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## TAIL BEHAVIOUR OF THE AREA UNDER A RANDOM PROCESS, WITH APPLICATIONS TO QUEUEING SYSTEMS, INSURANCE AND PERCOLATIONS

RAFAŁ KULIK AND ZBIGNIEW PALMOWSKI

ABSTRACT. The areas under workload process and under queueing process in a single server queue over the busy period have many applications not only in queueing theory but also in risk theory or percolation theory. We focus here on the tail behaviour of distribution of these two integrals. We present various open problems and conjectures, which are supported by partial results for some special cases.

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### 1. INTRODUCTION

In the past two decades there has been done enormous amount of work on asymptotics for queueing systems. Tail behaviour of steady state queueing process  $\{Q(t), t \geq 0\}$ , workload  $\{W(t), t \geq 0\}$  or busy period  $\tau$  in standard systems has been well-understood in both light and heavy tailed case. Surprisingly, however, very little is known on tail behaviour of integral functionals of the form

$$(1) \quad I_f(T) := \int_0^T f(X(u)) du,$$

where  $\{X(t), t \geq 0\}$  is a stochastic process (typically,  $X = Q$  or  $X = W$ ),  $f$  is a deterministic function and  $T$  is either  $\tau$  or deterministic (finite or infinite). Such integrals appear naturally in analysis of ATM. The reader is referred to references given in [9] and [22]. Recently, in [2], the authors connected mean bit rate in time varying  $M/M/1$  queue with moments of the integral  $\int_0^\tau Q(u) du$  in a corresponding standard  $M/M/1$  system.

However, applications of such integrals go beyond queueing systems. Let  $\{S(t), t \geq 0\}$  be a standard risk process. Integrals  $\int_0^T 1_{\{S(u) < 0\}} S(u) du$ , where  $T$  is deterministic (i.e. integrated negative part of the risk process), are suggested in [8] as possible risk measures. Further extensions are given in a multivariate setting. Furthermore, as in [17], integrals  $\int_0^\tau Q(u) du$  in  $Geo/Geo/1$  queue and (as a limit) in  $M/M/1$  system have particular interpretation in compact percolations. Last but not least, if  $X$  is Lévy process, integrals  $\int_0^\infty \exp(-X(u)) du$  have applications in financial mathematics, see [25]. Another applications are coming from the actuarial science, where very often regulated processes are considered and integral functionals from a regulation random mechanism are investigated.

### 2. SUBEXPONENTIAL ASYMPTOTICS

Consider a stable  $GI/GI/1$  queue. Denote by  $\{T, T_i, i \geq 0\}$  and  $\{S, S_i, i \geq 0\}$  two stationary i.i.d. and mutually independent sequences of interarrival and service times, respectively. Let  $\lambda_T = 1/E[T]$ ,  $\lambda_S = 1/E[S]$  and  $\rho = \lambda_T/\lambda_S$ . Let  $\{Q(t), t \geq 0\}$  be a stationary queueing process and  $\tau = \inf\{t \geq 0 : Q(t) = 0\}$  the corresponding busy period. We shall assume that the distribution  $F$  of service time  $S$  is subexponential (denotes as  $F \in \mathcal{S}$ ). The distribution  $G$  is subexponential when

$$\lim_{x \rightarrow \infty} \overline{G^{*2}}(x)/\overline{G}(x) = 2,$$

where  $G^{*2}$  is the convolution of  $G$  with itself and  $\overline{G}$  denotes the tail distribution given by  $\overline{G}(x) = 1 - G(x)$ .

Heuristically, the large area  $\int_0^\tau Q(u) du$  is realized by a customer with large service time  $S_0$ , say, who arrives at the very beginning of the busy period and blocks the server. During that time  $S_0$ , according to the Law of Large Numbers (LLN), approximately  $\lambda_T S_0$  customers arrive and the queue length process increases linearly. Hence the area under queueing process before reaching maximum is asymptotically equivalent to the area of a triangle:  $\frac{1}{2\lambda_T} \overline{Q}(\tau)^2$ , where  $\overline{Q}(\tau)$  is maximum of the queue length process over the busy period (see Figure 1). After passing the maximum, the queue length process behaves according to the LLN, decreasing almost linearly to 0 with the slope  $\lambda_S - \lambda_T$ . Thus, the area under the queueing process after reaching the maximum is equivalent to  $\frac{1}{2(\lambda_S - \lambda_T)} \overline{Q}(\tau)^2$  and  $\overline{Q}(\tau)$  is equivalent to  $\rho(\lambda_T - \lambda_S)\tau$ . This heuristic leads to the following conjecture.

**Conjecture 2.1.** *If  $F \in \mathcal{S}$ , then*

$$(2) \quad P\left(\int_0^\tau Q(u) du > x\right) \sim P\left(\tau > \sqrt{\frac{2x}{\rho(\lambda_S - \lambda_T)}}\right).$$

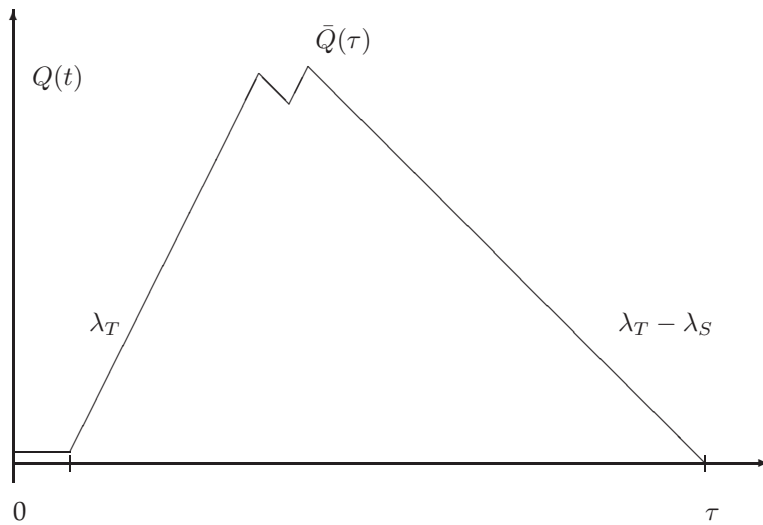


FIGURE 1. A typical behaviour of a heavy tailed queueing process.

Similarly, for the workload process the heuristic is as follows. The most likely way for the area to be large is that one early big service time occurs and apart from this, everything in the cycle develops normally. Using LLN and ignoring random fluctuations, this leads to the conclusion that the workload goes to zero with negative rate  $-(1 - \rho)$ . Thus the area exceeds level  $x$  iff the area of the triangle with the sides  $\tau(1 - \rho)$  and  $\tau$  is greater than  $x$ , hence when

$$\frac{1}{2}\tau^2(1 - \rho) > x ,$$

which suggests the following conjecture.

**Conjecture 2.2.** *If  $\bar{F} \in S$ , then*

$$(3) \quad P\left(\int_0^\tau W(u) du > x\right) \sim P\left(\tau > \sqrt{\frac{2x}{1 - \rho}}\right).$$

The statements (2) and (3) were proven in [22] and [9], respectively, under regularly varying assumption of the service time, that is  $\bar{F}(x) = x^{-\alpha}L(x)$ , where  $\alpha > 1$  and  $L$  is slowly varying at infinity. Furthermore, in [22] one needs additionally that

$$(4) \quad \lim_{t \rightarrow \infty} \frac{t^{1+\varsigma}P(T > t)}{\bar{F}(t)} = 0$$

holds with  $\varsigma > 0$ .

The tail behaviour of the busy period  $\tau$  can be identified in terms of  $\bar{F}$  for a large subclass of  $S$ :

$$(5) \quad P(\tau > x) \sim \frac{1}{1 - \rho} \bar{F}((1 - \rho)x),$$

see [5], [16] and [30]. In particular, using (5) in the regularly varying case, together with (2) and (3) yields exact asymptotics for area under queueing process and workload, respectively (see [22] and [9]).

### 3. LIGHT-TAILED ASYMPTOTICS

As in Section 2 we consider a stable  $GI/GI/1$  queue. Here, we assume that the service time  $S$  is light-tailed, that is there exists  $\theta > 0$  such that  $E[\exp(\theta S)] < \infty$ .

Under the above assumptions for the queueing process, we have the following open problem:

**Open Problem 3.1.** *Find exact asymptotics of*

$$P\left(\int_0^\tau Q(u) du > x\right).$$

We suppose that

$$P\left(\int_0^\tau Q(u) du > x\right) \sim Cx^{-1/4} \exp(-\psi\sqrt{x})$$

for some constants  $\psi$  and  $C$ .

We suggest above asymptotics, believing that it should be the same like for  $M/M/1$  queue, which was found in [14] under two conjectures on p. 391 and it is in the following form:

$$(6) \quad P\left(\int_0^\tau Q(u) du > x\right) \sim \frac{1-\rho}{\rho\sqrt{2\pi\psi}} x^{-1/4} \exp(-\psi\sqrt{x}),$$

where

$$\psi = 2\sqrt{-2(1-\rho) + (1+\rho)\log\rho}.$$

In the proof authors used the Laplace transform method. We are not aware of any probabilistic proof of this result and we do not know if Conjectures 1 and 2 in [14] hold true.

Unfortunately, we have not managed to produce any heuristic for this result either. The idea of the piecewise linear most likely trajectory seems to produce wrong expression. In particular, define the new probability measure  $\tilde{P}$ :

$$(7) \quad \frac{d\tilde{P}_{\mathcal{F}_n}}{dP_{\mathcal{F}_n}} = e^{\gamma \sum_{i=1}^n (S_i - T_{i-1})},$$

where  $\mathcal{F}_n = \sigma(T_1, S_1, \dots, T_n, S_n)$  and  $\gamma$  solves the equation  $E[\exp(\gamma(T - S))] = 1$ . Let  $\tilde{\rho} = \frac{E[T]}{E[S]} > 1$ . Consider the most likely path coming from large deviation theory for large cycle maxima, that is trajectory that develops along the line with the slope  $\tilde{\rho} - 1$  and after getting maximum behaves 'normally', that is goes to zero linearly with negative rate  $-(1 - \rho)$ . This trajectory produces wrong asymptotics for  $M/M/1$  queue since by Kyprianou [19] we have then:

$$P\left(\int_0^\tau Q(u) du > x\right) \sim P\left(\tau > \sqrt{2\frac{1+\rho}{1-\rho}x}\right) \sim Cx^{-3/4} e^{-\gamma\sqrt{2\frac{1+\rho}{1-\rho}x}}$$

for some constant  $C$  and  $\gamma = (1 - \sqrt{\tilde{\rho}})^2\mu$ . The explanation might come from papers [12, 26, 27], where the optimal trajectories in a sense of large deviation theory for the mean value of a reflected random walk  $W_n$  are considered. The relationship between area under the queue length and sum of  $W_n$  is clear when observing queue length process at arrival and departure epochs. These papers prove that for deterministic terminal times the optimal path, though still concave in general, might not be piecewise linear. In fact, these papers show that the optimal trajectory  $q(t)$  of the queueing process for the large value of the area on the cycle should solve the following equation:

$$(8) \quad \nabla I\left(\frac{d}{dt}q(t)\right) = \lambda^*(\tau - t),$$

where  $\lambda^* > 0$  and  $I$  is a rate function for the increment process. The proof of Problem 3.1 would then require an extension of this result to the case of a random terminal time being the end of busy

period. Apart of it, solving (8) explicite seems to be a difficult task and transferring it into finding the asymptotic tail distribution of  $\tau$  and the integral even more cumbersome. This problem shows also the need of simulations of the area  $\int_0^\tau Q(u) du$  to check if (8) indeed produces non-piecewise-linear optimal path. In general, we are not aware of any simulations results concerning area under a random process.

Similar open problem one can pose for  $P\left(\int_0^\tau W(u) du > x\right)$ .

#### 4. EXTENSIONS

**4.1. Transient case.** Let us consider the  $M/M/1$  queue with  $\rho = 1$ . In [17] the author established the following asymptotics for the critical case  $\rho = 1$ :

$$(9) \quad P\left(\int_0^\tau Q(u) du > x\right) \sim \frac{3^{1/3}}{\Gamma(1/3)\psi_0\psi_1} x^{-1/3},$$

where  $\psi_0$  and  $\psi_1$  are explicit constants.

**Open Problem 4.1.** Consider a transient  $GI/GI/1$  queue such that  $\bar{F}(x)$  is regularly varying. What is the tail asymptotics of  $\int_0^\tau Q(u) du$  and  $\int_0^\tau W(u) du$  ?

As mentioned above in the Introduction, this type of questions should have connections with heavy-tailed critical percolations.

**4.2. Utility functions, discounting.** Consider again a stable  $GI/GI/1$  queue. Assume that  $f$  is a deterministic function. We are interested in the tail behaviour of  $\int_0^\tau f(W(u)) du$ . In an insurance context,  $f(\cdot)$  may play a role of the utility function. Of course, one can formulate the corresponding problem for the queuing process, where  $f$  gives the costs of maintaining the system.

The heuristic given in the subexponential case in Section 2 suggests that studying the asymptotics of latter integral is equivalent to study the behaviour of

$$P\left(\int_0^\tau f((\tau - u)(1 - \rho)) du > x\right),$$

when  $x$  becomes large. For example, if one considers  $f(x) = x^k, k \geq 0$ , it leads to the following conjecture.

**Conjecture 4.1.** Assume that  $\bar{F} \in \mathcal{S}$  and  $f(x) = x^k$ . Then,

$$P\left(\int_0^\tau f(W(u)) du > x\right) \sim P\left(\tau > \sqrt[k+1]{\frac{(k+1)x}{(1-\rho)^k}}\right).$$

However, this is not clear for us what should be expected for a very general function  $f$ .

Now, assume that  $\theta > 0$  and consider the process  $X(t) = \exp(-\theta t)f(W(t)), t \geq 0$ . Similarly, the corresponding integral can be interpreted in an insurance context:  $\exp(-\theta t)$  is the discounting factor and  $f(\cdot)$  is the utility function.

The subexponential heuristic leads us again to

$$P\left(\int_0^\tau \exp(-\theta u) f((\tau - u)(1 - \rho)) du > x\right).$$

For further heuristic, consider  $f(x) = x^k$ . Then

$$\begin{aligned} \int_0^\tau \exp(-\theta u) f((\tau - u)(1 - \rho)) du &= (1 - \rho)^k \int_0^\tau \exp(-\theta u) (\tau - u)^k du \\ &= (1 - \rho)^k \exp(-\theta \tau) \int_0^\tau \exp(\theta s) s^k ds \\ &= (1 - \rho)^k \exp(-\theta \tau) \left[ \frac{1}{\theta} \exp(\theta \tau) \tau^k - \frac{k}{\theta} \int_0^\tau \exp(\theta s) s^{k-1} ds \right]. \end{aligned}$$

Integrating further by parts the last integral we see that the leading term will be  $O(\exp(\theta \tau) \tau^{k-1})$ . This leads to the following conjecture.

**Conjecture 4.2.** *Assume that  $F \in \mathcal{S}$  and  $f(x) = x^k$ . Then,*

$$P \left( \int_0^\tau \exp(-\theta u) f(W(u)) du > x \right) \sim P \left( \tau > \sqrt[k]{\frac{\theta x}{(1 - \rho)^k}} \right).$$

This conjecture should be compared with Conjecture 4.1, showing how discounting leads to the change in the asymptotic behaviour.

Let us consider now a light tailed case. The large area for the integral  $\int_0^\tau W(u) du$  is strictly connected to a large  $\bar{W}(\tau)$ . It does not seem to be the case if one considers  $\int_0^\tau \exp(-\theta u) f(W(u)) du$ , i.e. large values of  $W(u)$  may be killed by the discount factor.

**Open Problem 4.2.** *Under the conditions of Open Problem 3.1, find the asymptotics for*

$$P \left( \int_0^\tau \exp(-\theta u) f(W(u)) du > x \right).$$

**4.3. Finite horizon.** All the conjectures and open problems above may be formulated in case of  $\int_0^T$ , where  $T$  is finite and deterministic. In particular, if  $S(t) = v + ct - \sum_{i=1}^{N(t)} S_i$ ,  $t \geq 0$ , is the classical risk insurance process, then moments of

$$\int_0^T S(u) 1_{\{S(u) < 0\}} du$$

are proposed in [8] as particular risk measures. In that paper the authors established the asymptotics for the expected value of the integral, when  $T$  is fixed and the initial capital  $v$  becomes large. Clearly, this problem can be re-formulated for the dual workload process. Furthermore, all the above mentioned extensions (discounting, utility functions) may be considered.

**4.4. Multivariate case.** As in Section 4.3, let us consider insurance context. We assume that a company has two lines of business and customers arrive according to a renewal process  $N(t)$ ,  $t \geq 0$ , with generic interarrival time  $T$ . The  $i$ th customer has a claim  $(S_{i,1}, S_{i,2})$ . For example, a car accident may cause a claim for driving and liability insurance (see for example [3], [8] and references therein). If two lines of business are considered separately, we are interested in the tail behaviour of

$$\left( \int_0^T S_1(u) 1_{\{S_1(u) < 0\}} du, \int_0^T S_2(u) 1_{\{S_2(u) < 0\}} du \right),$$

where  $S_1$  and  $S_2$  are risk process associated with the corresponding lines of business. One can consider the tail behaviour when  $T$  is fixed and a vector of initial capital  $(x_1, x_2)$  becomes large (in a particular sense, usually dependence is linear, that is  $x_1 = ax_2$  for some fixed  $a > 0$ ).

In queuing theory one can consider e.g. parallel queues, where customers are coming into the system according to renewal process  $N(t)$  and each service time (we assume here that all of them are i.i.d.) is proportionally divided into two servers (see e.g. [23] and [24]). In this case,  $S_{i,1} = bS_{i,2}$  for some fixed  $b$ . Let  $W_i(t)$  and  $Q_i(t)$  will be the workload and queueing process on  $i$ th server ( $i = 1, 2$ ). The busy period  $\tau$  is understood here as the minimum of the busy periods on both servers. We could analyze the tail behaviour of the following "tail" probability:

$$P \left( \int_0^\tau W_1(u) du > x \quad \text{and} \quad \int_0^\tau W_2(u) du > ax \right) \quad \text{as } x \rightarrow \infty,$$

under assumption that distribution  $F$  of  $S_{i,1}$  is subexponential or light-tailed. Similar considerations could be analyzed for the bivariate occupation process.

**4.5. Regulated processes.** Apart of original process  $X(t)$  in (1), one can consider regulated process:

$$U(t) = X(t) - L(t),$$

where  $L(t)$  is left-continuous, increasing process, adapted with respect to the natural filtration of  $X$ . For example, if  $X$  is a spectrally negative Lévy risk process, then  $L(t)$  might be cumulative dividends payment up to time  $t$  paid according to some strategy. The most often used strategy is so-called barrier strategy in which all surpluses above a given level  $a$  are transferred (possibly subject to a discount rate) to a beneficiary. In this case  $L(t) = a \vee \sup_{s \leq t} X(s) - a$  and for given utility function we are interested in the following random variable:

$$(10) \quad I_f(\tau) = \int_0^\tau e^{-\theta u} f(L(u)) du,$$

(possibly with  $\theta = 0$ ), where  $\tau = \inf\{t \geq 0 : U(t) < 0\}$  is a ruin time.

We can also analyze discounted cumulative dividends paid up to ruin time:

$$(11) \quad \int_0^\tau e^{-\theta u} dL(u),$$

which is equivalent to finding the tail asymptotics of integral:

$$\int_0^\infty e^{-\theta Z_1(t)} \mathbf{1}_{\{\Delta Z_2(u) < a\}} du,$$

where  $(Z_1(t), Z_2(t))$  is a bivariate subordinator and  $\Delta Z_2(u) = Z_2(u) - Z_2(u-)$  is a size of jump of  $Z_2$  at time  $u$  (see [1, 4, 20, 29] for details and other references). In the case if  $a = \infty$  we end up with integral from exponential function of Lévy process, see e.g. [6, 7, 10, 25] for this kind of functional. Note that in particular  $I = \int_0^\infty e^{-\theta Z_1(t)} du$  solves the following equation:  $I \stackrel{D}{=} \int_0^\tau e^{-\theta Z_1(t)} du + e^{-\theta Z_1(\tau)} I$  and analyzing  $I$  is strongly related then with properties of  $e^{-\theta Z_1(\tau)}$  (see e.g. [13, 18]).

With this problem there is also related integral with respect to the general Lévy process:

$$\int_0^T Y(u) dX(u)$$

for appropriate predictable integrand  $Y$  and fixed  $T$  (see [15] for multivariate regularly varying setting).

In queueing systems with single server and finite capacity  $a$ ,  $L(t)$  corresponds then to cumulative lost of work. If only proportion of information is lost, then we can consider e.g.  $L(t) = c \mathbf{1}_{\{U(t) > a\}}$  for some  $c > 0$ . In this case formally so-called refracted process  $U$  solves the following stochastic



equation:

$$dU(t) = dX(t) - c\mathbf{1}_{\{U(t)>a\}} dt;$$

see [21]. In this case we can also try to find asymptotic tail of (10) and (11).

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