  $\psi_r$  profile at  $x = 0.4$

(b) Time evolution of  $\psi_r$  profile at  $x = 0.4$

Figure 5: Time evolution of magnetic flux  $\psi_r$ .

control algorithms on physics relevant model. The METIS code includes a full fast current diffusion solver and takes into account various nonlinear couplings between physical quantities. Such numerical simulator is quite time consuming (simulation for a 20s long Tore Supra tokamak plasma discharge takes about 3 hours). In the following, METIS is used jointly with the *Matlab/Simulink*<sup>TM</sup> toolbox to simulate Tore Supra plasma discharges. Preliminary METIS open loop simulation based on real pulse engineering inputs ( $V_0 = 0V$ ,  $P_{lh} = 4.17MW$  and  $N_{lh} = 1.7$ ) were performed in order to find a reachable target profile

The sliding mode controller feedback is then implemented with  $\mathcal{N} = 3 \times 10^5$ ,  $L = 10^5$  and  $N_d = 10$  (parameters previously adjusted in infinite dimension see 5a and Fig. 5b). The global feedback control scheme described in section 2

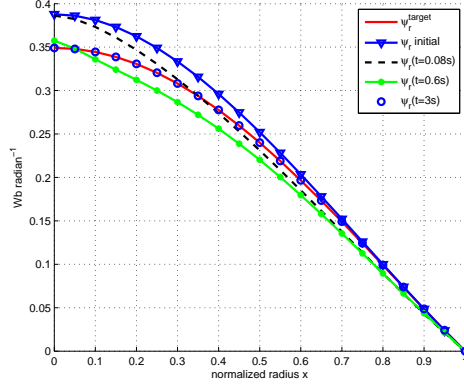


Figure 6: Evolution of  $\psi_r$  at some instants with  $\mathcal{N} = 3 \times 10^5$ ,  $L = 10^5$  and  $N_d = 10$

(see Fig. 3) is taken into account. In order to illustrate the method robustness, disturbance is added (see (29)).

An example of simulation results is given at Fig. 7, 8a, 8b, 9a and 9b. At  $t = 1$  s, the controller is activated in order to force the magnetic flux profile to reach the desired target. The time evolution of the magnetic flux  $\psi_r$  at point  $x = 0.2$ ,  $x = 0.6$  and  $x = 0.8$  are shown on Fig. 8b, 9a and 9b. Time evolution of the engineering plant inputs is plotted in Fig. 8a. The  $P_{lh}$ ,  $N_{lh}$  and  $V_0$  parameters behave as expected and experimental constraints for the engineering inputs are satisfied.

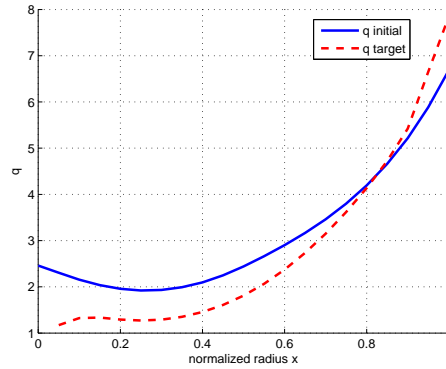
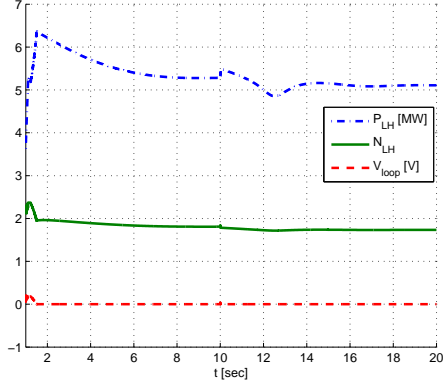
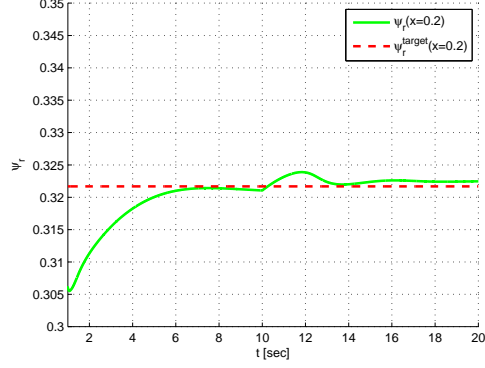


Figure 7: Initial and target profile of safety factor  $q$  with METIS code

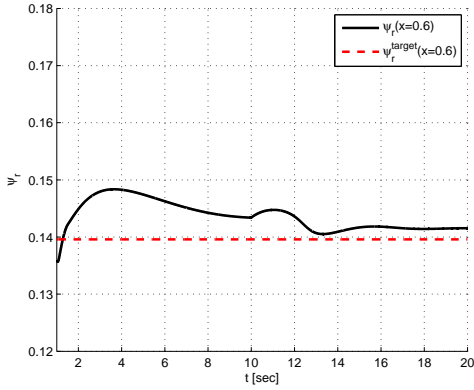


(a) Time evolution of engineering inputs

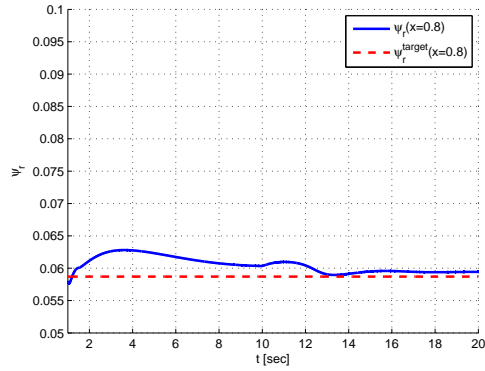


(b) Time evolution of  $\psi_r$  at  $x = 0.2$

Figure 8: Time evolution of engineering inputs and  $\psi_r$  at  $x = 0.2$



(a) Time evolution of  $\psi_r$  at  $x = 0.6$



(b) Time evolution of  $\psi_r$  at  $x = 0.8$

Figure 9: Time evolution of  $\psi_r$  at  $x = 0.6$  and  $x = 0.8$

## 6. Conclusion and future work

In this paper, a sliding mode control in the infinite dimensional setting has been designed for the control of tokamak plasma current profiles using 1D resistive diffusion equation of the magnetic flux. The investigated PDE system was reformulated so as to exhibit a Sturm-Liouville operator. Recent results on sliding mode control in infinite dimension were further developed to this control problem.

Then a model-based optimisation process was used to derive the engineering plant inputs related to both inductive and non-inductive current drive means. Numerical simulations were performed using typical Tore Supra values that



provide quite positive results with promising robustness properties. The practical interest of the new robust controller based on sliding mode approach in infinite dimension is demonstrated using METIS (Minute Embedded Tokamak Integrated Simulator). Future work will consist in preparing implementation on real Tore Supra experiments.

### Acknowledgment

This work was partially supported by ANR-12-BS03-008-03.

### References

- Argomedeo, F. B., Witrant, E., Prieur, C., Georges, D., & Brémond, S. (2010). Model-based control of the magnetic flux profile in a tokamak plasma. In *Proceedings of the 49th IEEE Conference on Decision and Control*. Atlanta, GA, USA.
- Ariola, M. et al. (2008). Integrated plasma shape and boundary flux control on jet tokamak. *Fusion Science and Technology*, 53, 789–805.
- Artaud, J. (2008). *Metis User's Guide*. Technical Report CEA, IRFM.
- Artaud, J. F. et al. (2010). The cronos suite of codes for integrated tokamak modeling. *Nuclear Fusion*, 50, 1–25.
- Baranov, Y. F., Garbet, X. et al. (2004). On the link between the q-profile and internal transport barriers. *Plasma Physics and Controlled Fusion*, 46, 1181–1196.
- Becoulet, A. et al. (2011). Steady state long pulse tokamak operation using Lower Hybrid Current Drive. *Fusion Engineering and Design*, 86, 490–496.
- Blum, J. (1989). *Numerical Simulation and Optimal Control in Plasma Physics with Application to Tokamaks*. Wiley / Gauthier-Villars Series in Modern Applied Mathematics.
- Cannarsa, P., Fragnelli, G., & Vancostenoble, J. (2006). Regional controllability of semilinear degenerate parabolic equations in bounded domains. *Math. Anal. Appl.*, 320, 804–818.
- Cannarsa, P., Martinez, P., & Vancostenoble, J. (2004). Persistent regional controllability for a class of degenerate parabolic equations. *Commun. Pure Appl. Anal.*, 4, 607–635.
- Challis, C. D., Litaudon, X. et al. (2002). Influence of the q-profile shape on plasma performance in jet. *Plasma Physics and Controlled Fusion*, 44, 1031–1055.
- Coron, J. M. (2007). *Control and Nonlinearity*. American Mathematical Society.

- Ekedahl, A. et al. (2010). Validation of the ITER-relevant passive-active-multijunction LHCD launcher on long pulses in Tore Supra. *Nuclear Fusion*, *50*, 1–5.
- Fisch, N. J. (1987). Theory of current drive plasmas. *Reviews of Modern Physics*, *59*, 175–234.
- Freidberg, J. (2007). *Plasma Physics and Fusion Energy*. Cambridge, U. K.: Cambridge Univ. Press.
- Gaye, O., Autrique, L., Orlov, Y., Moulay, E., Brémond, S., , & Nouailletas, R. (2013).  $\mathcal{H}_\infty$  Stabilization of the Current Profile in Tokamak Plasmas via LMI Approach. *To appear in Automatica*, .
- Gaye, O., Moulay, E., Brémond, S., Autrique, L., Nouailletas, R., & Orlov, Y. (2011). Sliding mode stabilization of the current profile in tokamak plasmas. In *Proceedings of the 50th IEEE Conference on Decision and Control*. Orlando, Florida, USA.
- Hirshman, S. (1998). Finite-aspect-ratio effects on the bootstrap current in tokamaks. *Phys. Fluids*, *10*, 3150–3152.
- Mazon, D., Litaudon, X. et al. (2002). Real time control of internal transport barriers in JET plasma. *Phys. Control. Fusion*, (pp. 1087–1104).
- Moreau, D., Mazon, D. et al. (2008). A two-scale dynamic model approach for magnetic and kinetic profile control in advanced tokamak scenarios on JET. *Nuc. Fusion*, *48*, 1–38.
- Naylor, A. W., & Sell, G. R. (1982). *Linear Operator Theory in Engineering and Science*. New York: Springer.
- Orlov, Y. (2000). Discontinuous Unit Feedback Control of Uncertain Infinite-Dimensional Systems. *IEEE transactions on automatic control*, *45*, 834–843.
- Ou, Y., & Schuster, E. (2009a). Controllability analysis for current profile control in tokamaks. In *IEEE Conference on Decision and Control*. Shanghai, China.
- Ou, Y., & Schuster, E. (2009b). Model predictive control of parabolic PDE systems with dirichlet boundary conditions via galerkin model reduction. In *American Control Conference*. Hyatt Regency Riverfront, St. Louis, MO, USA.
- Ou, Y., Xu, C., & Schuster, E. (2010). Robust Control Design for the Poloidal Magnetic Flux Profile Evolution in the Presence of Model Uncertainties. *IEEE Transactions on Plasma Science*, *38*, 375–382.

- Ou, Y., Xu, C., Schuster, E., Luce, T. C., Ferron, J. R., & Walker, M. L. (2007). Extremum-Seeking Finite-Time Optimal Control of Plasma Current Profile at the DIII-D Tokamak. In *Proceedings of the 2007 American Control Conference*. New York City, USA.
- Ou, Y., Xu, C., Schuster, E. et al. (2011). Optimal tracking control of current profile in tokamaks. *IEEE Transaction on Control Systems Technology*, *19*, 432–441.
- Ouarit, H., Bremond, S., Nouailletas, R., & Autrique, L. (2011). Validation of plasma current profile model predictive control in tokamaks via simulations. *Fusion Engineering and Design*, *86*, 1018–1021.
- Rosier, L. (2007). A survey of controllability and stabilization results for partial differential equations. *Journal Européen des Systèmes automatisés*, *41*, 365–411.
- Saoutic, B., Chatelier, M., & Michelis, C. D. (2009). Tore supra : toward steady state in a superconducting tokamak. *Fusion Science and Technology*, *56*, 1079–1091.
- Smyshlyaev, A., & Krstic, M. (2010). *Adaptive Control of Parabolic PDEs*. Princeton University Press.
- Taylor, T. S. (1997). Physics of advanced tokamaks. *Plasma Physics and Controlled Fusion*, *39*, B47.
- Wesson, J. (2004). *Tokamaks*. Tokamaks 3rd Edition by John Wesson, Oxford University Press.
- Wijnands, T., Houtte, D. V., Martin, G., Litaudon, X., & Froissard, P. (1997). Feedback control of the current profile on Tore Supra. *Nucl. Fusion*, *37*, 777–791.
- Witrant, E., Brémond, S., Giruzzi, G., Mazon, D., Barana, O., & Moreau, P. (2007). A control-oriented model of the current profile in tokamak plasma. *Plasma Phys. Control. Fusion*, *49*, 1075–1105.
- Wolf, R. C. (2003). Internal transport barriers in tokamak plasmas. *Plasma Physics and Controlled Fusion*, *45*, 1–91.
- Xu, C., Dalessio, J., Ou, Y., Schuster, E., Luce, T., Ferron, J., Walker, M., & Humphreys, D. (2010). Ramp-up phase current profile control of tokamak plasmas via nonlinear programming. *IEEE Transactions on Plasma Science*, *38*, 163–173.