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## A framework for the construction of state surfaces of unsaturated soils in the elastic domain

P. Devillers,<sup>1</sup> M. S. El Youssoufi,<sup>2</sup> and C. Saix<sup>2</sup>

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[1] This paper deals with volume changes of an unsaturated soil in the elastic domain. Two constitutive relations are established in the general framework of unsaturated porous media mechanics. They express the variations of the void ratio  $e$  and the water content  $w$  with respect to the mean net stress  $(\sigma - p_g^*)$  and the suction  $s$ . Associated with validated experimental results, these relations give access to the construction of the state surface  $e = f((\sigma - p_g^*), s)$  in the elastic domain. This surface shows two domains: the first, near saturation, is based on the hypothesis of a Biot's coefficient  $b_w$  equal to 1; the second, for high suctions, is based on an experimental value of  $b_w$ . Next, a preliminary construction of the state surface  $G_s w = g((\sigma - p_g^*), s)$  is proposed for a given soil. Although restricted to the elastic domain, these constructions give a new understanding of the suction role in important problems related to overconsolidation, shrinkage, or swelling of unsaturated soils.

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### 1. Introduction

[2] Many geotechnical problems concern coupling between the mechanical behavior of a soil and the transfer of water, in particular in the unsaturated zone. The behavior of soils in this zone remains one of the important subjects of soil mechanics which still presents some unknown and poorly understood aspects. Swelling and shrinkage problems of clayey unsaturated soils under water transfer are concerned by this study: after some drying/wetting cycles, unsaturated soils behave elastically. In France, for example, the disorders of constructions caused by dryness, or lashing rains, concern 5000 communes in 75 departments. The total damage caused by these disasters has been estimated to 3.3 billion euros from 1989 to 2002. The origin of these pathologies is a hydrous imbalance between the center and outsidewalls of constructions that produces differential movements. These movements can be large enough to create differential deformations which are at the origin of many damages. The objective of this article is to model the volume changes for an unsaturated soil along a hydromechanical loading path which does not cross the yield curve. The studies of overconsolidated soils, or swelling clays after some drying/wetting cycles, are directly concerned by this work.

[3] In geotechnical engineering, gravimetric water content  $w$  and void ratio  $e$  are commonly used to describe the volume changes of soils. In the case of a saturated soil, these two variables are linked (under the usual assumption of incompressibility of solid grains) by the relation  $e = G_s w$ ,

where  $G_s$  is the specific gravity of the grains of soil. Whatever is the hydromechanical loading applied to a saturated soil element, the variations of the void ratio are proportional to the variations of the water content. The description of the volume changes of a saturated soil element requires only one variable:  $e$  or  $w$ . The corresponding stress variable is the mean effective stress  $(\sigma - p_w^*)$ , where  $\sigma$  is the mean total stress (1/3 of the first invariant of stress tensor) and  $p_w^*$  is the pore water pressure.

[4] In the case of an unsaturated soil, the variables water content and void ratio are linked by the relation  $S_r e = G_s w$ , where  $S_r$  is the degree of saturation. Along any loading path, the variations of void ratio and water content are independent [Salager, 2007; Salager et al., 2007]. Variations in void ratio and water content are often represented according to the two variables: mean net stress  $(\sigma - p_g^*)$  and suction  $s = (p_g^* - p_w^*)$  [Coleman, 1962], with  $p_g^*$  as the pore gas pressure. For that reason, many authors [Bishop and Blight, 1963; Matyas and Radhakrishna, 1968; Delage and Cui, 2001] suggested to represent every experimental test with a loading or suction change path in a space  $(e, (\sigma - p_g^*), s)$ . When all the loading paths belong to the same surface, this surface is called state surface. These representations were resumed more recently by Ho et al. [1992], Bolzon and Schrefler [1995], Gatmiri and Delage [1995], Gatmiri et al. [1998], and Fleureau et al. [2002], who proposed a representation from the model of Alonso et al. [1990]. Matyas and Radhakrishna [1968], Fredlund and Morgenstern [1976], Gatmiri and Delage [1995], and Gatmiri et al. [1998] proposed the formulation of another state surface as a representation of the water volume variation.

[5] In this paper, a rigorous method of representation of these state surfaces is proposed. This method, which owes its development to Biot's works [Biot, 1972] and to those of Coussy [1995], was applied to the study of unsaturated soils in the framework of the thermoporoplasticity [Lassabatère,

<sup>1</sup>Centre des Matériaux de Grande Diffusion, Ecole des Mines d'Alès, Alès, France.

<sup>2</sup>Laboratoire de Mécanique et Génie Civil, UMR5508, Université Montpellier 2, CNRS, Montpellier, France.

1994; Devillers, 1998; Laloui and Nuth, 2005]. At first, the volume changes of an unsaturated soil element are modeled, along a loading path inside the elastic domain. The approach followed allows two relations to express the variations of void ratio and water content according to the variations of the mean net stress and the suction.

[6] Then, these two relations are represented in two three-dimensional spaces. Experimental results allow to propose a practical construction of the  $e = f((\sigma - p_g^*), s)$  state surface on the basis of a reference state. The method of construction is then applied to the  $G_s w = f((\sigma - p_g^*), s)$  state surface to propose a preliminary construction.

## 2. Thermodynamics Formulation in Poroelasticity

[7] In a macroscopic description, an unsaturated soil element is considered as the superposition of four continuous media in interaction: (1) a solid skeleton, which includes the solid constituent plus its geometry (denoted  $s$ ); (2) a liquid water constituent (denoted  $w$ ); (3) a vapor water constituent (denoted  $v$ ); and (4) a gas constituent called air, regrouping all the gases others than the vapor (denoted  $a$ ).

[8] From a thermodynamics point of view, a representative elementary volume (REV) of the unsaturated soil, for which the kinematics and deformation are located by that of its skeleton, is considered as an open system exchanging mass and energy with the outside, during the transformations which it undergoes.

[9] The state variables chosen to model the behavior of an unsaturated soil, along a loading path, are the volumetric strain  $\varepsilon_v$ , the deviatoric strain tensor  $\gamma$ , the water mass variation  $m_w$ , the vapor mass variation  $m_v$  and the air mass variation  $m_a$  per unit of the initial skeleton volume. These Lagrangian state variables are more convenient for the description of the deformations than the Eulerian variables.

[10] At time  $t$ , the material elementary volume  $dV_0$  becomes  $dV$  after deformation, the partial density of fluid  $i$ ,  $\rho_{i0}$  becomes  $\rho_i$ . The Lagrangian mass variation of fluid  $i$  per unit of initial volume  $m_i$ , is defined by

$$(\rho_{i0} + m_i)dV_0 = \rho_i dV \quad \text{where } i = w, a \text{ or } v \quad (1)$$

[11] In this study, only small volume changes of the soil element, i.e., small deformations and small mass variations of water, air and vapor, are considered. Transformations are supposed to be isothermal. The non linear poroelastic behavior of an unsaturated porous media (considering compressive stresses as positive) is defined by the following relations [Coussy, 1995]:

$$d\sigma = -Kd\varepsilon_v + b_i dp_i^* \quad (2)$$

$$dp_i^* = -M_{ij} \left( b_j d\varepsilon_v - \frac{dm_j}{\rho_{j0}^*} \right) \quad (3)$$

$$d\tau = 2Gd\gamma \quad (4)$$

where  $\sigma$  is the mean total stress and  $\tau$  the deviatoric stress tensor. The characteristics of the unsaturated soil element

can be interpreted in the following way:  $K$  is the drained modulus of compressibility;  $G$  is the shear modulus,  $b_i$  are the Biot's coefficients which express coupling between pressure  $p_i^*$  of fluid  $i$  and mechanical stress;  $M_{ij}$  is the matrix of Biot's moduli which express the influence of the mass variation of the fluid  $j$  on the pressure of fluid  $i$ .

[12] The use of these behavior equations to determine volume changes of an unsaturated soil element requires the determination of the following ten coefficients:

$$K \quad \mathbf{b} = \begin{bmatrix} b_w \\ b_a \\ b_v \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} M_{ww} & M_{wa} & M_{wv} \\ M_{wa} & M_{aa} & M_{av} \\ M_{wv} & M_{av} & M_{vv} \end{bmatrix} \quad (5)$$

[13] Linking the microscopic physical properties of the four constituents (soil grains incompressible, water incompressible, air and vapor as perfect gases) to the coefficients of the constitutive equations was carried out by Devillers [1998]. His development allowed the reduction of the number of coefficients of the constitutive equations from ten to five:

$$K \quad b_v = b_a = 1 - b_w \quad \mathbf{M} = \begin{bmatrix} M_{ww} & \frac{p_{a0}^*}{n_{g0}} & \frac{p_{v0}^*}{n_{g0}} \\ \frac{p_{a0}^*}{n_{g0}} & \frac{p_{a0}^*}{n_{g0}} & 0 \\ \frac{p_{v0}^*}{n_{g0}} & 0 & \frac{p_{v0}^*}{n_{g0}} \end{bmatrix} \quad (6)$$

where  $n_g$  is the volumetric fraction of the gas phase.

## 3. Introduction of the "Mean Net Stress" and "Suction" Variables

[14] Using Dalton's law ( $p_g^* = p_a^* + p_v^*$ ), the state equation of the perfect gases and the relations (6), the expressions of the variations of the mean total stress, the water pressure and the gas phase pressure become [Devillers, 1998]

$$d\sigma = -Kd\varepsilon_v + b_w dp_w^* + (1 - b_w) dp_g^* \quad (7)$$

$$dp_w^* = - \left( M_{ww} b_w + (1 - b_w) \frac{p_{g0}^*}{n_{g0}} \right) d\varepsilon_v + M_{ww} \frac{dm_w}{\rho_{w0}^*} + \frac{RT}{n_{g0}} \left( \frac{dm_a}{M_a} + \frac{dm_v}{M_v} \right) \quad (8)$$

$$dp_g^* = - \left( \frac{p_{g0}^*}{n_{g0}} \right) d\varepsilon_v + \frac{p_{g0}^*}{n_{g0}} \frac{dm_w}{\rho_{w0}^*} + \frac{RT}{n_{g0}} \left( \frac{dm_a}{M_a} + \frac{dm_v}{M_v} \right) \quad (9)$$

[15] The variations of the water pressure and the gas phase pressure depend on the volumetric strain and on mass variations of water, air and vapor. In relations (8) and (9), the air and vapor mass variations play an identical role. By introducing suction  $s = (p_g^* - p_w^*)$ , difference between the gas phase pressure and the water pressure, the relations (8) and (9) give

$$ds = \left( M_{ww} - \frac{p_{g0}^*}{n_{g0}} \right) \left( b_w d\varepsilon_v - \frac{dm_w}{\rho_{w0}^*} \right) = M \left( b_w d\varepsilon_v - \frac{dm_w}{\rho_{w0}^*} \right) \quad (10)$$

where

$$M = \left( M_{ww} - \frac{\rho_{g0}^*}{n_{g0}} \right) \quad (11)$$

similarly to Biot's modulus, expresses mutual influence between the water mass variation and the suction variation.

[16] The suction variation is then independent from the air and vapor mass variations. On the other hand, it depends on the water mass variation and on the volumetric strain.

[17] For a given soil, volume variations or water content variations are going to induce suction variations. The introduction of the suction in the relation (7) introduces a new variable, the mean net stress, as difference between the mean total stress and the gas phase pressure.

[18] The volume changes of an unsaturated soil element are given by the volumetric strain and the water mass variation which are expressed using the mean net stress variation and the suction variation:

$$d\varepsilon_v = -\frac{1}{K}d(\sigma - p_g^*) - \frac{b_w}{K}ds \quad (12)$$

$$\frac{dm_w}{\rho_{w0}^*} = b_w d\varepsilon_v - \frac{1}{M}ds = -\frac{b_w}{K}d(\sigma - p_g^*) - \left( \frac{b_w^2}{K} + \frac{1}{M} \right) ds \quad (13)$$

[19] To describe volume changes of unsaturated porous media, four state variables are necessary:  $\sigma$ ,  $p_w^*$ ,  $p_a^*$  and  $p_g^*$ . The previous developments show that only two stress state variables, mean net stress and suction, are sufficient to describe the volume changes of an unsaturated soil element in isothermal conditions. Before studying in more detail the representation of the volume changes of an unsaturated soil element, it is advisable to substitute for the volumetric strain, and for the water mass variation, two variables more widely used in soil mechanics, the variation of the void ratio and the variation of the water content. Relation (14) expresses volumetric strain according to the void ratio  $e$  or porosity  $n$  variation and relation (15) allows one to obtain an expression of the water mass variation according to the water content variation (relation (16)):

$$d\varepsilon_v = \frac{de}{1 + e_0} = \frac{dn}{1 - n_0} = \frac{dn_w - dn_g}{1 - n_0} \quad (14)$$

where  $n_w$  and  $n_g$  are volume fractions of water and gas respectively.

$$dm_w = d\varepsilon_v n_{w0} \rho_{w0}^* + \rho_{w0}^* dn_w \quad (15)$$

$$dm_w = \frac{de}{(1 + e_0)} \frac{w_0 \rho_s^*}{(1 + e_0)} + \rho_s^* d \left( \frac{w}{1 + e} \right) = \rho_s^* \frac{dw}{(1 + e_0)} \quad (16)$$

[20] By replacing volumetric strain by the variation of the void ratio, and the water mass variation by the variation of the water content, the volume changes equations of an unsaturated soil element become

$$de = -\frac{(1 + e_0)}{K} [d(\sigma - p_g^*) + b_w ds] \quad (17)$$

$$G_s dw = -\frac{b_w}{K} (1 + e_0) \left[ d(\sigma - p_g^*) + \left( b_w + \frac{K}{Mb_w} \right) ds \right] \quad (18)$$

[21] Relations (17) and (18) allow to calculate the volume changes of an unsaturated soil element starting from an equilibrium state characterized by  $e_0$ ,  $w_0$ ,  $(\sigma - p_g^*)_0$  and  $s_0$ . The three coefficients of the model are the drained modulus of compressibility  $K$ , the Biot's coefficient  $b_w$  and the modulus  $M$ . These coefficients depend on the initial state.

[22] Relation (17) can be compared to that proposed by *Fredlund and Morgenstern* [1976]. This relation gives the void ratio between an initial state ( $i$ ) and a final state ( $f$ ) in the elastic domain:

$$\Delta e = -C_{M1} \log \left[ \frac{(\sigma - p_g^*)_f}{(\sigma - p_g^*)_i} \right] - C_{H1} \log \left[ \frac{s_f}{s_i} \right] \quad (19)$$

where  $C_{M1}$  and  $C_{H1}$  are compressive or swelling indices with respect to mean net stress and suction. An example of experimental determination of these indices is given by *Salager et al.* [2008]. Within the framework of infinitesimal transformations around a reference state  $e_0$ ,  $(\sigma - p_g^*)_0$  and  $s_0$ , [Devillers, 1998] expressed relation (19) as

$$de = -\frac{C_{M1}}{(\sigma - p_g^*)_0 \ln(10)} d(\sigma - p_g^*) - \frac{C_{H1}}{s_0 \ln(10)} ds \quad (20)$$

[23] To obtain the relation (20) from the relation (19), a limited development to the order 1, of Fredlund's relations in the neighborhood of the reference state, is applied. The identification of relations (17) and (20) allows one to express Biot's coefficient  $b_w$  and drained modulus of compressibility  $K$  according to the indices  $C_{M1}$  and  $C_{H1}$ :

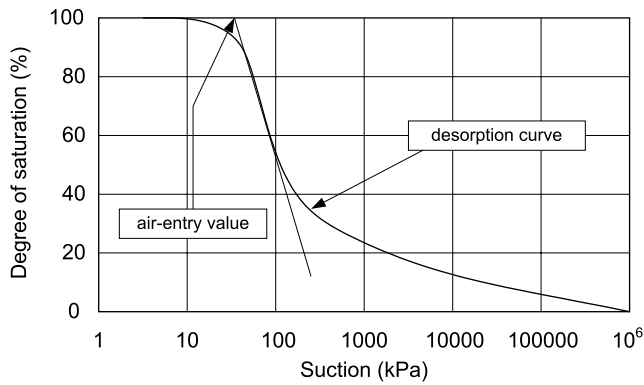
$$b_w = \frac{C_{H1} (\sigma - p_g^*)_0}{C_{M1} s_0} \quad (21)$$

$$K = \frac{\ln(10)(1 + e_0)(\sigma - p_g^*)_0}{C_{M1}} \quad (22)$$

[24] These relations between parameters  $b_w$  and  $K$ , on one hand, and  $C_{M1}$  and  $C_{H1}$ , on the other hand, establish a bridge between a theoretical modeling (relation (17)) and an experimental modeling (relation (19)). Very few experimental data on  $b_w$  and  $K$  are available. On the other hand more experimental data are available in the literature for  $C_{M1}$  and  $C_{H1}$  that allows for different soils, using relations (21) and (22), to calculate  $b_w$  and  $K$ . Moreover these two relations allow to clarify the variations of these two coefficients according to the state variables mean net stress and suction.

#### 4. Toward Construction of the State Surfaces in the Elastic Domain

[25] The objective of this part is to specify the form of the state surfaces in the elastic domain. These surfaces give access to the void ratio or the water content variations



**Figure 1.** Typical soil water retention curve [after Vanapalli *et al.*, 1999].

according to the mean net stress and suction variations, starting from the equations (17) and (18) of the previously presented model.

#### 4.1. State Surface in the $[e, (\sigma - p_g^*), s]$ Space

[26] An unsaturated soil element which underwent mechanical loading and drying/wetting cycles is considered. The current equilibrium state is characterized by the values of the void ratio  $e_0$ , the water content  $w_0$ , the mean net stress  $(\sigma - p_g^*)_0$  and the suction  $s_0$ .

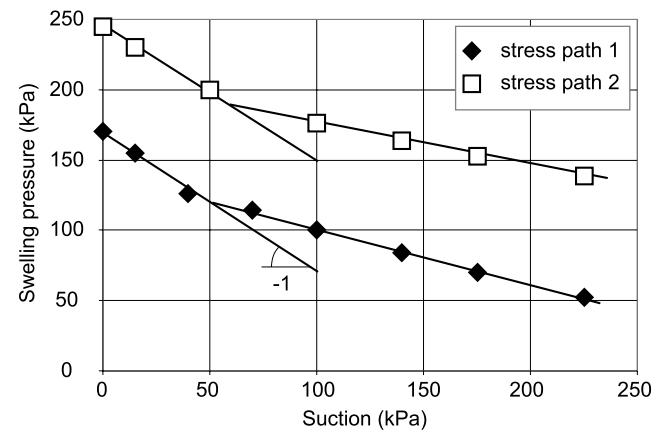
[27] This equilibrium state is represented by point P of coordinates  $(e_0, (\sigma - p_g^*)_0, s_0)$  in the  $[e, (\sigma - p_g^*), s]$  space. From this point, first we propose to determine all of the possible states  $(e_0, (\sigma - p_g^*), s)$ .

[28] In the  $[e = e_0]$  plane of this space, equation (17) gives the mean net stress variation according to the suction variation:

$$d(\sigma - p_g^*) = -b_w ds \quad (23)$$

[29] Nine stress paths of an unsaturated remolded gray-greenish clay ( $w_L = 68\%$ ;  $I_p = 39\%$ ) at constant volume have been published by Escario [1969]. These tests have been performed using the mercury apparatus, in the field of low suction (0–90 kPa). Each one of these stress paths may be represented by a straight line near saturation. These results associated with the relation (23) make it possible to identify a Biot's coefficient for each straight line. All of the straight lines, in the field of low suctions, near saturation, present a slope of  $-1$ . This result is in agreement with physics considerations since for a saturated soil, the Biot's coefficient is worth 1. Those results show that mechanical behavior is then identical to the behavior of the fully saturated material with a slightly compressible fluid. In those tests, the suction is smaller than air expulsion value which separates the saturated and unsaturated behaviors.

[30] The transition between the saturated and unsaturated behaviors corresponds to a particular value of suction  $s_e$ ,



**Figure 2.** Constant volume stress paths for an unsaturated expansive clay [after Blight, 1965].

referred to as air entry value for drying paths and air expulsion value for wetting paths. The suction  $s_e$  is usually defined using the soil water retention curve [Vanapalli *et al.*, 1999] as shown in Figure 1. Air entry value is around 3 kPa for silty clayey sand but may reach 1000 kPa or more for some clay. The air entry pressure seems to be a fundamental parameter which constitutes a border between two domains of behavior for unsaturated soils. For suction values lower than the air entry pressure, close to saturation, the behavior is identical to that of saturated soils. For suction values higher than the air entry pressure, the behavior depends both on the two variables mean net stress and suction.

[31] Escario [1969] also performed three constant volume stress paths using the air pressure apparatus, in the field of high suctions (0–4000 kPa). The three stress paths correspond to samples compacted at the same dry density but at three different initial water content. Each one of these stress paths may be represented by a straight line for high suctions. These results associated with the relation (23) make it possible to identify a Biot's coefficient for each straight line (Table 1). The Biot's coefficient seems to be independent of the initial water content. The insufficient number of experimental points, for low suctions, near saturation, does not allow the identification of the air entry value.

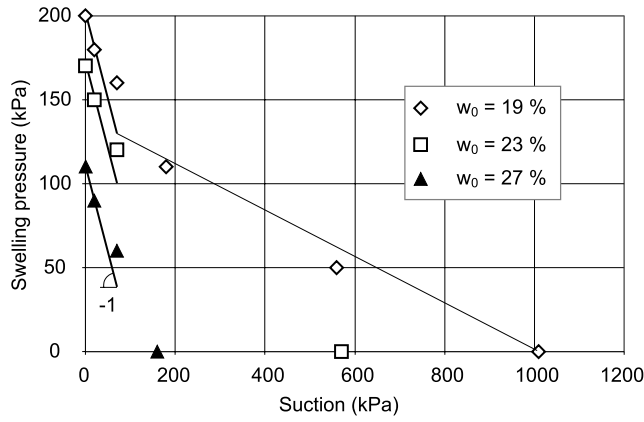
[32] Two stress paths of an unsaturated expansive clay ( $w_L = 55\%$ ;  $I_p = 35\%$ ) at constant volume have been published by Blight [1965] and are reproduced in Figure 2. Each one of these stress paths may be represented by two straight lines. The intersection of the two straight lines let us identify the suction  $s_e$ , which is around 50 kPa for this clay. It is possible to identify a Biot's coefficient for each straight line (Table 2). The first straight line, in the field of low suctions, near saturation, presents a slope of  $-1$ . The second straight line, in the field of higher suctions, allows the

**Table 1.** Biot's Coefficients for Escario's Tests

	Stress Path A	Stress Path B	Stress Path C
$w_0(\%)$	4.72	11	16.6
$b_w$	0.14	0.13	0.18

**Table 2.** Biot's Coefficients for Blight's Tests

Stress Path 1	Stress Path 1	Stress Path 2	Stress Path 2
$b_{w1}$	$b_{w2}$	$b_{w1}$	$b_{w2}$
1	0.40	1	0.34



**Figure 3.** Swelling pressure stress paths for a montmorillonite clay [after *Kassif and Shalom*, 1971].

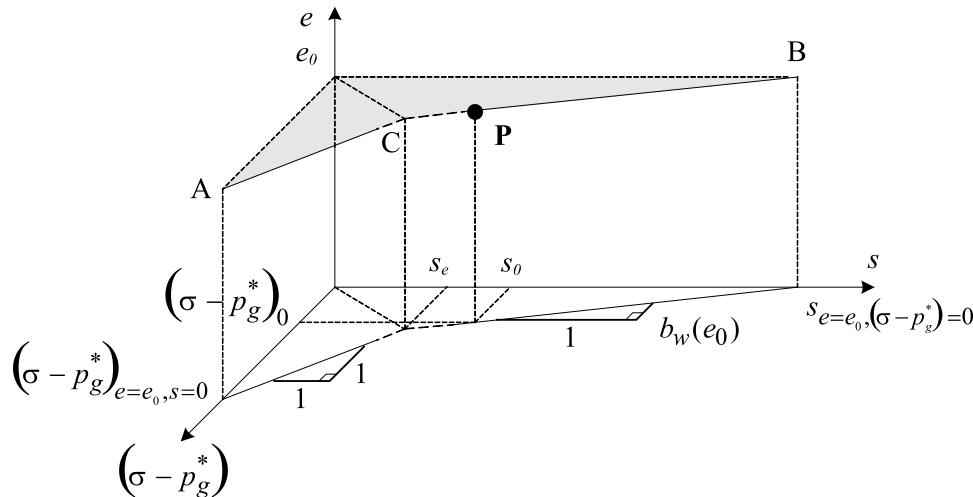
identification of a Biot's coefficient  $b_{w2}$ , which, in accordance with physics considerations, is lower than 1.

[33] Three constant volume stress paths of an unsaturated montmorillonitic clay ( $w_L = 78-85\%$ ;  $I_P = 58-63\%$ ) at three different water content have been published by *Kassif and Shalom* [1971] and are reproduced in Figure 3. Each one of these stress paths can be modeled by a straight line, for low suctions, near saturation. The slopes of these straight lines are about  $-1$ . The few number of points for high suctions does not allow the identification of a Biot's coefficient  $b_{w2}$ .

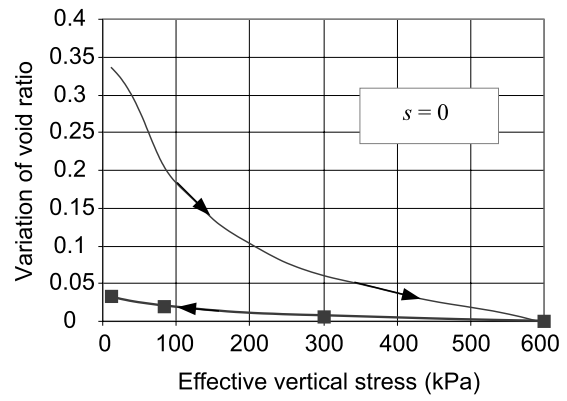
[34] In the first approach, we choose to preserve only two straight lines to model constant volume stress paths. This additional approximation, valid in the field of higher suctions and near saturation, becomes questionable around the air expulsion value.

[35] All the possible states  $(e_0, (\sigma - p_g^*), s)$  are built as follows (Figure 4):

[36] If  $s_0 \leq s_e$  the straight line running through P and with a slope of  $-1$  is plotted. This line cut the  $[s = 0]$  plane at point A of coordinate  $(\sigma - p_g^*)_{e=e_0, s=0}$  and the  $[s = s_e]$  plane at point C. The straight line running through C and with a slope of  $-b_w$  is then plotted. This line cut the  $[(\sigma - p_g^*) = 0]$  plane at point B of coordinate  $s_{e=e_0, (\sigma - p_g^*)=0}$ .



**Figure 4.** Illustration of the first stage of the state surface construction for  $s_0 > s_e$ .



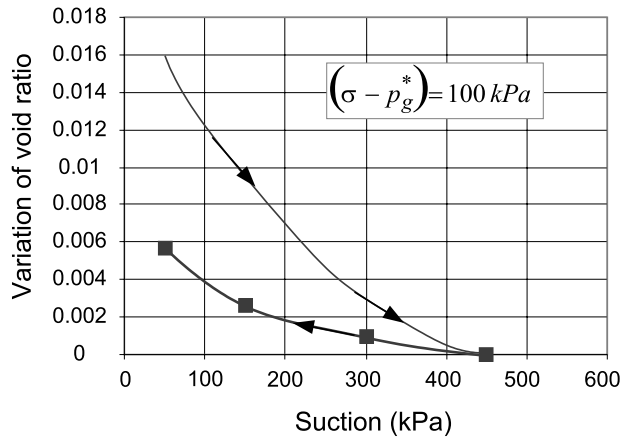
**Figure 5.** Void ratio variation for a saturated clayey silty sand [after *Devillers*, 1998].

[37] If  $s_0 > s_e$  the straight line running through P and with a slope  $-b_w$  is plotted. This line cut the  $[(\sigma - p_g^*) = 0]$  plane at point B of coordinate  $s_{e=e_0, (\sigma - p_g^*)=0}$  and the  $[s = s_e]$  plane at point C. The straight line running through C and with a slope of  $-1$  is then plotted. This line cut the  $[s = 0]$  plane at point A of coordinate  $(\sigma - p_g^*)_{e=e_0, s=0}$ .

[38] This first stage is illustrated by Figure 4. In Figure 4 we can also notice that  $(\sigma - p_g^*)_{e=e_0, s=0}$  is lower than  $s_{e=e_0, (\sigma - p_g^*)=0}$  in accordance to physical considerations which lead to a coefficient  $b_w$  lower than 1.

[39] The intersection curve between the state surface and the  $[s = 0]$  plane is the traditional swelling curve of a saturated soil. All the swelling curves, whatever the void ratio  $e_0$  value, are almost all parallel. They can be linearized by using a semilogarithmic scale [*Lambe and Whitman*, 1979]. This curve is thus of logarithmic form in the  $[s = 0]$  plane. Figure 5, which gives the consolidation curve of saturated clayey silty sand [*Devillers*, 1998], shows an example of the swelling curve.

[40] In the same way, the intersection curve between the state surface and the  $[(\sigma - p_g^*) = 0]$  plane can be linearized by using a semilogarithmic scale [*Ho et al.*, 1992]. This curve too will thus be of logarithmic form in the  $[(\sigma - p_g^*) = 0]$  plane. Figure 6 gives an example of a wetting path



**Figure 6.** Example of void ratio variation along a wetting path for a clayey silty sand [after Devillers, 1998].

carried out under a relatively low mean net stress of 100 kPa on the same clayey silty sand [Devillers et al., 1997].

[41] In the  $[s = 0]$  plane, the swelling curve which passes by point A is plotted (Figure 7). Equation (17) makes it possible to express the slope of this curve at point A equal to  $(\frac{1+e_0}{K})$ .

[42] In the same way, in the  $[(\sigma - p_g^*) = 0]$  plane, the wetting path which passes by point B is plotted. Equation (17) makes it possible to express the slope of this curve at point B equal to  $(-\frac{(1+e_0)b_w}{K})$ . Figures 5 and 6 show that the incidence of a suction variation on the void ratio variation is weak compared to the incidence of a mean net stress variation. Consequently, the swelling coefficient  $(\frac{1+e_0}{K})$ , with

respect to the mean net stress variations, must be higher than the swelling coefficient  $(\frac{(1+e_0)b_w}{K})$ , with respect to the suction variations.

[43] It must be noted that these two swelling curves end in the same value of the final void ratio, noted  $e_f$  for a null mean net stress for the first and a null suction for the second (Figure 7). This remark is related to the definition of a state surface which requires that, on the basis of a void ratio  $e_i$ , two different loading paths between  $((\sigma - p_g^*)_i, s_i)$  and  $((\sigma - p_g^*)_f, s_f)$ , end to the same void ratio  $e_f$ .

[44] The three borders of the surface are now defined. To generate the surface we need more information about the variation of the suction  $s_e$  and the Biot's coefficient with the void ratio.

**4.2. State Surface in the  $[G_s, w, (\sigma - p_g^*), s]$  Space**

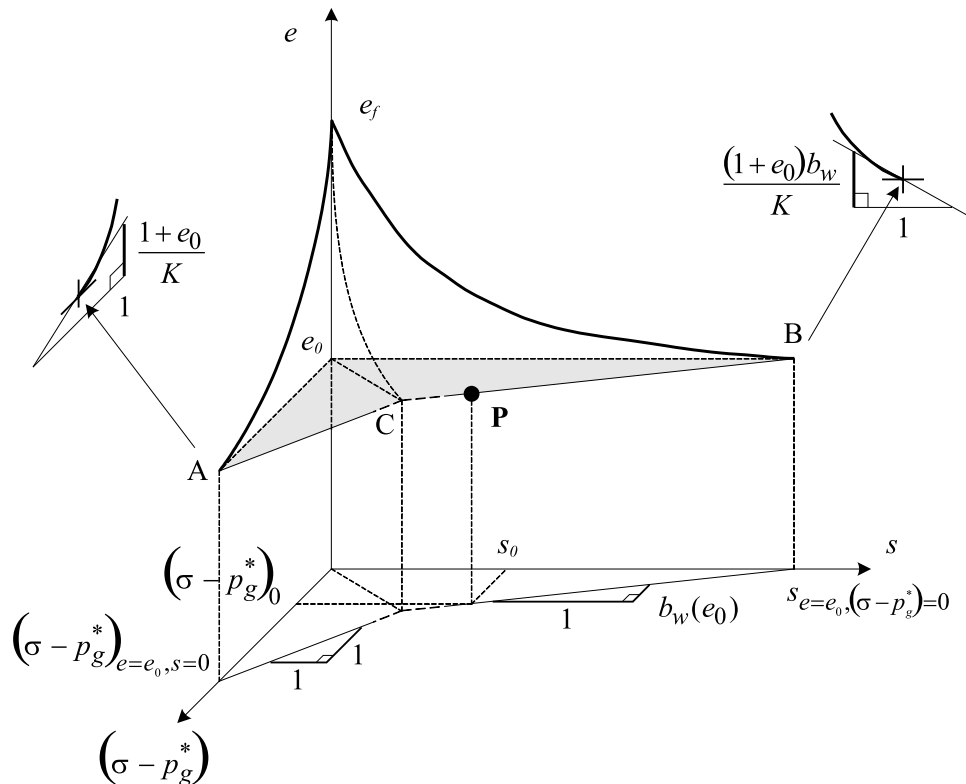
[45] There is very little information in the literature on the water content variations which occur with swellings of an unsaturated soil element. Nevertheless taking into account the similarity of the roles which the void ratio and the water content play, we can propose an approach of the  $[G_s, w = g((\sigma - p_g^*), s)]$  state surface starting from the  $[e = f((\sigma - p_g^*), s)]$  state surface.

[46] The justification of this similarity is partly based on the following points.

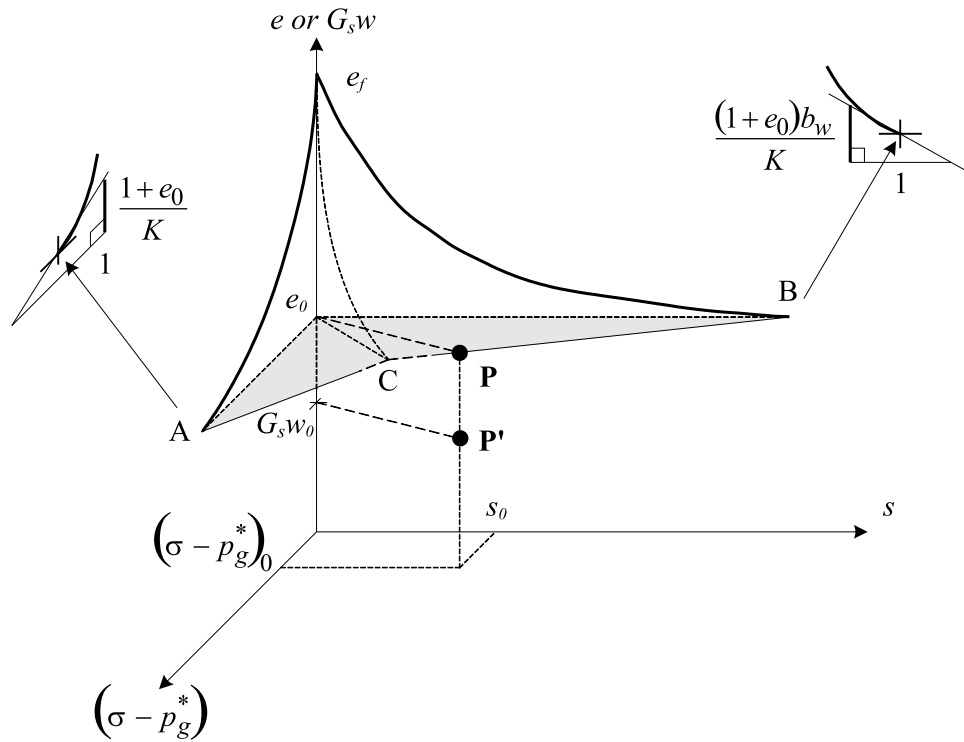
[47] 1. In the saturated case, which constitutes a borderline case, the two variables  $e$  and  $G_s, w$  are identical.

[48] 2. Fredlund [1979, 2000] proposed a constitutive law for the water content variation similar to the constitutive law suggested for the void ratio variation.

[49] 3. At the theoretical level, the similarity is confirmed by equations (17) and (18).



**Figure 7.** Swelling curves on the  $[s = 0]$  and  $[(\sigma - p_g^*) = 0]$  planes.



**Figure 8.** Representation of points P and P' corresponding to the same initial state.

[50] The same unsaturated soil element, whose current equilibrium state is characterized by the values of the void ratio  $e_0$ , the water content  $w_0$ , the mean net stress  $(\sigma - p_g^*)_0$  and the suction  $s_0$ , is considered.

[51] The current equilibrium state is represented by the point P' of coordinates  $(G_s w_0, (\sigma - p_g^*)_0, s_0)$  in the  $[G_s w, (\sigma - p_g^*), s]$  space (Figure 8).

[52] In the  $[s = 0]$  plane which corresponds to a saturated soil, we have  $(G_s w = e)$ . The intersection curve between the state surface  $[G_s w = g((\sigma - p_g^*), s)]$  and the  $[s = 0]$  plane is the swelling curve of a saturated soil which passes by point A, as already represented on Figure 4. Of course, comments then made remain valid. Equation (18) makes it possible to express the slope of this curve at point A equal to  $(-\frac{1+e_0}{K})$  because, in this plane,  $b_w = 1$ .

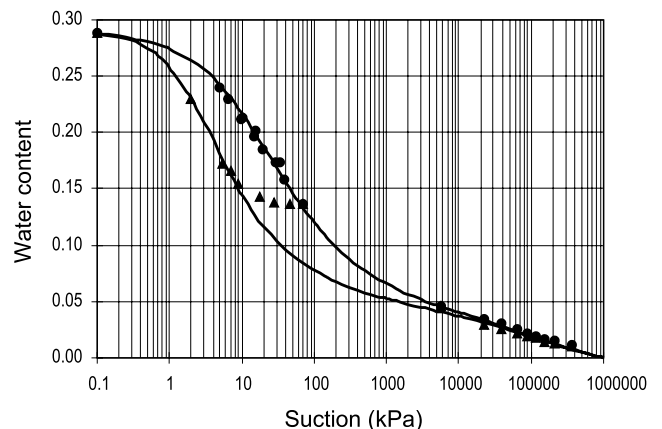
[53] In the  $[(\sigma - p_g^*) = 0]$  plane which corresponds to an unsaturated soil, mechanically nonloaded, the intersection curve is given by the soil water retention curve. An example of this type curve is given on Figure 9 for the same clayey silty sand [Devillers, 1998] with a semilogarithmic scale in which it is possible to linearize it by segments. Figure 10 gives, in an arithmetic scale, an example of a wetting path which corresponds to the experimental points located below a suction of 100 kPa. These points were obtained from a pressure plate test. For suctions higher than 100 kPa, the experimental points were obtained starting from the desorption hygroscopic equilibrium curve.

[54] The curves of Figures 9 and 10 express variations of water content  $w$  according to suction  $s$ . In the literature, the soil water retention curves usually express the variation of the degree of saturation  $S_r$ , according to  $s$  without taking into account the variations of volume which accompany the variations of  $s$ . If this step can be allowed for non plastic soils, the case of very plastic soils justifies the use of relations such as (16) which indicates that the characteristic

curve would, in this case, be parameterized by also the void ratio. By introducing relations (14) and (16) into relation (10), one ends up with

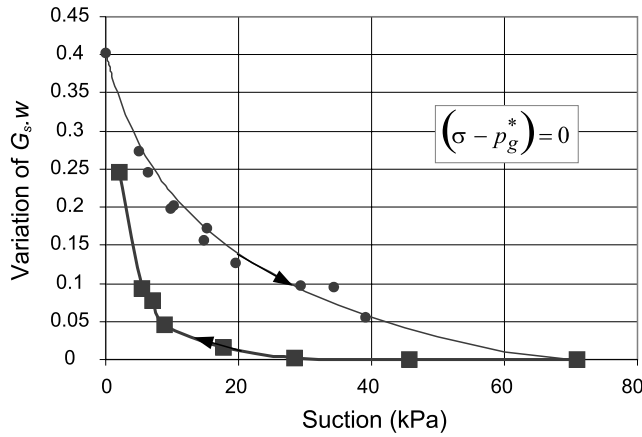
$$dS_r = -\frac{1+e_0}{Me_0} ds + \left(\frac{b_w - S_{r0}}{e_0}\right) de \quad (24)$$

[55] Relation (24) explicitly reveals the influence of the void ratio variation on the expression of the soil water retention curve, as shown by experiments [Sugii *et al.*, 2002; Salager, 2007]. This influence disappears in two situations; either the void ratio variation is too weak to be taken into account; or the parameter  $b_w$  is equal to the degree of saturation.



**Figure 9.** Soil water retention curve for a clayey silty sand on a semilogarithmic scale [after Devillers, 1998].





**Figure 10.** Example of suction variation along a wetting path for a clayey silty sand [after Devillers, 1998].

[56] In the  $[(\sigma - p_g^*) = 0]$  plane, the wetting path which passes by the point of coordinates  $((G_s w_f = e_f), 0, 0)$  is plotted. Indeed the two curves end in the same value for a null mean net stress for the first and a null suction for the second (Figure 11). Just as for the void ratio, on the basis of one  $G_s w_i$ , two different unloading paths from  $((\sigma - p_g^*)_i, s_i)$  to  $((\sigma - p_g^*)_f, s_f)$ , lead to same  $G_s w_f$ . To the B point of the  $[e = f((\sigma - p_g^*), s)]$  surface corresponds the B' point in the  $[G_s w, (\sigma - p_g^*), = s]$  space. This point, of coordinates  $(G_s w_B, 0, = s_{e=e_0; (\sigma - p_g^*)=0})$  corresponds to the third node of the state surface (Figure 11). Equation (18) makes it possible to express the slope of the wetting curve at the B' point equal to  $(-\frac{(1+e_0)}{K}((b_w)^2 + \frac{K}{M}))$ .

[57] In the case of low suctions, near saturation, Figures 5 and 10 seem to show that the incidence of a suction

variation on the  $G_s w$  variation is more significant than the incidence of a mean net stress variation. Consequently, the water content coefficient  $(\frac{(1+e_0)}{K})$ , with respect to the mean net stress variations seems here lower than the water content coefficient  $(\frac{(1+e_0)}{K}((b_w)^2 + \frac{K}{M}))$  with respect to the suction variations.

[58] The combination of the two equations (17) and (18) makes it possible to express the water content variation according to the void ratio and suction variations. On an elementary constant volume path ( $de = 0$ ) relation (25) expresses the water content variations according to the suction variation:

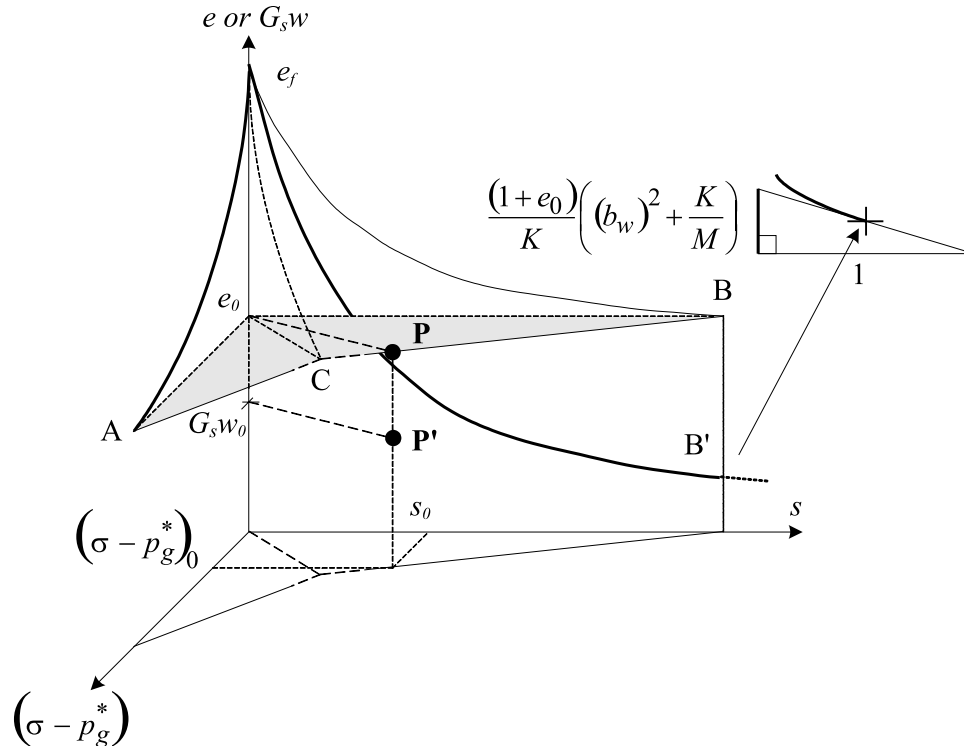
$$G_s dw = -\frac{1 + e_0}{M} ds \tag{25}$$

[59] Along the constant volume path PB, the relation (25) makes it possible to determine the water content  $w_{B'}$ :

$$\int_{w_0}^{w_{B'}} G_s dw = -\int_{s_0}^{s_{B'}} \frac{1 + e_0}{M} ds \tag{26}$$

[60] The third border of the state surface corresponds with the curve AP'B' which is located in two vertical planes containing the straight lines AC and CB respectively. The void ratio of all the points of this curve is  $e_0$ . The measurement of the water content variations along a constant volume loading path would make it possible to approach the geometrical shape of this curve.

[61] To generate this surface, a thorough experimental study of undrained loading paths is necessary. This study would make it possible to specify the shape of the curve of



**Figure 11.** Wetting curves on the  $[s = 0]$  and  $[(\sigma - p_g^*) = 0]$  planes.

intersection between the  $[G_{s,w} = g((\sigma - p_g^*), s)]$  state surface and constant water content planes.

[62] The generation of the state surface  $[G_{s,w} = g((\sigma - p_g^*), s)]$  thus appears much more complex than the generation of the state surface  $[e = f((\sigma - p_g^*), s)]$ , and additional experimental investigations prove to be necessary.

## 5. Conclusion

[63] A general method of construction of the state surfaces in the elastic domain was clarified within the poroelasticity framework of unsaturated soils. The model presented gives two relations respectively expressing the variation of the void ratio and the water content according to the variations of the mean net stress and the suction. These two relations allowed, on the basis of a state of reference and being based on experimental results, to precise the shape of the state surfaces. This construction has a general range in unsaturated soils by having the advantage of being simple of use. It is based on theoretical considerations, common on unsaturated soils, and on a reduced number of tests specific to the studied soil.

[64] This surface reveals two domains: the first, near saturation is based on a Biot's coefficient value equal to 1, the second, for high suctions is based on an experimental value of the Biot's coefficient. The border between these two domains allow to find the air entry or air expulsion value usually defined by the soil water retention curve.

[65] This study, although restricted in the elastic domain, makes it possible to bring a new lighting on the role of suction in significant problems involved in overconsolidation or swelling and recompression in the case of unsaturated soils.

[66] To model the deformations of a given unsaturated soil in the elastic domain, the identification of three coefficients is necessary: the drained modulus of compressibility  $K$ , the Biot's coefficient  $b_w$  and the Biot's modulus  $M$ . The direct application of this approach to the real case study requires a projection in the acquisition of sufficient experimental results, especially with regard to the Biot's coefficient variations which occur with void ratio variations. The proposed construction method rests at the same moment on experimental and theoretical considerations. This new approach presents certain interest and allows the description of particular loading paths. It allowed advancing the important rule of air entry pressure, including as regards the mechanical behavior of unsaturated soils.

## Notation

$e$	void ratio.
$w$	gravimetric water content.
$G_s$	specific gravity of solid.
$S_r$	degree of saturation.
$p_i^*$	pore pressure of fluid $i$ ( $i = w, a$ or $v$ ).
$p_g^*$	pore gas pressure.
$s$	suction ( $p_g^* - p_w^*$ ).
$s_e$	air entry value of suction.
$\gamma$	deviatoric strain tensor.
$\varepsilon_v$	volumetric strain.
$\tau$	deviatoric stress tensor.
$\sigma$	mean total stress.
$(\sigma - p_g^*)$	mean net stress.

$(\sigma - p_g^*)$	mean effective stress.
$m_i$	mass variation of fluid $i$ ( $i = w, a$ or $v$ ).
$dV$	elementary volume.
$\rho_i$	partial density of fluid $i$ ( $i = w, a$ or $v$ ).
$\rho_i^*$	specific density of fluid $i$ ( $i = w, a$ or $v$ ).
$T$	temperature.
$K$	drained isothermal modulus of compressibility.
$G$	shear modulus.
$b_i$	general term of Biot's coefficients vector $b$ ( $i = w, a$ or $v$ ).
$M_{ij}$	general term of Biot's moduli matrix $M$ ( $i$ and $j = w, a$ or $v$ ).
$n$	porosity.
$n_w$	volumetric fraction of liquid phase.
$n_g$	volumetric fraction of gas phase.
$R$	perfect gases constant.
$M_a$	molar mass of air.
$M_v$	molar mass of vapor.
$C_{M1}$	compressive index with respect to mean net stress.
$C_{H1}$	compressive index with respect to suction.
$w_L$	liquid limit.
$I_P$	plasticity index.
$V$	total volume.

## References

- Alonso, E. E., A. Gens, and A. Josa (1990), A constitutive model for partially saturated soils, *Geotechnique*, 40, 405–430.
- Biot, M. A. (1972), Theory of finite deformations of porous solids, *Indiana Univ. Math. J.*, 21, 597–620.
- Bishop, A. W., and G. E. Blight (1963), Some aspects of effective stress in saturated and partly saturated soils, *Geotechnique*, 13, 177–197.
- Blight, G. E. (1965), The time-rate of heave of structures on expansive clays, in *Moisture Equilibria and Moisture Changes in Soils Beneath Covered Areas*, edited by G. D. Aitchison, pp. 78–87, Butterworths, Sydney, N. S. W., Australia.
- Bolzon, G., and B. A. Schrefler (1995), State surfaces of partly saturated soils: An effective pressure approach, *Appl. Mech. Rev.*, 48, 643–649.
- Coleman, J. D. (1962), Stress/strain relation for partly saturated soil, *Corresp. Geotech.*, 12, 348–350.
- Coussy, O. (1995), *Mechanics of Porous Media*, John Wiley, New York.
- Delage, P., and Y. J. Cui (2001), Comportement mécanique des sols non saturés, *Tech. Ing.*, C302-19, 1–19.
- Devillers, P. (1998), Couplages thermohydromécaniques en milieux poreux—Application à la consolidation d'un sol non saturé, Ph.D. thesis, Univ. Montpellier 2, Montpellier, France.
- Devillers, P., M. S. El. Youssoufi, and C. Saix (1997), Identification des indices de compression hydraulique d'une loi de comportement thermohydromécanique pour les sols non saturés, in *Actes du 13ème Congrès Français de Mécanique, Poitiers, France*, pp. 313–316, Texas A&M Univ. Press, College Station, Tex.
- Escario, V. (1969), Swelling of soils in contact with water at a negative pressure, in *Proceedings 2nd International Research and Engineering Conference on Expansive Soils*, pp. 207–217, Texas A&M Univ., College Station, Tex.
- Fleureau, J. M., J. C. Verbrugge, P. J. Huergo, A. Gomes Correia, and S. Kheirbek-Saoud (2002), Aspects of the behaviour of compacted clayey soils on drying and wetting paths, *Can. Geotech. J.*, 39, 1341–1357.
- Fredlund, D. G. (1979), Appropriate concepts and technology for unsaturated soils, *Can. Geotech. J.*, 16, 121–139.
- Fredlund, D. G. (2000), The 1999 R. M. Hardy lecture: The implementation of unsaturated soil mechanics into geotechnical engineering, *Can. Geotech. J.*, 37, 963–986.
- Fredlund, D. G., and N. R. Morgenstern (1976), Constitutive relations for volume change in unsaturated soils, *Can. Geotech. J.*, 13, 261–276.
- Gatmiri, B., and P. Delage (1995), Nouvelle formulation de la surface d'état en indice des vides pour un modèle non linéaire élastique des sols non saturés, in *Proceedings of the 1st International Conference on Unsaturated Soils, Paris*, vol. 2, edited by E. E. Alonso and P. Delage, pp. 1049–1056, A. A. Balkema, Rotterdam, Netherlands.

- Gatmiri, B., P. Delage, and M. Cerrolaza (1998), UDAM: A powerful finite element software for the analysis of unsaturated porous media, *Adv. Eng. Software*, 29, 29–43.
- Ho, D. Y. F., D. G. Fredlund, and H. Rahardjo (1992), Volume change indices during loading and unloading of an unsaturated soil, *Can. Geotech. J.*, 29, 195–207.
- Kassif, G., and A. B. Shalom (1971), Experimental relationship between swell pressure and suction, *Geotechnique*, 21, 249–255.
- Laloui, L., and M. Nuth (2005), An introduction to the constitutive modeling of unsaturated soils, *Rev. Eur. Genie Civ.*, 9, 651–669.
- Lambe, T. H., and R. V. Whitman (1979), *Soil Mechanics*, John Wiley, New York.
- Lassabatère, T. (1994), Couplages hydromécaniques en milieu poreux non saturé avec changement de phase. Application au retrait de dessiccation, Ph.D. thesis, Ecole Nat. des Ponts et Chaussées, Paris.
- Matyas, E. L., and H. S. Radhakrishna (1968), Volume change characteristics of partly saturated soils, *Geotechnique*, 18, 432–448.
- Salager, S. (2007), Etude de la rétention d'eau et de la consolidation de sols dans un cadre thermo-hydro-mécanique, Ph.D. thesis, Univ. Montpellier 2, Montpellier, France.
- Salager, S., M. S. El Youssoufi, and C. Saix (2007), Experimental study of the water retention curve as a function of void ratio, paper presented at New Peaks in Geotechnics, GEI Consultants, Denver, Colo.
- Salager, S., B. Francois, M. S. El Youssoufi, L. Laloui, and C. Saix (2008), Experimental investigations of temperature and suction effects on the mechanical behaviour of a sandy silt, *Soils Found.*, 44(4), 453–466.
- Sugii, T., K. Yamada, and T. Kondou (2002), Relationship between soil-water retention curve and void ratio, in *Proceedings of the 3rd International Conference on Unsaturated Soils*, vol. 1, edited by J. F. T. Jucá, T. M. P. de Campos, and F. A. M. Marinho, pp. 209–214, Swets and Zeitlinger, Lisse, Netherlands.
- Vanapalli, S. K., D. G. Fredlund, and D. E. Pufahl (1999), The influence of soil structure and stress history on the soil-water characteristic of a compacted till, *Geotechnique*, 49, 143–159.

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P. Devillers, Centre des Matériaux de Grande Diffusion, Ecole des Mines d'Alès, 6 avenue de Clavières, F-30319 Alès, France. (philippe.devillers@ema.fr)

M. S. El Youssoufi and C. Saix, Laboratoire de Mécanique et Génie Civil, UMR5508, Université Montpellier 2, CNRS, cc 048, Place Eugène Bataillon, F-34095 Montpellier CEDEX 5, France. (elyous@lmgc.univ-montp2.fr; saix@lmgc.univ-montp2.fr)