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Elastocapillary Snapping

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We report on the capillary-induced snapping of elastic beams. We show that a millimeter-sized water drop gently deposited on a thin buckled polymer strip may trigger an elastocapillary snap-through instability. We investigate experimentally and theoretically the statics and dynamics of this phenomenon and we further demonstrate that snapping can act against gravity, or be induced by soap bubbles on centimeter-sized thin metal strips. We argue that this phenomenon is suitable to miniaturization and design a condensation-induced spin-off version of the experiment involving an hydrophilic strip placed in a steam flow.

Elastic arches and spherical shells can sustain large loads but they all eventually fail through an elastic instability, called snapping or snap-through buckling, see [1, 2] for early references on the subject. This phenomenon is central to the failure of arches and vaults but has also been exploited to actuate bistable switches or valves [3] with point force [4], electrostatic [5], piezoelectric [6], or vibrational [7] loading. Snapping is also a useful mechanism in the design of responsive surfaces with applications to on-demand drug delivery, optical surface properties modification, or on-command frictional changes [8]. Nature provides examples of practical applications of snapping in prey capturing by carnivorous plants [9], fast ejection of spores [10], or underwater plant suction trap [11]. Similarly, polymersomes [12] or malaria infected blood cells [13] also exhibit snapping events (or fast shell eversion) that promote fast ejection of drug components or parasites. These examples differ in their triggering mechanisms, but they all involve a snapping instability including fast movements and curvature reversals that are a consequence of the sudden release of stored elastic energy and its transfer into kinetic energy.

Here we show how capillary forces may be used to trigger snap-through instabilities: a drop deposited on a thin buckled elastic strip induces snapping, possibly even against gravity, as illustrated Fig. 1. Our experiments consist in loading buckled elastic strips with either transverse point forces or water droplets. Initially flat elastic strips of length L and width w were carefully cut out of a thin polymer film made of polydimethyl-siloxane (PDMS, Sylgard 184 Elastomer base blended with its curing agent in proportion 10:1), spin-coated and cured at 60°C for two hours. The resulting thickness h of the samples was quantified with an optical profilometer. The Young's modulus of our samples, measured using a Shimadzu testing machine, was found to be $E = 1.50 \pm 0.05$ MPa, enabling us to evaluate the bending rigidity $EI = Eh^3w/12$ of our samples. Experiments were carried out with two different strips whose geometrical and mechanical properties are reported in Table I. These PDMS strips were clamped at both ends with mi-

#	L (mm)	w (mm)	h (μm)	λ ($\mu\text{g}/\text{mm}$)	Δ/L
S1	5.0	1.07	68.3	83.3	0.95
S2	3.5	0.98	33.7	42.9	0.90

TABLE I. Length L , width w , thickness h , and mass per length λ of the strips used in the two experimental setups. The axial compression Δ is illustrated in Fig. 2.

croscope slides with cut edges. In point-force induced snapping, force-displacement data were gathered with a micro-force sensor using capacitive deflection measurement [14] (Femtools FT-S270) and a nano-positioner (SmarAct SLC-1730). Capillary snapping was investigated by depositing water drops with Hamilton syringes or syringe pump (Harvard Apparatus) with PTFE coated needles. Elastic strips were taced to promote contact line pinning. Video have been captured with an ultrafast Photron SA-5 camera.

When compressed axially a straight beam buckles when the load exceeds a critical limit and adopts an arched shape. As the load is further increased so does the height of the arch, see Fig. 2a. If one now fixes the axial compression and applies a downward shear force at the middle point of the beam, the height starts to decrease. As this lateral force reaches a threshold the arch snaps to a downward configuration. The threshold value for snap-through is known to depend on the position x of the applied shear force and reaches a local maximum when $x/\Delta = 1/2$ [15]. In Fig. 2 a comparison is made between experiments and theory. Theoretical bifurcation curves were computed using Kirchhoff equations for the equilibrium and stability of elastic rods [16]. In the present case the lateral displacement (as opposed to the force F) is controlled. Consequently configurations in the secondary bifurcated path (part of the response curve between $F = F_{\text{max}}$ and $F = 0$) are stable and snap-through really only occurs as the lateral force F reaches zero. We will nevertheless refer to the point $F = F_{\text{max}}$ as the snapping threshold. It should be noted that the fixed compression $\Delta = 0.95L$ is small enough for the precise way

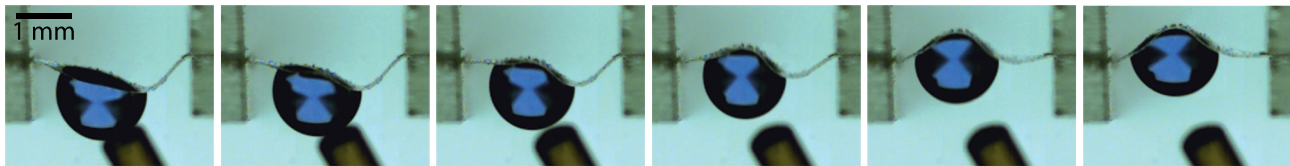


FIG. 1. Snapping against gravity. Using a PTFE coated needle, a drop is gently deposited under a downward buckled PDMS strip. Within a few milliseconds, capillary forces induce a snap-through elastic instability of the strip which jumps to the upward buckled state. Note that in this setup surface tension overcomes both elastic forces and gravity. The liquid is tap water dyed with blue ink for visualization purposes. The geometrical and mechanical properties of the strip correspond to sample S2 described in Table I. The time interval between each snapshot is 5 ms.

with which the shear force loading is applied to be disregarded [17], and conversely large enough for extension effects to be negligible [18].

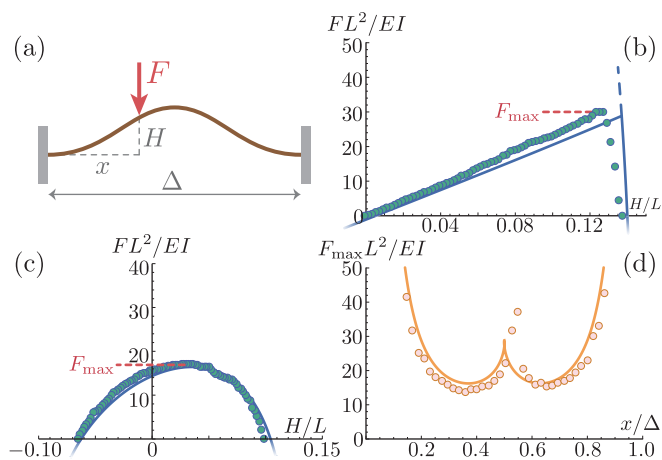


FIG. 2. Snap-through instability with point force. (a) Elastic strip S1 clamped at both end, with fixed $\Delta = 0.95L$. As h is gradually reduced, we measure F . (b) Comparison between experiments (filled circles) and model (continuous curves) when the point load is centered on the beam, $x/\Delta = 1/2$. We measured $F_{\max} = 55 \mu\text{N}$. (c) Similar bifurcation diagram in a non-symmetric case $x/\Delta = 1/3$. Note that the snapping threshold F_{\max} is significantly lowered. (d) Evolution of the snapping threshold F_{\max} as a function of the point load position x , showing the two preferential positions along the beam where the threshold is minimal $x/\Delta \simeq 0.37$ and 0.63 .

We now replace the point load with a water drop. We performed experiments where drops of increasing volume were deposited or hung on the same strip (case S1 in Table I). The height of the arch H was recorded as a function of the total weight F of the drop, see Fig. 3. As the volume of the drop is increased, the height of the arch decreases until a limit is reached where snap-through occurs. We remark (i) that the critical weight necessary to snap the strip is appreciably lower when the drop is deposited above the strip than when the drop is hanging below the strip, and (ii) that the critical lateral force necessary to snap the strip in the classical indentation experiment presented in Fig. 2(b) is intermediate between

these two new critical weights. We conclude that only considering the weight of the drop is not enough, i.e. capillary forces have a strong influence on snap-through. As known in shell indentation, the response of elastic structures to external loads strongly depends on whether the loading is performed through point forces or distributed pressure loads [19]. In our case the water drop applies distributed hydrostatic and Laplace pressures as well as localized meniscus forces, as shown in Fig. 3. The combined action of Laplace and meniscus forces can be seen as two opposite effective bending moments, promoting the eversion of the strip [20] when the drop is located above, and hindering it when located below.

To further inquire relative strengths of capillarity, weight, and elastic forces we study the following setup: an elastic strip (case S2 in Table I) is buckled downward ($h < 0$ in Fig. 2(a)) and a drop is hung at a given location under the strip, see Fig. 4(c). Parameters are the total weight F of the drop and the abscissa x_M of the middle point of the wetted region of the beam. Experiments show that snapping only occurs for specific values of F and x_M , see Fig. 4(b). For small drops (i.e. small F), capillary forces exceed self-weight (a drop deposited under a rigid surface is stable if small enough) but are not powerful enough to overcome elastic forces, mainly because the lever arm of the effective bending moments discussed earlier is not large enough: the wet length is indeed a key factor determining the behavior of elasto-capillary systems [21]. Consequently the system stays in the downward configuration. For moderate drops (with larger wet lengths) we see in Fig. 4(b) that provided the location of the drop is carefully chosen snapping occurs, resulting in a final state where the strip is bent upward: in this case capillary forces overcome both weight and elastic forces. For large drops capillarity still beats elasticity but self-weight is too large and the system stays in the downward configuration.

To understand the different regions of the (x_M, F) phase diagram we numerically computed equilibrium and stability of the drop-strip system in the following way. We consider a 2D setting where a liquid drop of a given volume is hung under an elastic strip of length L , thickness h , width w , mass per unit length

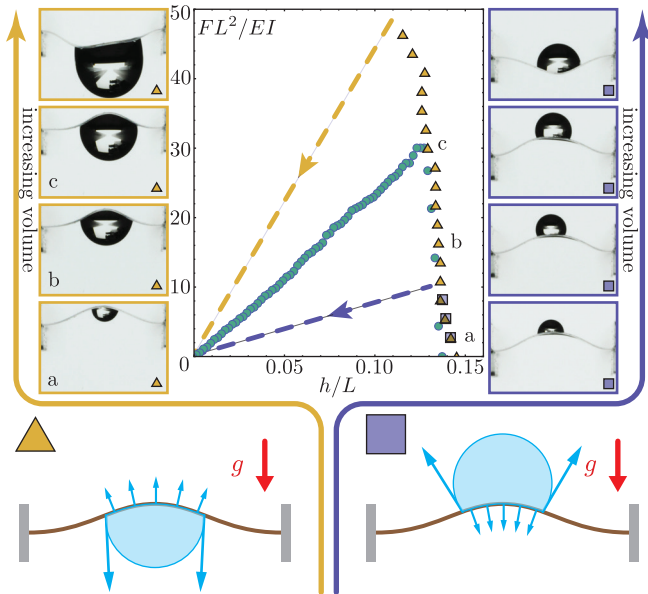


FIG. 3. Influence of capillarity on the bifurcation diagram of Fig. 2(b). Drops of increasing volume are hung below (orange triangles) or deposited above (purple squares) the elastic strip S1, buckled upwards with $\Delta = 0.95L$. As the non-dimensional drop weight FL^2/EI increases the deflection h/L of the strip midpoint decreases, up to a point where snapping occurs (indicated by the dashed lines on the diagram). For comparison we plot the data of Fig. 2(b), filled circles, performed on the same S1 strip. Much heavier drops are required to trigger the snap-through instability in the hanging drop setup as compared to the sitting drop setup, the ‘dry’ setup being intermediate. For both square and triangle sets, the volume increase between each measure is $0.5 \mu\text{l}$, corresponding to a non-dimensional force increase of 2.73. The right (orange) panel shows hanging configurations with, from top to bottom, $V = F/\rho g = 0.5 \mu\text{l}, 3 \mu\text{l}, 5.5 \mu\text{l}, 9 \mu\text{l}$, with $\rho = 1000 \text{ kg/m}^3$. The left (purple) panel shows sitting configurations with, from top to bottom, $V = 0.5 \mu\text{l}, 1 \mu\text{l}, 1.5 \mu\text{l}, 2 \mu\text{l}$. Note that the present dead loading (squares and triangles) makes the secondary bifurcated path (part of the response curve between $F = F_{\text{max}}$ and $F = 0$) unstable, contrary to the rigid loading setup of Fig. 2.

λ , and bending rigidity EI . The strip is clamped at both ends which are separated by a fixed distance Δ . We use the arc-length s along the strip to parametrize its position $\mathbf{r}_s(s) = (x_s(s), y_s(s))$. The unit tangent, $\mathbf{t}_s(s) = d\mathbf{r}_s/ds$, makes an angle $\theta_s(s)$ with the horizontal: $\mathbf{t}_s = (\cos \theta_s(s), \sin \theta_s(s))$. The drop lies between positions $s = s_A$ and $s = s_B$ on the strip, and the shape of the liquid-air interface, parametrized with its own arc-length σ , is $\mathbf{r}_i(\sigma) = (x_i(\sigma), y_i(\sigma))$ and has a total contour length ℓ , see Fig. 4(c). The elastic and gravity energies of the strip are:

$$E_{\text{strip}} = \frac{1}{2}EI \int_0^L [\theta'_s(s)]^2 ds + \int_0^L \lambda g y_s(s) ds \quad (1)$$

The gravity potential energy of the water is:

$$E_{\text{water}} = \rho g w \iint_{\mathcal{A}} y dA \quad (2)$$

where $\mathcal{A} = \int_0^\ell y_i(\sigma) x'_i(\sigma) d\sigma - \int_{s_A}^{s_B} y_s(s) x'_s(s) ds$ is the area between the strip and the liquid-air interface. The energy per unit area of solid-liquid (respectively solid-air, and liquid-air) interface is noted $\gamma_{\ell s}$ (resp. γ_{sv} and γ). The total interface energy is then:

$$E_\gamma = w(s_B - s_A)\gamma_{\ell s} + w[L - (s_B - s_A)]\gamma_{sv} + \gamma w \ell \quad (3)$$

Due to the constraint on the water volume, the energy contribution of the end-caps $2\gamma\mathcal{A}w$ is constant and therefore discarded. We minimize total potential energy $U = E_{\text{strip}} + E_{\text{water}} + E_\gamma$ under the constraints of inextensibility $\mathbf{r}'_s(s) = \mathbf{t}_s$, constant volume $V = \mathcal{A}w$, and matching conditions $\mathbf{r}_s(s_A) = \mathbf{r}_i(0)$ and $\mathbf{r}_s(s_B) = \mathbf{r}_i(\ell)$. This constrained minimization problem is solved by considering the following Lagrangian functional:

$$\mathcal{L}[\mathbf{r}_s(s), \theta_s(s), s_A, s_B, \mathbf{r}_i(\sigma), \theta_i(\sigma), \ell] = U - \boldsymbol{\mu} \cdot \boldsymbol{\psi} \quad (4)$$

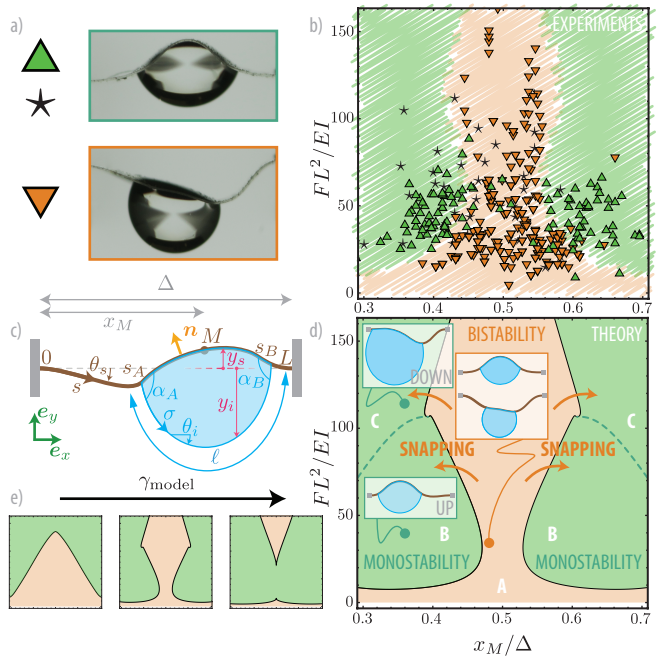


FIG. 4. Phase diagram for elastocapillary snapping. A drop is deposited on the lower side of a buckled strip. (a) Possible final states of the system. (b) Experimental phase diagram plotted in the (x_M, F) plane. Triangles (respectively \star) correspond to experiments where the drop is deposited on an initially downward (resp. upward) buckled strip. (c) Model notations and (e) evolution of the phase diagram as the surface tension used in the model γ_{model} takes the values 0.38γ , 0.67γ , and 0.96γ (from left to right). (d) Theoretical phase diagram showing bistable (A) and monostable (B) and (C) regions. The continuous curve corresponds to the loss of stability of an equilibrium configuration, and the dashed curve to a smooth transition between a downward state and an upward state.

where the vector ψ comprises all the constraints and μ is the vector of associated Lagrange multipliers. Classical minimization and continuation techniques are used to track equilibrium states along branches in bifurcation diagrams, see [22]. Note that in this 2D model the effective surface of the drop is not minimal because of its cylindrical shape. To counterbalance this effect we have used a reduced surface tension $\gamma_{\text{model}} = 0.67\gamma$, analogous to the surface correction coefficient introduced in [23]. In the computation, sliding of the drop was prevented by constraining the mean position $s_M = (s_A + s_B)/2$ and the mean contact angle $(\alpha_A + \alpha_B)/2 = 110^\circ$ deg. Results are shown in Fig. 4(d) where the theoretical (x_M, F) phase diagram is plotted. The continuous curve, latter refereed as the instability curve, corresponds to loss of the stability (at a fold point) of an equilibrium configuration. The dashed curve corresponds to the smooth transition from downward buckled states ($y_M < 0$) to upward buckled states ($y_M > 0$). These two curves divide the (x_M, F) plane in three regions. In the first region (A), which lies below the instability curve, there are two stable configurations: downward and upward buckled configurations are both found to be stable. As the crossing of the instability curve is associated with the loss of stability of one of the configurations, in the two regions above the instability curve there is only one stable configuration: upward for region (B), below the dashed curve, and downward for region (C), above the dashed curve. We remark that the shape of the instability curve and hence the topology of the phase diagram is altered by changes in the value of γ , as shown in Fig. 4(c). These numerical results shed light on experimental findings: in the bistable region (A), a drop deposited under a downward buckled strip will lead to a downward final state unless the perturbation created during the deposition is too large and the system jumps to upward final state, whereas in the monostable region (B) the final state is always an upward configuration. As a cross-check we have experimentally hanged drops under upward buckled strips and found that in regions (A) and (B) the system stays in the upward configuration, thereby confirming the bi-stability of region (A), see markers \star in Fig. 4(b).

We next show that snapping may be induced remotely. The lower side of a PDMS strip is treated with an hydrophilic coating. The strip is then buckled downward and placed in a steam flow. Water droplets nucleate on the hydrophilic side of the strip, coalesce and eventually induce snapping, see Fig. 5. This phenomenon could be used to build moisture sensors that would snap once ambient humidity has reached a given threshold.

We finally investigate time scales involved in the dynamics of the snapping instability. The shape of the beam as it leaves the unstable equilibrium is recorded with a high-speed camera and the vertical position $y_s(s = L/2, t)$ of the midpoint of the beam is extracted from the image sequence. From the fit $y_s(L/2, t) = y_0 + y_1 e^{\mu t}$ we



FIG. 5. Condensation-induced snapping. A thin PDMS sample, made hydrophilic on the lower face, is placed in a steam flow. Tiny droplets nucleate on the hydrophilic face and coalesce. When the resulting drop is sufficiently large, elastocapillary snapping takes place. The experiment approximately lasts three minutes.

obtain the growth rate μ . From this growth rate μ we

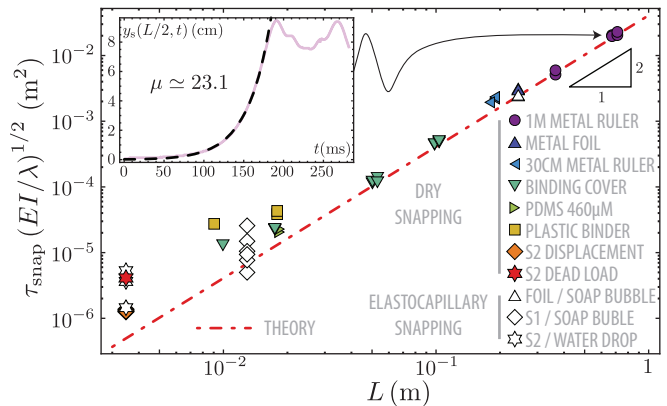


FIG. 6. Snapping dynamics. Typical time τ_{snap} for snapping in different setups. The dashed line is the theoretical prediction for ‘dry’ snapping $\tau_{\text{snap}} = (L^2/24) \sqrt{\lambda/(EI)}$.

define a snapping time $\tau_{\text{snap}} = 1/\mu$ and plot τ_{snap} as a function of the length L of the beam. For ‘dry’ snapping and in the case of controlled lateral displacement the instability occurs as the force reaches zero. At this point the beam has a unstable equilibrium shape corresponding to the second buckling mode of the planar Elastica and we numerically computed the growth rate to be $\mu = 24.26/T$ for $\Delta = 0.95L$ and $\mu = 24.42/T$ for $\Delta = 0.9L$, where $T = L^2 \sqrt{\lambda/(EI)}$ is the typical time of bending dynamics, see also [18]. In Fig. 6 we compare the theoretical prediction $\tau_{\text{snap}} = T/24$ for ‘dry’ snapping to snapping times extracted from experiments with various setups, e.g. ‘dry’ setups involving $L = 0.7$ m metal beams, capillary setups with S1 and S2 strips, but also setups with soap bubbles actuating $L = 0.25$ m metal foil strips. Apart from a deviation at small lengths attributed to viscous effects, we see that the snapping time seems to be the same for ‘dry’ and ‘wet’ snapping. From this we conclude that the elastocapillary dynamics is mainly driven by elastic forces and that fluid forces and fluid inertia play only a minor role. Capillary forces are driving the system toward instability, but elastic forces are ruling the subsequent dynamics.

As in adhesive film separation [24] or in the pull-out of a soft object from a liquid bath [25], the elastic energy

stored in the system before the instability is suddenly released in the form of kinetic energy and is mainly ‘lost’. We nevertheless showed in our setup that part of the energy could be used to lift the liquid drop. The typical scaling of surface forces makes elastocapillary snapping a good candidate to miniaturization and its use as a micro-actuator might be envisaged. In any case the present study is an example of the constructive use of capillarity at small scales.

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