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# Effect of harmonic heat flux variation on solid material piloted ignition

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## Abstract

This study aims at modeling the effect of incoming heat flux fluctuations on solid material ignition. In order to propose a general methodology that can be applied to any kind of solid target, kernels accounting for the target temperature response regarding an incoming heat flux are developed for thermally thick and thin solids with low or high thermal inertia. The expected behavior of these different target types during ignition is therefore discussed, based on the kernel analysis. A Fourier decomposition of the incoming heat flux is then used to calculate the target response to harmonic heat fluxes, allowing description of the different target type behavior. Finally, the practical meaning of these harmonic fluctuations and their effects on target ignition is discussed based on the previous analytical results, allowing to discriminate situations where ignition time is predictable from situations where ignition time is unpredictable.

*Keywords:* Piloted ignition, harmonic time-varying heat flux, ignition predictability

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## Introduction

In the frame of fire protection study and especially in the Wildland-Urban Interface (WUI), modeling of a radiant heat flux coming from fire fronts on solid surfaces and its consequences are topics of major interest. Indeed, their understanding allow determination of safety distance, as studied analytically in Rossi et al. (2011), or ignition time of solid material, as studied analytically in Reska et al. (2012) or numerically in Porterie et al. (2007). In these studies, different radiant heat flux modeling are suggested: In Rossi et al. (2011), a

solid flame is used, which consists in substituting the flame front by a radiant surface with equivalent properties, as in Billaud et al. (2011) for the study of fire spread. In Reska et al. (2012), the radiant heat flux on the target is modeled using a linear time dependency in order to take into account the time-varying heat flux on a stationary target as suggested by Cohen (2004). The ignition time is then provided by an analytical relation based on the surface temperature evolution.

In these studies, only the global trend of the incoming radiant heat flux has been taken into account. However, turbulent motions are common in the fire front as explained by Morvan (2011) and often generate periodic or quasi-periodic flame behavior as demonstrated by Atkinson et al. (1995) and Dupuy et al. (2011), thus periodic or quasi-periodic fluctuations of the radiant heat flux can occur that are not taken into account in the previously described modeling. These fluctuations could be responsible for ignition unpredictabilities since they are not controlled in practical applications. Moreover, recent experimental results on fire propagation by Finney et al. (2013b) demonstrate the existence of quasi-periodic fluctuations of the heat flux on the solid particles composing the fuel layer which seems to be of great matter in particle ignition. Therefore, this study aims at analytically modeling the effect of harmonic variations of the heat flux on the piloted ignition of different solid targets of interest. The suggested target could be PMMA and wood slabs, insulating foams, excelsior or pine needles, which can be classified as thermally thick or thin with low or high thermal inertia. Hence ignition is modeled for these solid categories. Thermally thick targets are considered to be large surfaces while thermally thin targets are considered to be particles. Solutions are discussed for practical fire safety applications, thus use of “practical“ in this article will refer to material thermophysical properties of PMMA, wood and insulating foams, with ignition time ranging from a few seconds to a few hundred seconds. The limit (regarding the particle size) between thermally thin and thermally thick material will not be discussed in this study. However, it must be noticed that a classical Biot number cannot account for the thermal behavior transition since heating is here due to radiative heat transfer. Therefore, use of a radiative Biot number depending on the incoming radiant heat flux as suggested in Benkoussas et al. (2007), should be considered. In order to illustrate the incoming heat flux harmonic variation effect, an example considering the ignition under heat fluxes composed of a constant and a harmonic part is presented. Nevertheless, the methodology allows to extend the study to heat fluxes composed of any kind

of slow time-varying variations part adding any harmonic part.

The manuscript is organized as follow: The mathematical modeling of the ignition time regarding an arbitrary time-varying heat flux view as series of step is discussed and solutions are proposed for different solid targets, i.e. thermally thick and thin with low or high thermal inertia. The expected behavior at ignition is then discussed comparing the different kernels. The response of these targets regarding fluxes modeled using Fourier decomposition is calculated and the effect of the heat flux harmonic variations is discussed for each type of solid target. Finally, a conclusion is drawn on the effect of these variations on ignition.

## 1. Mathematical formulation

Flaming ignition can be described as two separated mechanisms: Heat and pyrolysis of the solid material followed by chemical reactions in the gas phase. If the pyrolysis gas flow is low with normal oxygen concentration and considering that ignition occurs when the solid surface temperature reaches a given ignition value (usually set as the pyrolysis temperature), the ignition time corresponds to the time needed by the solid to heat until it begin pyrolysing. This leads to model ignition as a temperature raise process in the solid material as suggested by Fernandez-Pello (1995). Ignition can then be modeled as a one-dimensional heat conduction problem in the solid material as proposed by Torero (2008) for thermally thick surfaces and as a zero-dimension heat transfer problem for thermally thin particles as suggested in Quintiere (2006).

### 1.1. Thermally thick solids

For thermally thick targets, a one-dimensional semi-infinite inert solid material is considered with one side exposed to an incoming heat flux  $\Phi(t)$ . The heat conduction equation is now expressed for the solid target, introducing  $\theta(x, t) = T(x, t) - T_0$  where  $T(x, t)$  is the solid temperature and  $T_0$  is the initial temperature of the solid and its surrounding air:

$$\frac{\partial \theta(x, t)}{\partial t} = \alpha \frac{\partial^2 \theta(x, t)}{\partial x^2} \quad (1)$$

Where  $\alpha = \lambda/\rho Cp$ .  $\lambda$ ,  $\rho$  and  $Cp$  represent respectively the solid heat conductivity, its density and its specific heat. The boundary conditions are:

$$-\lambda \frac{\partial \theta(x, t)}{\partial x} = \Phi(t) - h\theta(x, t) \quad , \quad x = 0 \quad (2)$$

$$-\lambda \frac{\partial \theta(x, t)}{\partial x} = 0 \quad , \quad x \rightarrow \infty \quad (3)$$

$$\theta(x, 0) = 0 \quad , \quad \forall x \quad (4)$$

Where  $h$  is a total heat transfer coefficient. For high thermal inertia solids, the associated surface heat loss can be neglected as shown in Reska et al. (2012) whereas it is taken into account for low thermal inertia solids.

### 1.1.1. High thermal inertia solids

In order to get a general solution of this equation for an arbitrary function  $\Phi(t)$ , let us first consider the response of Eq.(1) to a sudden constant heat flux  $\Phi(t) = \Phi_0 H(t)$ , where  $H(t)$  is the Heaviside function (surface losses are neglected in this section). Using Laplace Transforms, Eq.(1) can be solved along with its initial and boundary conditions, providing:

$$\theta(x, t) = \frac{\Phi_0}{\lambda} \left[ 2\sqrt{\frac{\alpha t}{\pi}} \exp\left(\frac{-x^2}{4\alpha t}\right) - x \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \right] \quad (5)$$

The kernel  $K(x, t)$  of Eq.(1) can then be identified in Eq.(5), where:

$$K(x, t) = \frac{2\sqrt{t}}{\sqrt{\pi\lambda\rho C_p}} \exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{x}{\lambda} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad (6)$$

Since the study focuses on the surface temperature evolution, the kernel  $K$  is calculated at  $x = 0$ , providing:

$$K(0, t) = \frac{2\sqrt{t}}{\sqrt{\pi\lambda\rho C_p}} \quad (7)$$

Now, considering an arbitrary function  $\Phi(\tau)$  as series of steps and using Eq.(1) linearity, the response  $\theta(0, t)$  can be expressed as a sum of response with time offsets for an arbitrary kernel  $K_0$ :

$$\theta(0, t) = \int_0^t \frac{d\Phi}{d\tau} K_0(0, t - \tau) d\tau \quad (8)$$

This result is similar to the solution suggested by Carslaw and Jaeger (1959) for Eq.(1) at  $x = 0$ . Setting  $\Phi$  as a Heaviside function and  $K_0(0, t) = K(0, t)$ , the classical solution of Torero (2008) used for a constant heat flux neglecting the surface heat losses is recovered. Setting  $\frac{d\Phi}{d\tau} = cst = m$  and  $K_0(0, t) = K(0, t)$ , the result of Reska et al. (2012) for a constant variation of the heat flux is recovered.

### 1.1.2. Low thermal inertia solids

A kernel  $K_H(0, t)$  that take into account the surface heat losses through a total heat transfer coefficient  $h$  is now provided. It has been calculated the same way as for high thermal inertia solids:

$$K_H(0, t) = \frac{1}{h} \left[ 1 - \exp\left(\frac{h^2}{\lambda\rho C_p} t\right) \operatorname{erfc}\left(\frac{h}{\sqrt{\lambda\rho C_p}} \sqrt{t}\right) \right] \quad (9)$$

Application of the kernel  $K_H(0, t)$  setting  $\Phi$  as an Heaviside function or setting  $\frac{d\Phi}{dt} = cst = m$  allows recovering solutions of Reska et al. (2012) for constant heat flux or constant heat flux variations, taking into account surface heat losses. The practical use of this kernel is however limited due to its mathematical form. Nevertheless it can be simplified assuming that even for material with low thermal inertia, the characteristic time  $\tau_{K_H} = \lambda\rho C_p/h^2$  of the kernel  $K_H(0, t)$  is always larger than the ignition time, allowing series expansion of the kernel. The latter is then re-written as  $K_H(0, t) = K(0, t) + K_h(0, t)$  with

$$K_h(0, t) \approx -\frac{ht}{\lambda\rho C_p} + \frac{4}{3} \frac{h^2 t^{3/2}}{\sqrt{\pi} (\lambda\rho C_p)^{3/2}} \quad (10)$$

Application of this simplified kernel to a linear time-dependent heat flux does not strictly allow recovering the series expansion result of Reska et al. (2012) since the series expansion has been made before calculating the convolution. The solution suggested here is nevertheless more accurate regarding the exact solution than the solution proposed in Reska et al. (2012) which has been experimentally validated, thus validating the simplified kernel. Indeed, in most practical cases, the ignition time is lower than the kernel characteristic time and the series expansion provides an efficient approximation. In order to assert this assumption, the characteristic time  $\tau_{K_H}$  is calculated for typical material as wood, polymer and insulating foam. Thermophysical properties of these material are extracted from Demharter (1998); Drysdale (1999); Tihay (2007) and Bartoli (2011) and are listed in table 1. In order to estimate the total heat transfer coefficient, different approaches can be suggested. For instance, in Simeoni et al. (2012), this coefficient stands for radiant re-emission only and its value is maximized, providing  $h = 22\text{W.m}^{-2}\text{.K}^{-1}$ . Linearized values of this coefficient provide  $h$  in the range  $18 - 20\text{W.m}^{-2}\text{.K}^{-1}$  regarding the reference temperature (ignition or mean temperature between ambient and ignition temperature). The convective heat transfer coefficient can also be added to this coefficient, as suggested in Quintiere (2006), however natural

convection on a flat surface is expected to be negligible regarding radiant re-emission. Finally, in Grishin (1997), this coefficient is set at  $10\text{W.m}^{-2}.\text{K}^{-1}$ . Thus, a value in the range  $10 - 20\text{W.m}^{-2}.\text{K}^{-1}$  is used in this study. Therefore, considering wood,  $\tau_{K_H} \in [270; 4000]\text{s}$ ; For polymer,  $\tau_{K_H} \in [260; 10000]\text{s}$  while  $\tau_{K_H} \in [2; 150]\text{s}$  for insulating foams. These ranges are summarized in table 1, showing (for wood and polymer) the relevancy of series expansion for ignition times ranging from a few seconds to a few hundred seconds. For insulating foam, the efficiency of series expansion observed in Reska et al. (2012) in case of ignition time up to 400s however suggests that the efficient characteristic time  $\tau_{K_H}$  for foam is much greater than table 1 values. Finally, a comparison of the different thermally thick kernels (low and high thermal inertia series expansion compared to the exact kernel  $K_H$ ) is shown on Fig.1 where a satisfactory agreement is found between the exact kernel and the high thermal inertia kernel for a dimensionless ignition time  $t^* = t_{ig}/\tau_{K_H}$  with  $t^* \leq 0.05$ , using a 20% error criterion ( $t^* \leq 0.015$  using a 10% error criterion). For the low thermal inertia kernel, a satisfactory agreement is found for  $t^* \leq 0.54$  with a 20% error criterion and for  $t^* \leq 0.34$  with a 10% error criterion. Therefore, it is suggested to define high thermal inertia solid regarding the dimensionless ignition time rather than using a criteria based only on  $\lambda\rho C_p$  or  $\rho C_p$ . Thanks to all these remarks, practical dimensionless ignition time for thermally thick solids will be considered lower or equal to one.

Material type	$\lambda$ [ $\text{W.m}^{-1}.\text{K}^{-1}$ ]	$\rho$ [ $\text{kg.m}^{-3}$ ]	$C_p$ [ $\text{J.kg}^{-1}.\text{K}^{-1}$ ]	$\tau_{K_H}$ [s]
Wood	0.12 – 0.17	500 – 800	1800 – 2850	270 – 4000
Polymer	0.11 – 0.35	940 – 1400	1000 – 2000	260 – 10000
Insulating foam	0.022 – 0.037	20 – 200	1400 – 2000	2 – 150

Table 1: Physical property ranges from Demharter (1998); Drysdale (1999); Tihay (2007) and Bartoli (2011) and thermally thick characteristic time ranges

### 1.2. Thermally thin solids

A model is now developed for thermally thin solid targets, such as solid particles composing a forest fuel layer. In that case, surface heat losses cannot be neglected as discussed later in this section. The efficient area involved in radiative and convective heat transfer is also different depending on the shape and orientation of the solid particles regarding the incident radiant

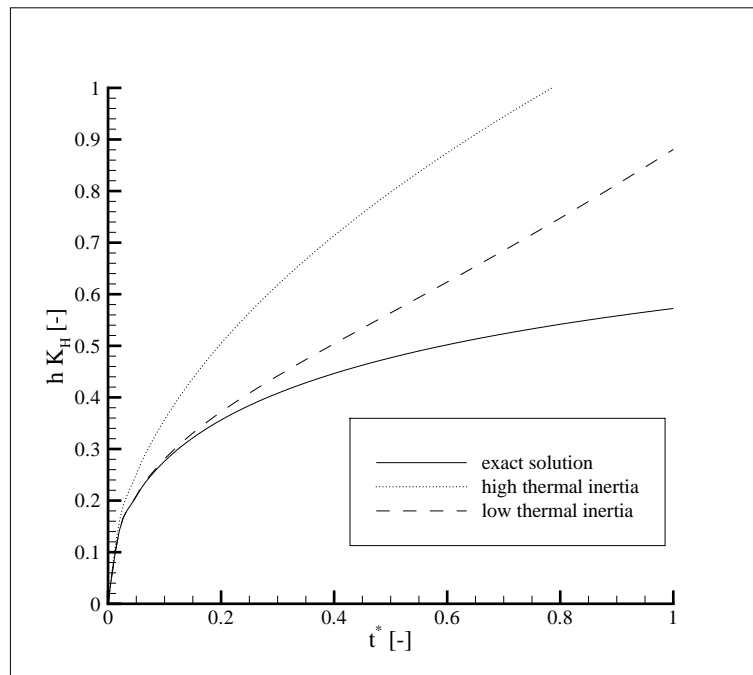


Figure 1: Kernel  $K_H$  compared to the series expansion for high and low thermal inertia



heat flux, introducing two coefficients  $R_\Phi$  and  $R_h$ : For a solid particle,  $A/R_\Phi$  is the area exposed to the radiant heat flux while  $A/R_h$  is the efficient area for heat losses with  $A$ , the solid particle area and  $R_\Phi \geq R_h$ . In order to include the fact that solid particles in a fuel bed can also be partially hidden by other particles, an efficient specific area  $\sigma_{eff}$  is also introduced with  $\sigma_{eff} \leq \sigma$ ,  $\sigma$  being the specific area of the solid particles. For instance in a needle litter, considering needles as horizontal cylinders,  $R_\Phi \sim \pi$  and  $R_h \sim 2$  for dense litter while  $R_h \sim 1$  for sparse litter where the needles could be considered as independent. This leads to the following expression for the thermally thin kernel  $k(t)$ :

$$k(t) = \frac{R_h}{hR_\Phi} \left[ 1 - \exp\left(-\frac{h\sigma_{eff}t}{R_h\rho C_p}\right) \right] \quad (11)$$

Where a characteristic time  $\tau_k = R_h\rho C_p/h\sigma_{eff}$  is appearing. This expression is similar to the solution suggested in Quintiere (2006). Since the ratio  $\tau_k/\tau_{K_H} = R_h h/\lambda\sigma_{eff}$  for a given material is similar to an efficient convective Biot number of the particle, it seems logical for the characteristic time  $\tau_k$  to be small compared to  $\tau_{K_H}$  using a thermally thin assumption (i.e. small Biot number value). Thanks to the expression of the efficient convective Biot number, the latter can be estimated for typical forest fuel layer particle, considering  $\sigma$  in the range  $600 - 12000\text{m}^{-1}$  using values from Bartoli (2011); Cheney et al. (1993) and Catchpole et al. (1998). It provides  $\tau_k/\tau_{K_H} \in [0.005; 0.555]$  for woody fuel particles. However, the thermally thin limit provided by Benkoussas et al. (2007) suggests that some of these particles can behave as thermally thick (for radiative Biot number  $Bi > 0.1$ ), which corresponds to  $\sigma < 2640\text{m}^{-1}$  for a  $10\text{kW.m}^{-2}$  radiant heat flux. Hence, the specific area will be considered to range approximately from  $3000\text{m}^{-1}$  to  $12000\text{m}^{-1}$  for particles behaving as thermally thin. Thus, the kernel characteristic time  $\tau_k$  is in the range  $1.35 - 20\text{s}$  for the thinner woody fuel particles and in the range  $30 - 444\text{s}$  for the thicker woody fuel particles that still behave as thermally thin. Consequently, in most practical applications no series expansion can be made on the kernel  $k(t)$ , contrary to what can be done with the kernel  $K_H(0, t)$ . It also means that for thermally thin solids the ignition temperature is reached at long time scale (regarding the kernel time scale). Therefore heat losses cannot be neglected for thermally thin particles as suggested earlier.

### 1.3. Target behavior during ignition

A few remarks are presented about the global behavior during ignition of thermally thin and thick target: Regardless of the target thermal inertia, effects of heat losses (radiative or convective) and thus effects of convective heating for a given material are much greater for thermally thin targets as explained by the characteristic time scale ratio and the relevancy of series expansions on the kernels. Indeed thermally thick particles with high thermal inertia are not affected by radiant heat loss variations or convective transfer variations since kernel  $K(0, t)$  does not depend on the total heat transfer coefficient. For thermally thick particles with low thermal inertia, the series expansion practical efficiency proves that these variations have only small effects compared to the radiative heating major role. Therefore, thermally thick particle ignition time is expected to depend almost exclusively on radiant exposure for high thermal inertia materials. Ignition time dependency on the particle surrounding temperature (gas and other particles) is however increasing as the thermal inertia decreases.

For thermally thin particles, the small efficient convective Biot number value suggests that radiative heat losses, convective heating and cooling strongly affect the ignition time. This means that considering a purely radiative heating, a given material is expected to ignite earlier if the particles made of this material are thermally thick. This result is in accordance with recent experimental measurements by Cohen and Finney (2010) on ignition induced by radiation. It is however contrary to what is commonly admitted in the frame of forest fire propagation, where thermally thin particles are considered responsible for fire propagation under the effect of an incoming radiant heat flux as suggested by Albini (1985) and Grishin (1997).

Moreover, according to the remark concerning the characteristic time ratio, for infinitely thin particles (i.e. the efficient convective Biot number is equal to zero), ignition is not possible anymore under a radiant heat flux as the ignition time is pushed forward to infinite for any finite ignition temperature. However, particles are expected to ignite instantly if the heat losses disappear, for instance if the surrounding air temperature is equal to the particle temperature with other surrounding particles at the same temperature as the surrounding air. In practical applications, the efficient convective Biot number is different from zero and the temperature response regarding heat loss modifications is not instantaneous but a stiff behavior is expected for very thin particles. Thus, fire propagation over fuel layers composed of thermally thin particles may not depend on long range radiative trans-

fer since the latter is not expected to heat particles because of convective cooling and radiant re-emission toward the atmosphere. However, the fire front proximity reduces these cooling processes and a stiff temperature raise should be observed. Therefore, fire propagation should mostly depend on short range processes, which is in accordance with the analysis of Finney et al. (2013a) on fuel particle heat exchange leading to ignition, based on an extensive comparison of forest fuel layers ignition model with experimental measurements.

Now, thanks to the previous analysis for convective Biot number different from zero, effects of radiative heating variations and convective transfer variations can both be represented by apparent incoming heat flux variations. Consequently, heat flux harmonic variations are suggested to mimic effects of flame turbulent behavior on the radiant heat flux, practical uncontrolled variations of the heat loss (convective and radiative) and periodic flaming contact (responsible for both incoming heat flux enhancement and heat loss decrease). To mimic convective heat transfer variations, the apparent incoming heat flux variation magnitude is increasing as the Biot number decreases, as suggested by previous remarks. Thus, the relative heat flux variation magnitude accounting for heat loss variations should be higher for low thermal inertia and thermally thin solids.

## 2. Temperature response to radiant heat fluxes viewed as Fourier series

Fourier decomposition is here proposed to mimic the periodic variations of the apparent incoming heat flux on solid targets. The temperature response to a Fourier decomposition of the radiant heat flux can be expressed for an arbitrary kernel  $K_0$  thanks to Eq.(8) and using Eq.(1) linearity, providing for a radiant heat flux  $\Phi(t) = \Phi_0 + \sum (a_n \cos(\omega t) + b_n \sin(\omega t))$ :

$$\theta(t) = \Phi_0 K_0(t_{ig}) + \omega \left( \sum_n I_c - \sum_n I_s \right) \quad (12)$$

Where  $\omega = 2\pi n/t_{ig}$ ,

$$I_{c\ n} = b_n \int_{t=0}^{t_{ig}} K_0(t_{ig} - \tau) \cos(\omega\tau) d\tau \quad (13)$$

And

$$I_{s\ n} = a_n \int_{t=0}^{t_{ig}} K_0(t_{ig} - \tau) \sin(\omega\tau) d\tau \quad (14)$$

Integrals  $I_{c\ n}$  and  $I_{s\ n}$  are solved analytically for the different kernels suggested in this study yet expressions are not provided here for clarity.

### 2.1. A criteria to estimate ignition time unpredictability

The solution  $\theta(t)$  is now studied for a radiant heat flux  $\Phi(t) = \Phi_0 + b_n \sin(\omega t)$ . In order to investigate the effect of the harmonic part, the following expression  $f$  is firstly suggested, accounting for the ratio of the temperature raise due to the harmonic part of the radiant heat flux and the temperature raise due to the constant part:

$$f(K_0, n) = \frac{\omega I_{c\ n}}{\Phi_0 K_0(t_{ig})} \quad (15)$$

An harmonic dephasing, which is obviously a parameter that is not controlled in a practical ignition case, should also be taken into account. This is why  $f$  possible variation range  $\Delta f = 2|f|$  will be considered to account for ignition unpredictability. This function is calculated for the different kernels and then studied regarding  $n$ , the perturbation relative magnitude  $\phi = b_n/\Phi_0$  (with  $0 < \phi < 1$ ) and the dimensionless time to ignition  $t^* = t_{ig}/\tau_{K_0}$ . As suggested earlier, for thermally thick solids,  $\phi$  is mostly accounting for radiant heat flux variations. Since experimental radiant heat fluxes from fire fronts in Chetehouna et al. (2008) exhibits only small magnitude temporal variations,  $\phi$  is practically expected to be small. On the contrary, for thermally thin solids,  $\phi$  can also account for heat loss variations which suggests that  $\phi$  can increase almost up to 1 when mimicking flame contact on thermally thin particles.

### 2.2. Thermally thick target with high thermal inertia

In the case of thermally thick target with high thermal inertia, function  $f$  is depending only on  $n$  and  $\phi$ :

$$f(K, n) = -\frac{1}{2}\phi \frac{S_{\text{Fresnel}}(2\sqrt{n})}{\sqrt{n}} \quad (16)$$

Where  $S_{\text{Fresnel}}$  is the sinusoidal Fresnel integral. For  $n \geq 1$ , the global trend exhibits a relatively slow yet always decreasing rate regarding  $n$  as

shown on Fig.2 (the curve  $t^* = 0$  corresponds to the high thermal inertia case). For  $n > 7$ ,  $\Delta f/\phi < 0.2$ , which means that the potential effect of any harmonic  $n > 7$  on the temperature raise leading to ignition is lower than 20% of the harmonic perturbation relative magnitude. For instance, setting a 50% perturbation relative magnitude, if this perturbation oscillates more than seven times before theoretical ignition (i.e. ignition if no perturbation occurs), the solid surface temperature raise error due to harmonic perturbations at theoretical ignition time is lower than 10% of the temperature raise needed to achieve ignition. For a 10% criterion on a harmonic potential effect ( $\Delta f/\phi < 0.1$ ), the limit harmonic is  $n > 28$ . Consequently, radiant heat flux slow time-varying variations are sufficient to calculate ignition time of such targets and this ignition time is expected to be rather predictable.

### 2.3. Thermally thick target with low thermal inertia

For thermally thick target with low thermal inertia, function  $f$  is expressed the following way:

$$f(K_H, n) = f(K + K_h, n) = f(K, n) + f(K_h, n) \quad (17)$$

$$f(K_h, n) = \frac{1}{2}\phi \frac{t^*}{n\pi} - \frac{1}{4}\phi \frac{t^* C_{\text{Fresnel}}(2\sqrt{n})}{n^{3/2}\pi} \quad (18)$$

Where  $C_{\text{Fresnel}}$  is the cosinusoidal Fresnel integral. As previously mentioned,  $t^*$  is practically expected to be lower or equal to one. Consequently, the first term in  $f(K_h, n)$  is always lowering the effect of the term  $f(K, n)$  and  $f(K_H, n)$  global trend is practically monotonic as shown on Fig.2 where  $\Delta f(K_H, n)$  is plotted for different values of  $t^*$  in the range  $t^* \in [0; 1]$ . Then, effects of the apparent incoming heat flux harmonic part on the ignition are lowered by surface losses which are damping the system response. For instance, when  $t^* = 0.5$ ,  $\Delta f/\phi < 0.2$  for  $n > 5$  and  $\Delta f/\phi < 0.1$  for  $n > 24$ : For a given  $\Delta f/\phi$  ratio, the limit harmonic is smaller for low thermal inertia than for high thermal inertia.

These results concerning thermally thick targets lead to neglect the effect of intermediate and rapid harmonic heat flux variations regarding the main constant contribution in the ignition process. Hence ignition time of thermally thick target is rather predictable and only slow time-varying variations of the incoming heat flux have to be taken into account.

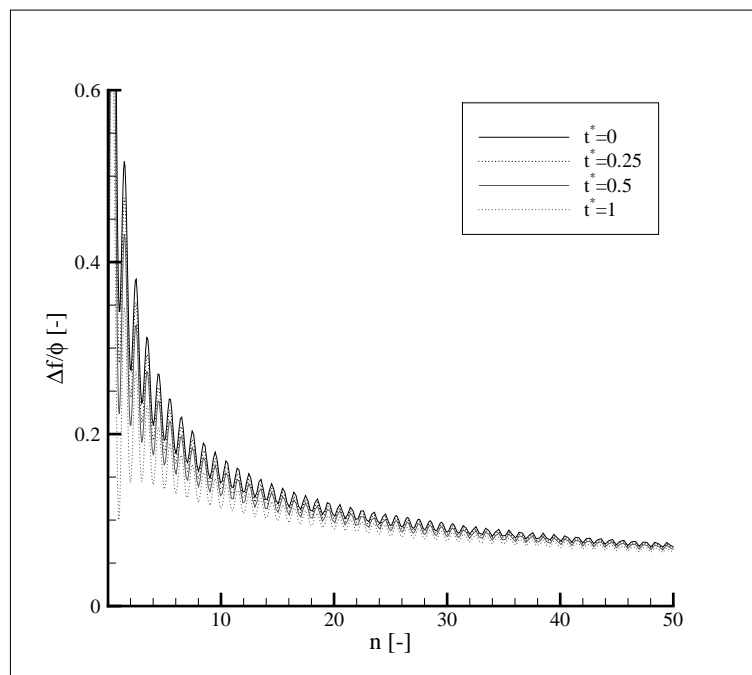


Figure 2:  $\Delta f(K_H, n)/\phi$  for  $t^* = 0$ ,  $t^* = 0.25$ ,  $t^* = 0.5$  and  $t^* = 1$

#### 2.4. Thermally thin target

For thermally thin target, function  $f$  is expressed the following way:

$$f(k, n) = -\phi \frac{2\pi n t^*}{4\pi^2 n^2 + t^{*2}} \quad (19)$$

This relation shows that a non-monotonic behavior of  $f$  can occur for  $n \geq 1$ . Indeed, the kernel and the heat flux solicitation can resonate since  $df(k, n)/dn = 0$  when  $n = t^*/2\pi$ . Hence  $\Delta f_{max} = \phi$ , showing clearly that some harmonic variations of the heat flux (with period in the range 1.35 – 40s for thin forest fuel particles) can greatly modify the ignition time of thermally thin targets. A plot of  $f(k, n)/\phi$  is provided on Fig.3 for  $t^* = 10$ ,  $t^* = 40$  and  $t^* = 100$  to show the width of the resonating band. For small to intermediate ignition time, this result demonstrates the strong potential effect of few frequencies (around the kernel characteristic frequency) on the ignition time. For instance, setting  $t^* = 40$  and comparing to the first example on thermally thick solids with high thermal inertia and a 50% perturbation relative magnitude, the previous limit harmonic  $n = 7$  now generates a 50% temperature raise relative error while harmonics  $n = 2$  and  $n = 30$  only generate a 20% temperature raise relative error. Therefore, the harmonic which was previously negligible is now responsible for a large contribution in the particle temperature raise. Fig.3 also explains the high unpredictability of ignition for  $\phi$  value of order near unity when the same perturbation frequencies are involved (leading to  $\Delta f \sim 1$ ), what could happen in practical case of flaming contact on thermally thin particles as suggested earlier. Moreover, it indicates why long ignition time under radiant heat flux is highly unpredictable for thermally thin particles: Indeed, for long ignition time, Fig.3 suggests that any frequency can modify the particle temperature. Hence ignition is sensitive to any kind of experimental heat loss perturbation. Finally, despite the fact that  $\Delta f(K_H, n)$  is globally decreasing as  $1/\sqrt{n}$  and  $\Delta f(k, n)$  is globally decreasing as  $1/n$ , this result proves that the thinner targets are practically the most sensitive to high frequency heat flux variations.

#### Concluding remarks

Using Fourier decomposition for the incoming heat flux on different kind of solid targets, this study demonstrates that rapid fluctuations of this heat flux have practically no effects regarding a main constant contribution on the ignition of a thermally thick target due to the characteristic time scale of

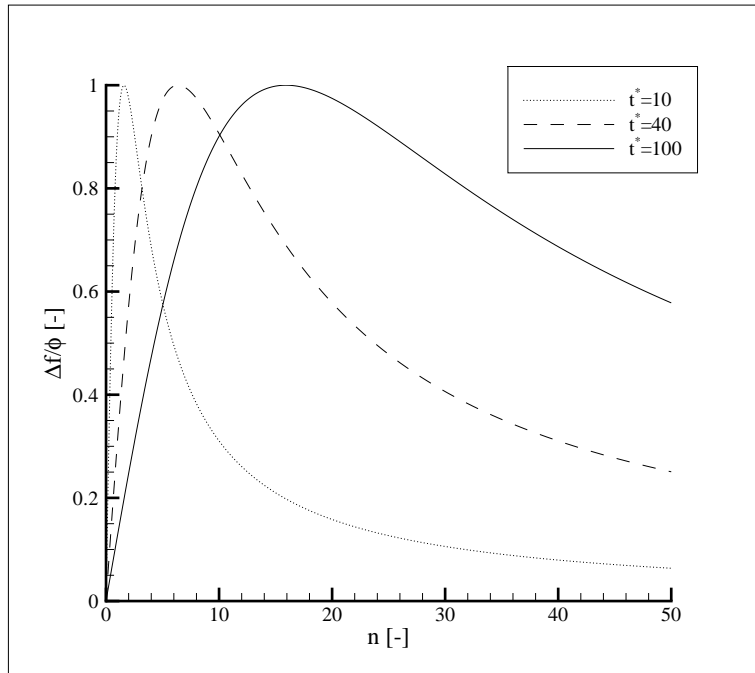


Figure 3:  $\Delta f(k, n)/\phi$  for  $t^* = 10$ ,  $t^* = 40$  and  $t^* = 100$



the associated kernel. However, for a thermally thin target, the characteristic time scale of the kernel allows the later to resonate with the solicitation, showing the existence of particular frequencies which interact actively with the ignition process. Making the assumption that these harmonic fluctuations can mimic the effect of a periodic flaming contact, this study demonstrates that such flaming contact have practically no effects on thermally thick targets while it can drive the ignition for thermally thin targets. This phenomenon can be responsible for great uncertainties in thermally thin particle ignition time and could also generate uncertainties in the fire propagation over fuel layers composed of thermally thin particles if the heat flux ahead of the fire front exhibits periodic fluctuations in the same frequency range as the particles kernel frequency. Since recent experimental results by Finney et al. (2013b) showed the existence of quasi-periodic fluctuations that seems to be driven by the flow upstream the fire front as suggested by Finney et al. (2013b) and Adam et al. (2013), the capability of these two phenomenon to resonate should now to be studied to see if it can generates large variations of the fire rate of spread for small variations of the upstream flow conditions. Moreover, this study suggests a noticeable behavior for ignition at low heat flux (i.e. less than twice the critical heat flux): For thermally thick solids, ignition near the critical conditions happens for dimensionless ignition time lower than 1 (and often lower than 0.5), allowing application of series expansions very close to the critical heat flux, thus the previous analysis on ignition predictability is still applicable. Therefore, external perturbations have reduced effects on the particle temperature raise and the ignition time provided by such model is rather robust even at low heat fluxes. For thermally thin solids, critical conditions are reached for high values of the dimensionless ignition time. Then, external perturbation can greatly modify the particle temperature raise and ignition becomes highly unpredictable.

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