



# STATISTICAL ENERGY ANALYSIS: CORRELATION BETWEEN DIFFUSE FIELD AND ENERGY EQUIPARTITION

Thibault Lafont, Alain Le Bot, Nicolas Totaro

## ► To cite this version:

Thibault Lafont, Alain Le Bot, Nicolas Totaro. STATISTICAL ENERGY ANALYSIS: CORRELATION BETWEEN DIFFUSE FIELD AND ENERGY EQUIPARTITION. MEDYNA, Apr 2013, Marrakech, Morocco. hal-00841466

**HAL Id: hal-00841466**

**<https://hal.science/hal-00841466>**

Submitted on 5 Jul 2013

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



## STATISTICAL ENERGY ANALYSIS: CORRELATION BETWEEN DIFFUSE FIELD AND ENERGY EQUIPARTITION

T. Lafont<sup>1\*</sup>, A. Le Bot<sup>1</sup> and N. Totaro<sup>2</sup>

<sup>1</sup>Laboratoire de Tribologie et Dynamique des Systèmes  
Ecole Centrale de Lyon, Ecully, FRANCE  
Email: [thibault.lafont@ec-lyon.fr](mailto:thibault.lafont@ec-lyon.fr), [alain.le-bot@ec-lyon.fr](mailto:alain.le-bot@ec-lyon.fr)

<sup>2</sup>Laboratoire Vibrations Acoustique  
INSA de Lyon, Villeurbanne, FRANCE  
Email: [nicolas.totaro@insa-lyon.fr](mailto:nicolas.totaro@insa-lyon.fr)

### ABSTRACT

*This paper studies two concepts used in Statistical Energy Analysis (SEA): the diffuse field and the energy equipartition. To study the equivalence between these concepts the example of a simply supported plate is taken. First, the degree of diffuseness of the field is quantified by a simple criterion. Then, extracting information from an analytical model, the standard deviation of modal energies is evaluated. Two maps are drawn on the frequency-damping plane in both cases. These maps allow correlating concepts of diffuse field and energy equipartition and validity diagrams defined by Le Bot.*

## 1 INTRODUCTION

Using the statistical energy analysis (SEA) method introduced by Lyon and DeJong [1] requires some specific assumptions which limit its range of application. These fundamental requirements have been clearly highlighted in the literature. Burroughs [2] summarised them: broadband excitation, conservative coupling, light damping loss factor, equipartition of energy, large number of modes and light coupling. Many authors have studied these hypotheses individually and proposed solutions to enhance the validity of SEA. For example Goyder [3] on power flow, Wöhle [4] on coupling.

The concept of diffuse field comes out with its assumptions even if it is not clearly mentioned. Indeed, broadband power flow and high modal overlap suggest diffuseness. Defining from a raw acoustical point of view, an ideal diffuse sound field is homogeneous and isotropic or, in other words, it is assumed that independent sound of equal strength arrives at a receptor from all directions. In the literature, Langley [5] presents several approaches when talking about diffuse field, the wave approach and the modal approach. It thus can lead to different types of definitions. As an example, Fahy [6] employs one using the energy density: “the average energy density is the same throughout the volume of the enclosure”. The computation of a diffuse field criterion in the next section is based on this latter definition.

Equipartition of the energy is another assumption used in SEA. Lesueur [7] proposed a clear definition in his chapter dedicated to SEA. Each subsystem has a finite number of modes and the coherent effect between them is ignored. In addition, each mode of subsystem A is coupled with all of the modes of subsystem B. Calling  $\langle E_{A,i} \rangle$  and  $\langle E_{B,k} \rangle$  the average energies of the mode  $i$  in subsystem A and mode  $k$  in subsystem B;  $E_A$  and  $E_B$  the global energies of subsystem A and B;  $N_A$  and  $N_B$  the number of modes in each subsystem, energy equipartition assumption can be written as

$$\langle E_{A,i} \rangle = \frac{E_A}{N_A}; \langle E_{B,k} \rangle = \frac{E_B}{N_B} \quad (1)$$

In the study, for each frequency band and with a varying damping coefficient, the modal energies are computed in order to analyse the energy distribution over all of the modes.

Alongside, recently A. Le Bot [8] proposed validity diagrams which leaned on some specific parameters: the number of modes, the modal overlap of the subsystem, the attenuation factor of wave per meter and the coupling strength. It permits to have an idea of how well the SEA method could be applied to a system. In this paper, a correlation is made between the validity diagrams and the assumptions of diffuse field and energy equipartition for a simply supported plate.

## 2 DEFINING A CRITERION

### 2.1 Diffuse field criterion

The example of a simply supported plate excited by a random point force  $F(t)$ . The power spectral density (PSD) of such a force is equal to the PSD of a white noise  $S_0$  confined to a specified frequency band  $\Delta\omega_i$ . The energy expectation  $W(x,y, \omega_i)$  of a receptor  $R(x,y)$  for a frequency band  $\omega_i$ , is given by

$$W(x, y, \omega) = m \langle \dot{u}^2 \rangle = \frac{mS_0}{2\pi} \int_{\Delta\omega_i} \omega^2 |H_{rs}|^2(\omega) d\omega \quad (2)$$

Where  $u$  is the deflexion,  $m$  the specific basis weight and  $H_{rs}$  the frequency response function (FRF) calculated between the excitation source  $s$  and a receiver  $r$ .

The definition of the criterion is based on the energy expectation distribution. For that, several energy receptors are placed on the plate. For each frequency band, a comparison is done between the values of all receivers.

The criterion is defined as the standard deviation normalized by the average,

$$\sigma_{diffuse} = \frac{\sqrt{\overline{W^2} - \overline{W}^2}}{\overline{W}}. \quad (3)$$

Where  $S$  is the surface of the plate, the average is defined by  $\bar{f} = \int_S f(x, y) dx dy$ .

When the criterion tends to zero, the field can be considered perfectly diffused. All of the energy densities are equal over the plate.

### 2.2 Energy equipartition criterion

A similar method is used to define the energy equipartition criterion. The deflexion  $u(x,y,t)$  at a receiver  $R(x,y)$  at a specific time  $t$  is given by

$$\begin{aligned} u(x, y, t) &= \sum_k A_k(t) \psi_k(x, y) \\ E(t) &= m \int \dot{u}^2 dx dy \Leftrightarrow E = \sum_k m \langle \dot{A}_k^2 \rangle \Leftrightarrow E_k = m \langle \dot{A}_k^2 \rangle \end{aligned} \quad (4)$$

Where  $A_k$  is the modal energy and  $\psi_k$  the eigenfunction of mode  $k$ . The criterion is defined as follow

$$\sigma_{equi} = \frac{\sqrt{\overline{E_k^2} - \overline{E_k}^2}}{\overline{E_k}}. \quad (5)$$

### 3 RESULTS

For the numerical simulation, several parameters must be fixed (number of modes, frequency step and number of receptors for spatial discretization). For that, a preliminary study on the convergence of these parameters is done. Then, two maps presenting the criterion of the diffuse field versus the validity diagrams of SEA and the computation of modal energies are showed.

On one hand, the validity domain of SEA defined by A. Le Bot and the mapping of the criterion fit well. On the other hand, a strong correlation between the two maps is done. Some conclusions on the SEA assumptions finally closed the presentation.

### REFERENCES

- [1] R. H. Lyon, G. DeJong. *Theory and application of Statistical Energy Analysis*. Butterworths-Heimann, 1995.
- [2] C. B. Burroughs, R.W. Fischer and F. R. Kern. An introduction to statistical energy analysis. *J. Acoustic. Soc. Am.* 101(4):1779-1789, 1997.
- [3] H. G. D. Goyder. Vibrational power flow from machines into built-up structures. *Journal of Sound and Vibration*. 68(1):59-75, 1980.
- [4] W. Wöhle. Coupling loss factors for statistical energy analysis of sound transmission at rectangular structural slab joints. *Journal of Sound and Vibration*. 77(3):335-344, 1981.
- [5] R. S. Langley and P. J. Shorter. Diffuse wavefields in cylindrical coordinates. *J. Acoustic. Soc. Am.* 112(4):1465-1470, 2002.
- [6] F. J. Fahy. *Sound and structural vibration*. Academic, 1985
- [7] C. Lesueur. *Rayonnement acoustique des structures*. Eyrolles, 1988.
- [8] A. Le Bot. Validity diagrams of statistical energy analysis. *Journal of sound and vibration*. 329(6):221-235, 2010.