

Supplementary information on ‘Elastocapillary Snapping’

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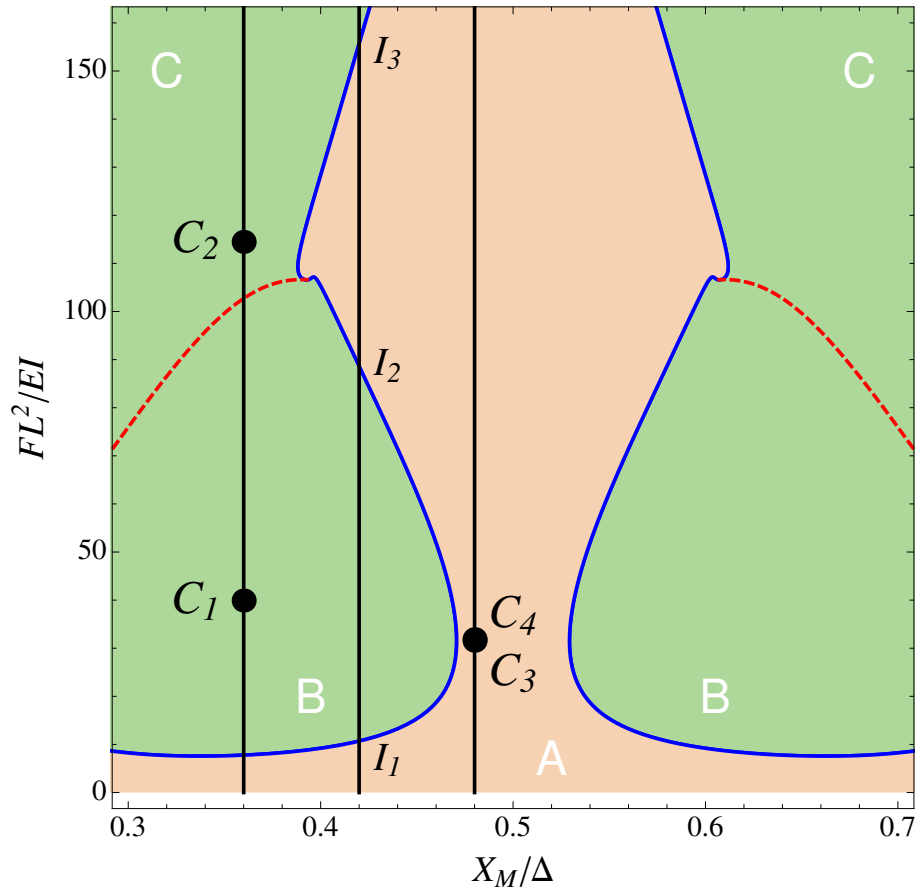


FIG. 1. Phase diagram as in Fig. 4(b) of main text, but with three additional vertical lines corresponding to $x_M = 0.36\Delta$, $x_M = 0.42\Delta$, and $x_M = 0.48\Delta$.

BIFURCATION DIAGRAMS ASSOCIATED TO FIG. 4

We here show some bifurcation diagrams in relation to the phase diagram presented in Fig. 4 of the main text. As in other problems where capillarity, weight, and elasticity are present the following quantities are found to be relevant:

$$\begin{aligned} L_{eg} &= \sqrt{EI/(\lambda g)} \\ L_{ec} &= \sqrt{EI/(\gamma w)} \\ L_c &= \sqrt{\gamma/(\rho g)} \end{aligned}$$

Using parameters from Table 1, we find $L_{eg} = 0.683L$, $L_{ec} = 0.0736L$, and $L_c = 0.774L$. Nevertheless in the numerical computations we made use of an effective value $\gamma_{\text{model}} = 0.67\gamma$, thereby changing L_{ec} and L_c to $L_{ec} = 0.09L$ and $L_c = 0.634L$. In Fig. 2 the bifurcation diagram for $x_M = 0.36\Delta$ is drawn with points corresponding to C_1 and C_2 configurations. Configuration C_1 (respectively C_2) has $Y_M > 0$ (resp. $Y_M < 0$) and hence is called an upward (resp. downward) buckled configuration. Along the bifurcation curve, at $FL^2/(EI) \simeq 100$, Y_M changes sign and this corresponds to the crossing of $x_M = 0.36\Delta$ with the dotted curve in Fig. 1. In Fig. 3 we have plotted the three fold points $I_{1,2,3}$, that are also shown in Fig. 1. These points are instability points and correspond to the loss of stability of an equilibrium as one leaves the bistable region A in Fig. 1. In Fig. 4 we have plotted the two points C_3 and C_4 , corresponding to stables configuration having the same value of $FL^2/(EI)$. These points lie in the bistable region A that extends up to $FL^2/(EI) \simeq 220$. For larger F values only the downward buckled configuration remains (in upper region C, out of the range of Fig. 1).

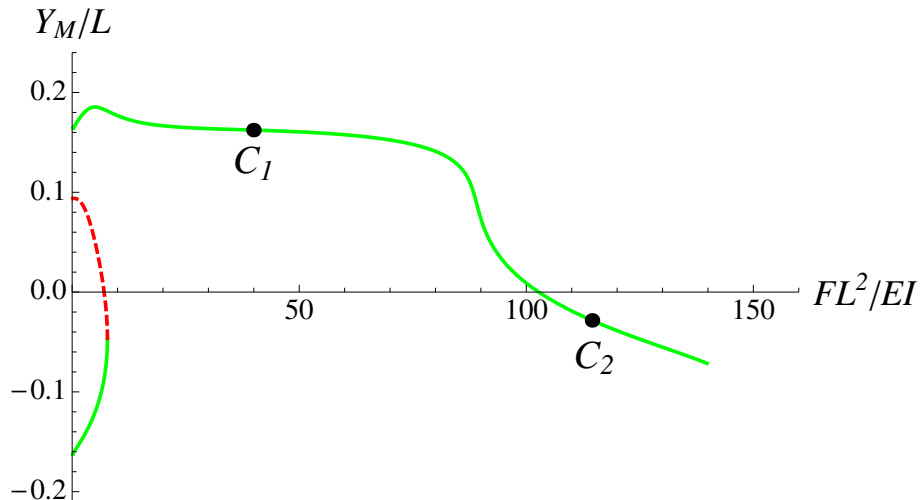


FIG. 2. Bifurcation diagram for $x_M = 0.36\Delta$ showing C_1 and C_2 configurations. Plain (respectively dotted) curves correspond to stable (resp. unstable) equilibria.

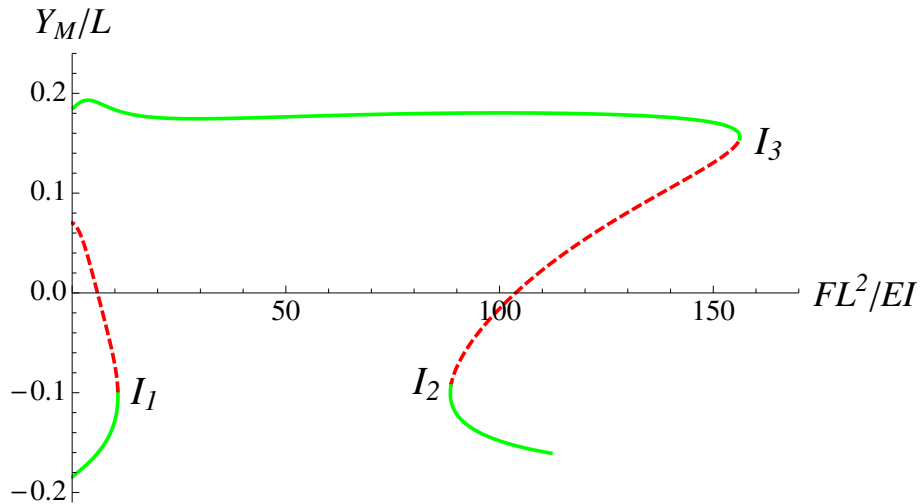


FIG. 3. Bifurcation diagram for $x_M = 0.42\Delta$. Plain (respectively dotted) curves correspond to stable (resp. unstable) equilibria. Also shown are the three fold points $I_{1,2,3}$ where stability changes.

SNAPPING DYNAMICS

The shape of the beam as it leaves the unstable equilibrium is recorded with a high-speed camera and the vertical position $y_s(s = L/2, t)$ of the midpoint of the beam is extracted from the image sequence. From the fit

$$y_s(L/2, t) = y_0 + y_1 e^{\mu t} \quad (1)$$

we obtain the growth rate μ . In Fig. 5 we plot $y_s(s = L/2, t)$ together with three snapshots showing the instantaneous shape of the beam during the snapping event.

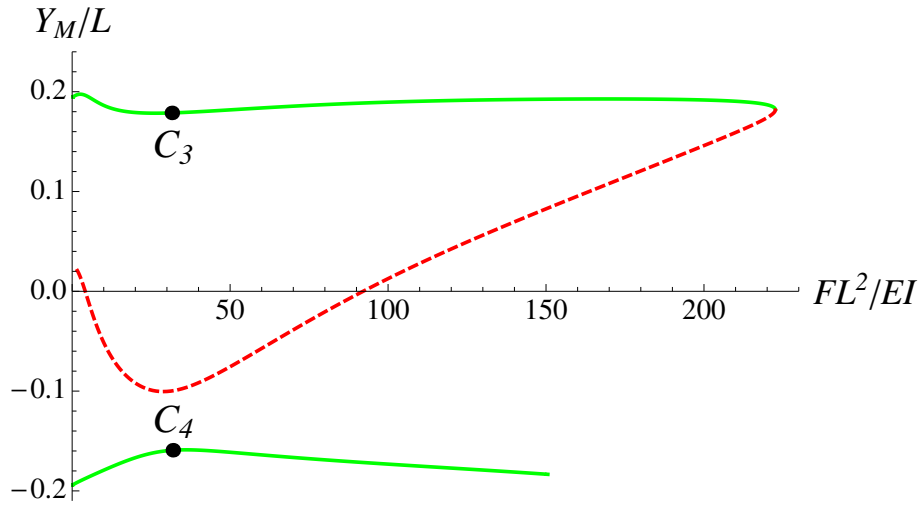


FIG. 4. Bifurcation diagram for $x_M = 0.48\Delta$ showing C_3 and C_4 configurations that share the same F value. Plain (respectively dotted) curves correspond to stable (resp. unstable) equilibria.

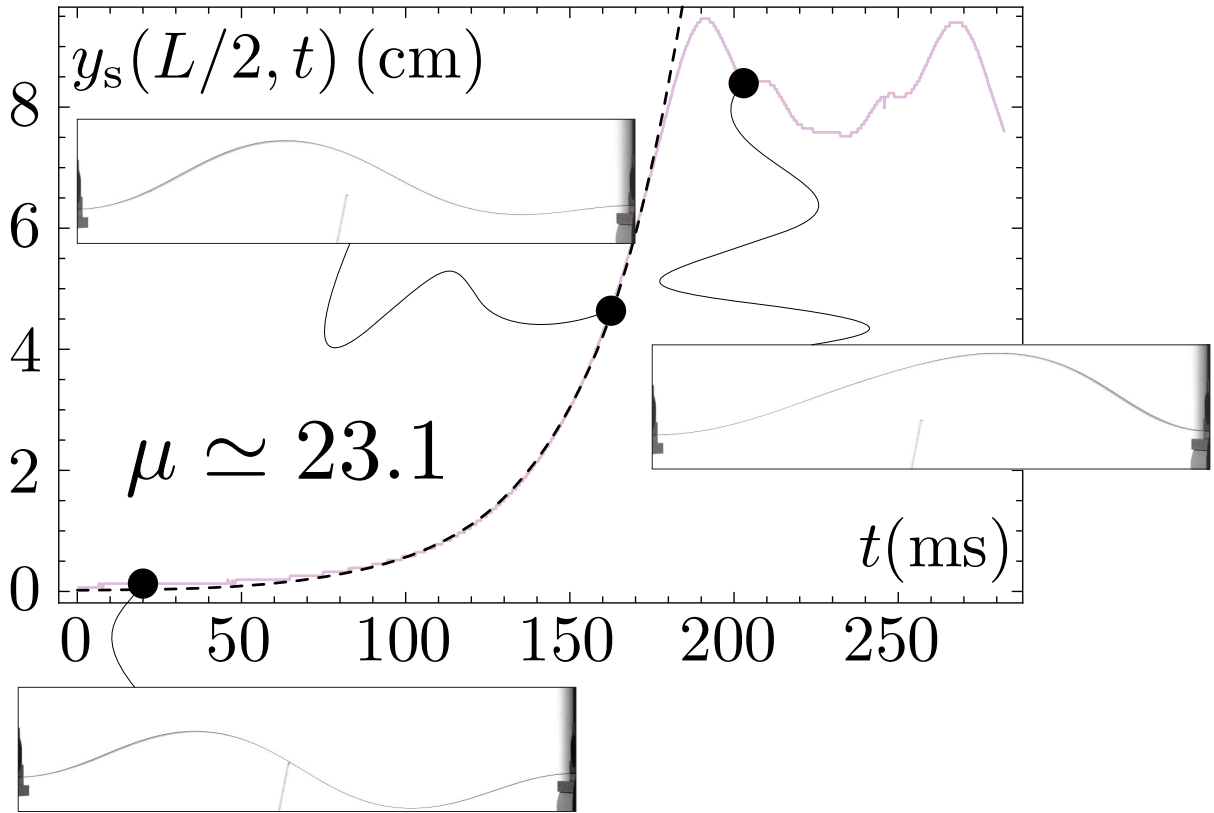


FIG. 5. Position on the beam mid-point $y_s(L/2, t)$ as function of the physical time t during a snapping event. Continuous curve corresponds to measured data, while dashed curve is Eq. 1 with fitted $\mu \simeq 23.1$.